

AdS/CFT and condensed matter

Reviews:

[arXiv:0907.0008](https://arxiv.org/abs/0907.0008)

[arXiv:0901.4103](https://arxiv.org/abs/0901.4103)

[arXiv:0810.3005](https://arxiv.org/abs/0810.3005) (with Markus Mueller)

Talk online: sachdev.physics.harvard.edu



Lars Fritz, Harvard
Victor Galitski, Maryland
Max Metlitski, Harvard
Eun Gook Moon, Harvard
Markus Mueller, Trieste
Yang Qi, Harvard
Joerg Schmalian, Iowa
Cenke Xu, Harvard

Frederik Denef, Harvard
Sean Hartnoll, Harvard
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington



Outline

A. “Relativistic” field theories of quantum phase transitions

1. Coupled dimer antiferromagnets
2. Triangular lattice antiferromagnets
3. Graphene
4. AdS/CFT and quantum critical transport

B. Finite density quantum matter

Outline

B. Finite density quantum matter

1. Graphene

Fermi surfaces and Fermi liquids

2. Quantum phase transitions of Fermi liquids

*Pomeranchuk instability and spin density waves;
Fermi surfaces and “non-Fermi liquids”*

3. AdS₂ theory

4. Cuprate superconductivity

Outline

B. Finite density quantum matter

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Fermi surfaces and Fermi liquids

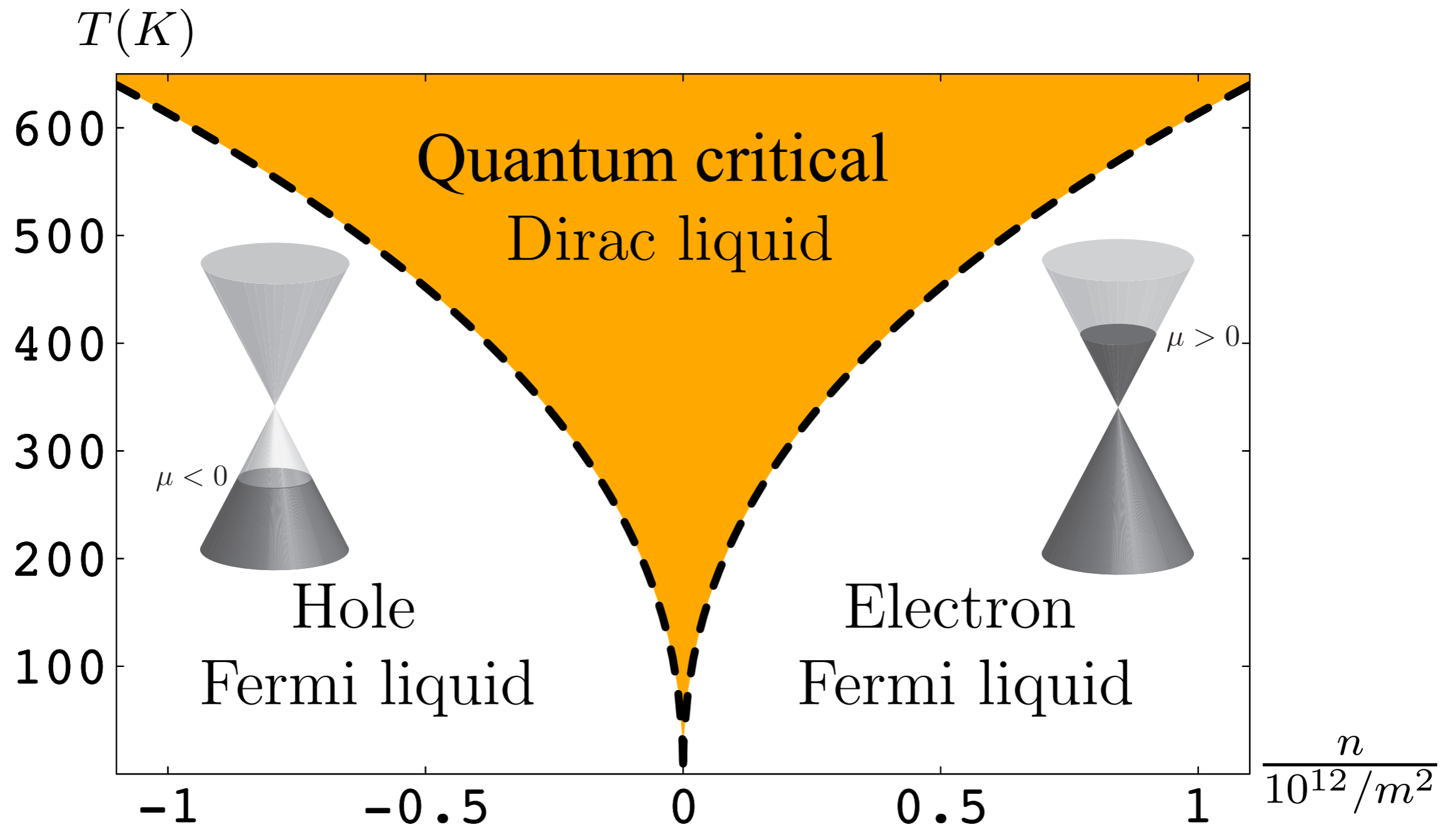
2. Quantum phase transitions of Fermi liquids

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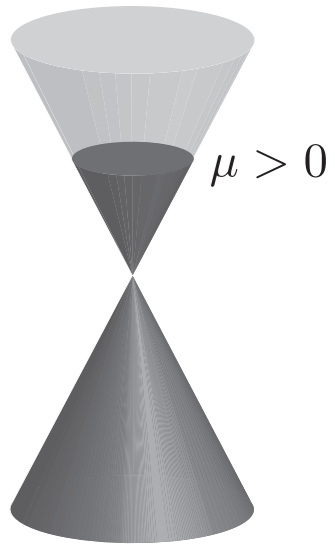
4. Cuprate superconductivity

Quantum phase transition in graphene



Electron Green's function in Fermi liquid (T=0)

$$G(k, \omega) = \frac{Z}{\omega - v_F(k - k_F) - i\omega^2 \mathcal{F}\left(\frac{k - k_F}{\omega}\right)} + \dots$$

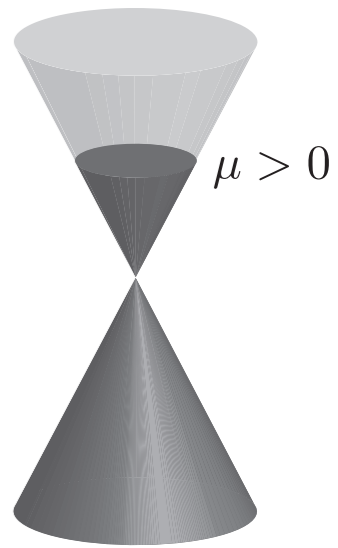


Electron Green's function in Fermi liquid (T=0)

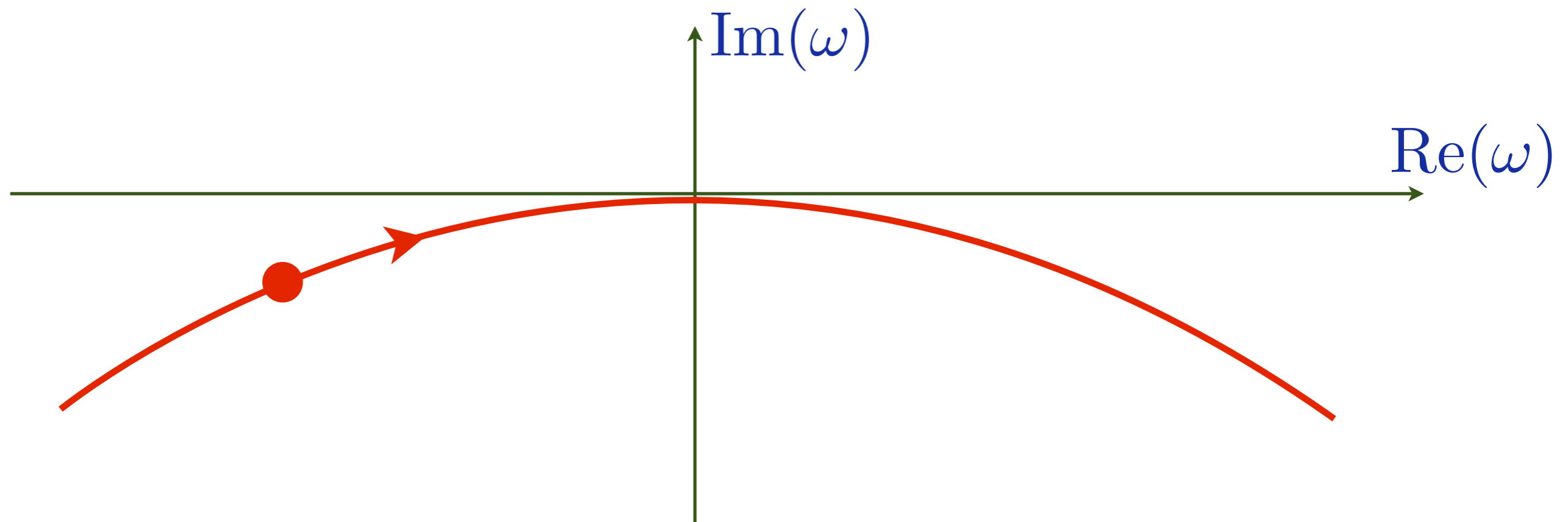
$$G(k, \omega) = \frac{Z}{\omega - v_F(k - k_F) - i\omega^2 \mathcal{F}\left(\frac{k - k_F}{\omega}\right)} + \dots$$

Green's function has a pole in the LHP at

$$\omega = v_F(k - k_F) - i\alpha(k - k_F)^2 + \dots$$



Pole is at $\omega = 0$ precisely at $k = k_F$ *i.e.* on a sphere of radius k_F in momentum space. This is the *Fermi surface*.



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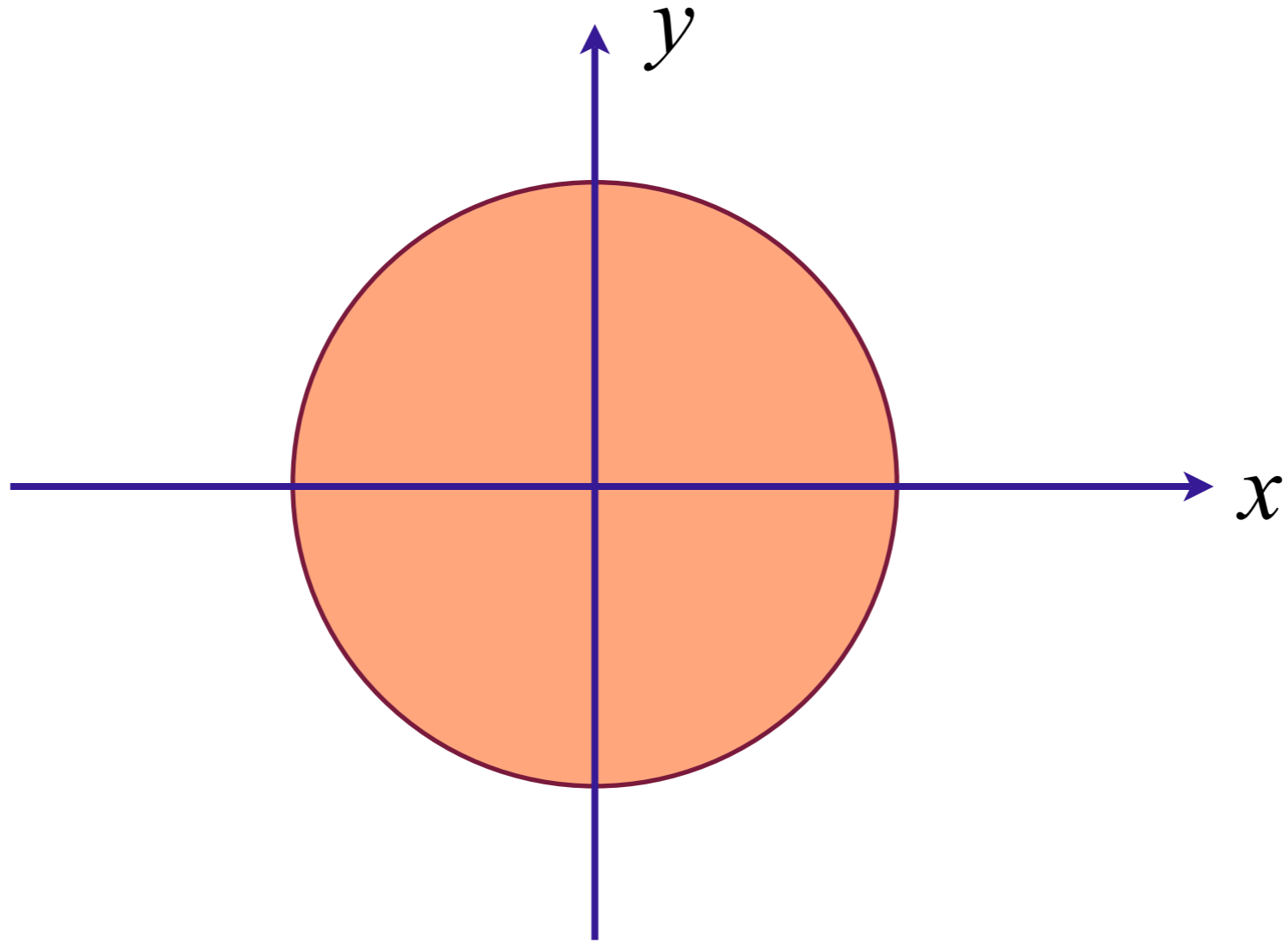
2. Quantum phase transitions of Fermi liquids

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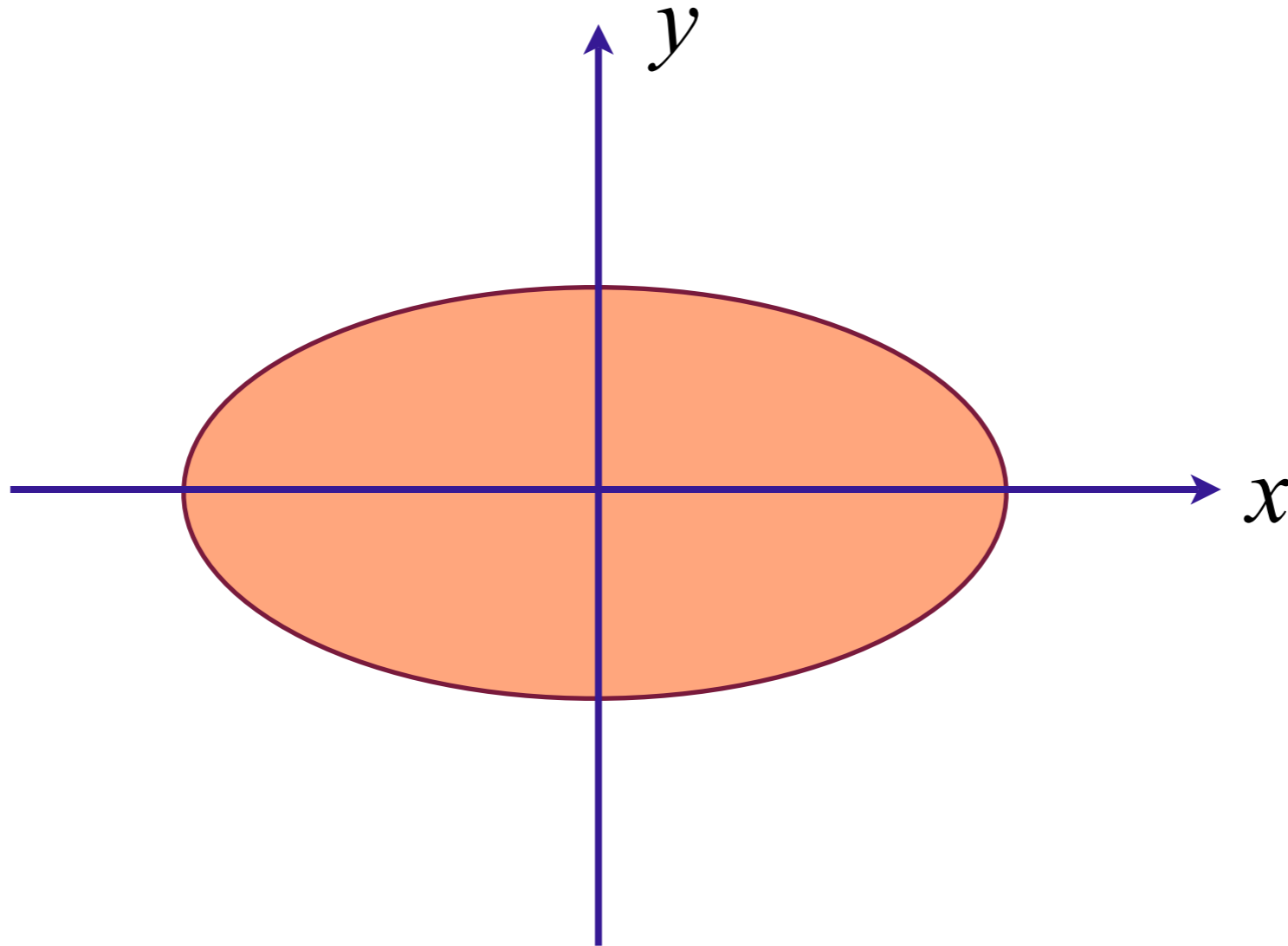
4. Cuprate superconductivity

Quantum criticality of Pomeranchuk instability



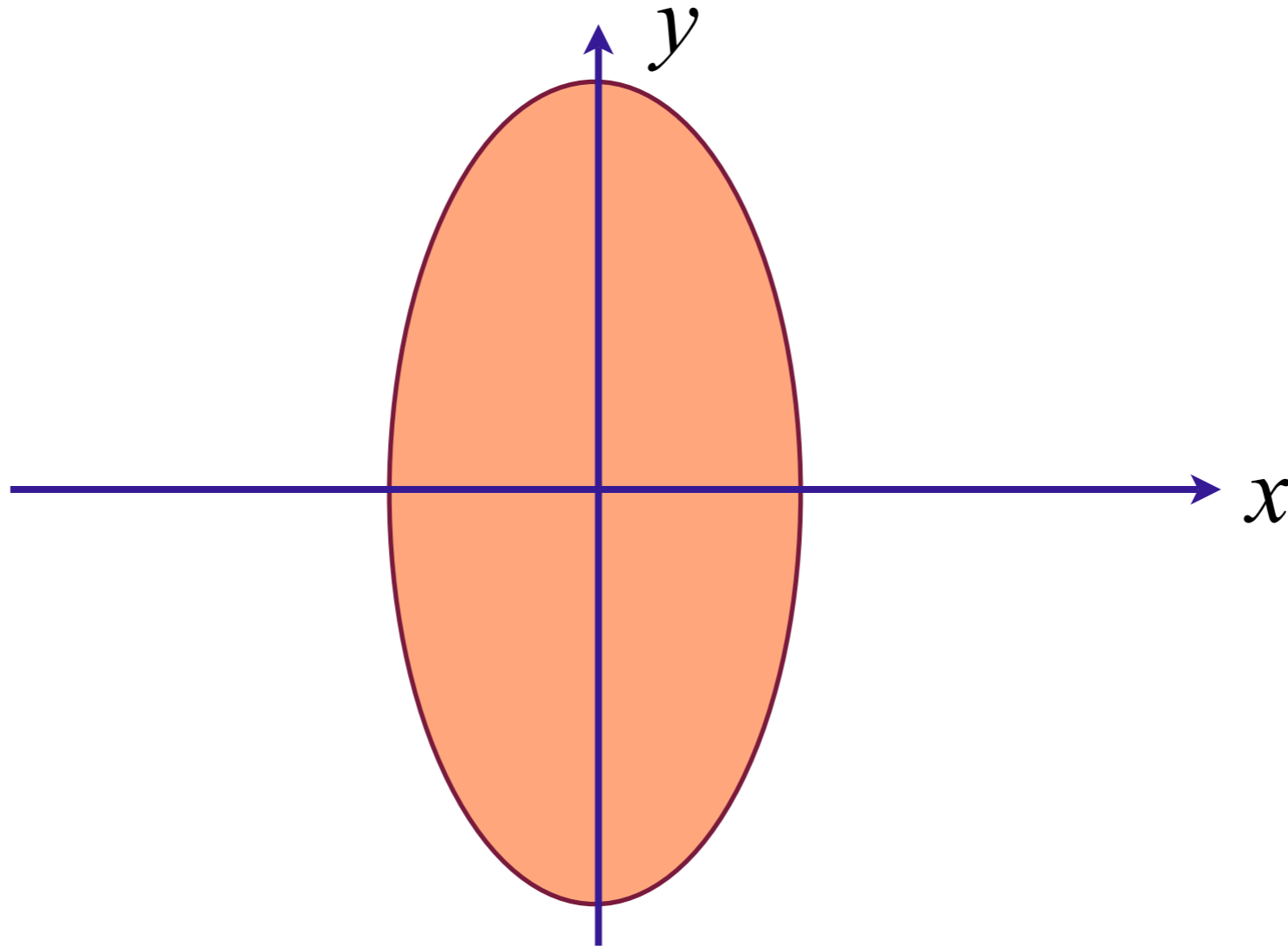
Fermi surface with full square lattice symmetry

Quantum criticality of Pomeranchuk instability



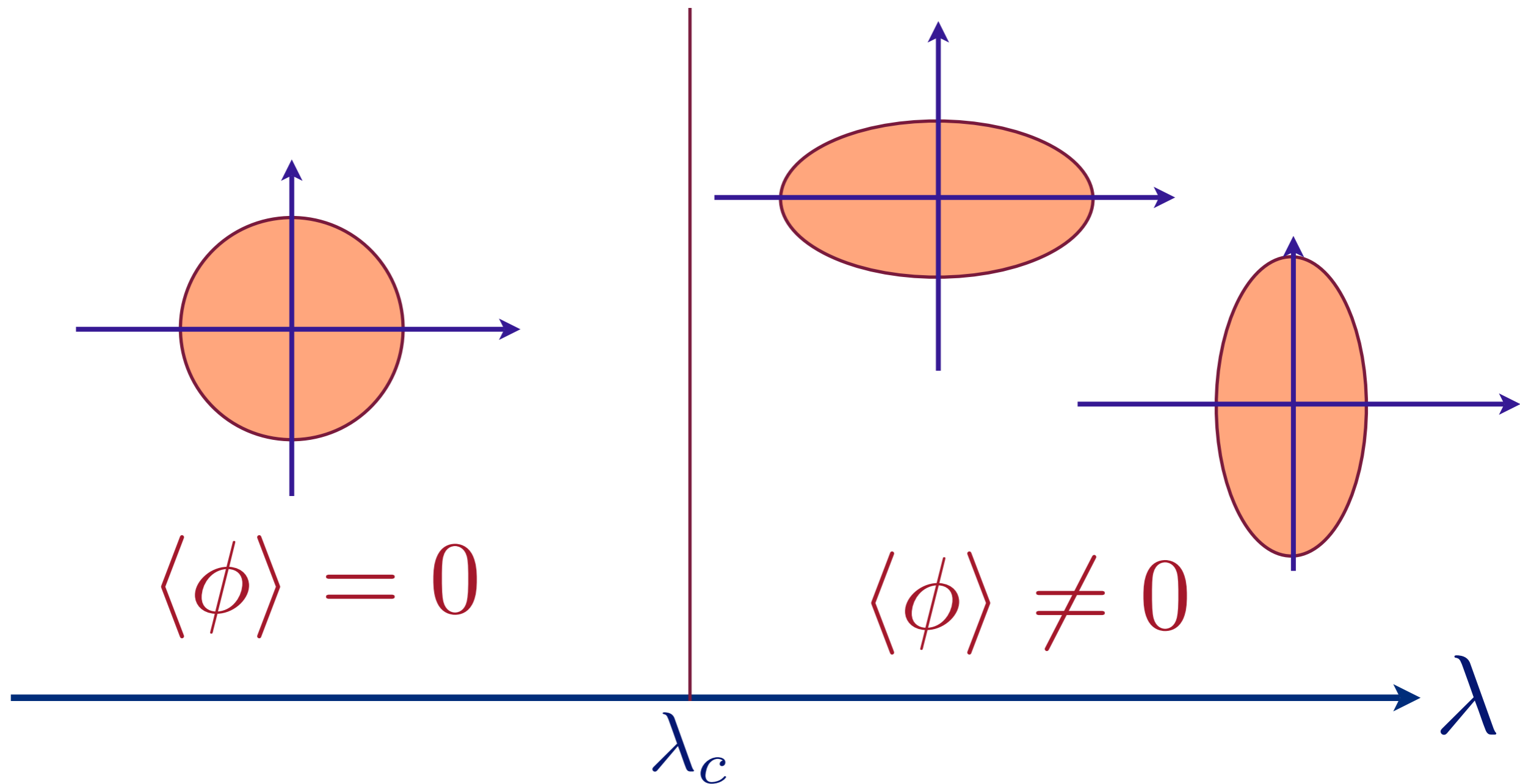
Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Pomeranchuk instability



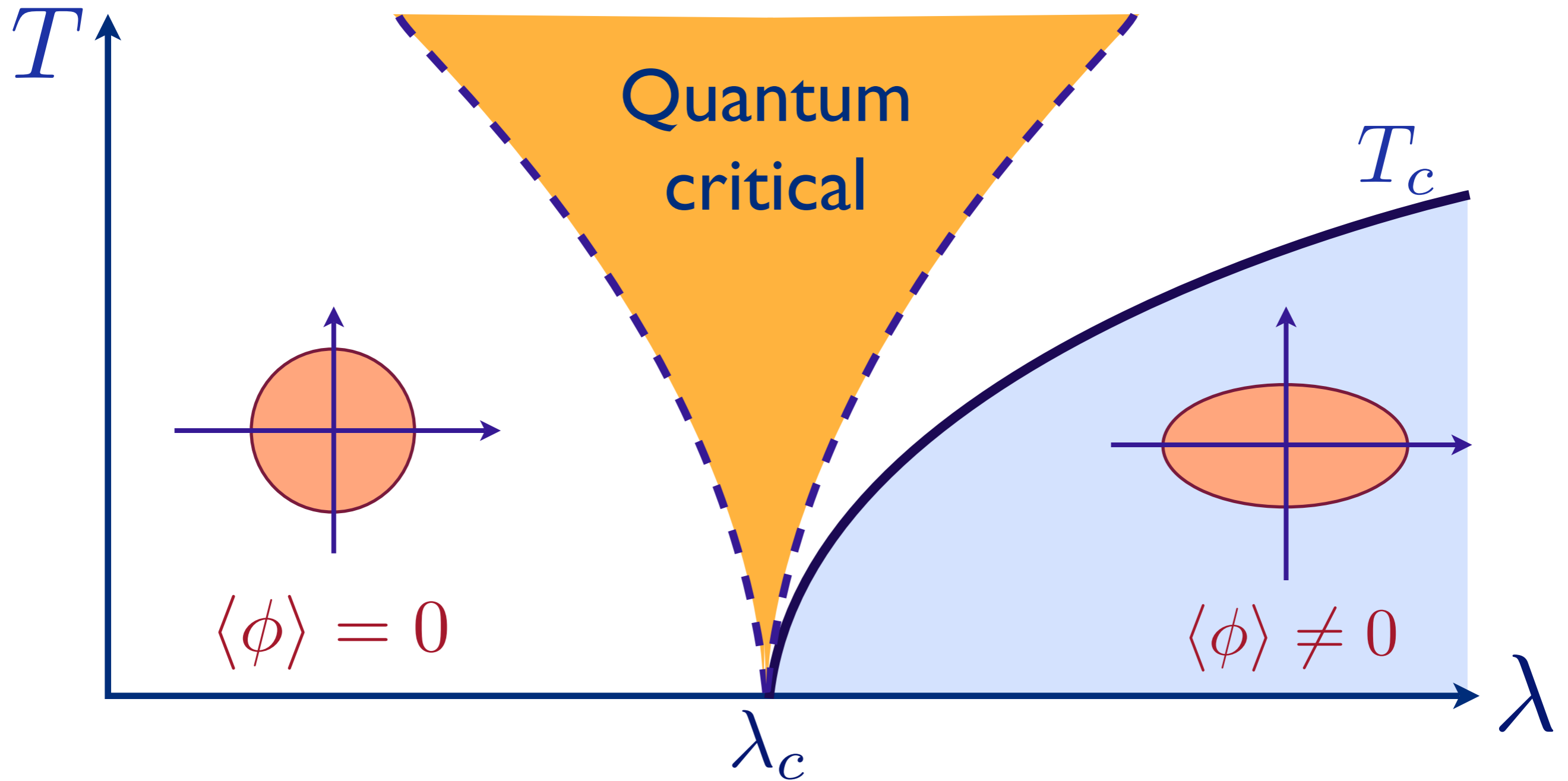
Spontaneous elongation along y direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Pomeranchuk instability



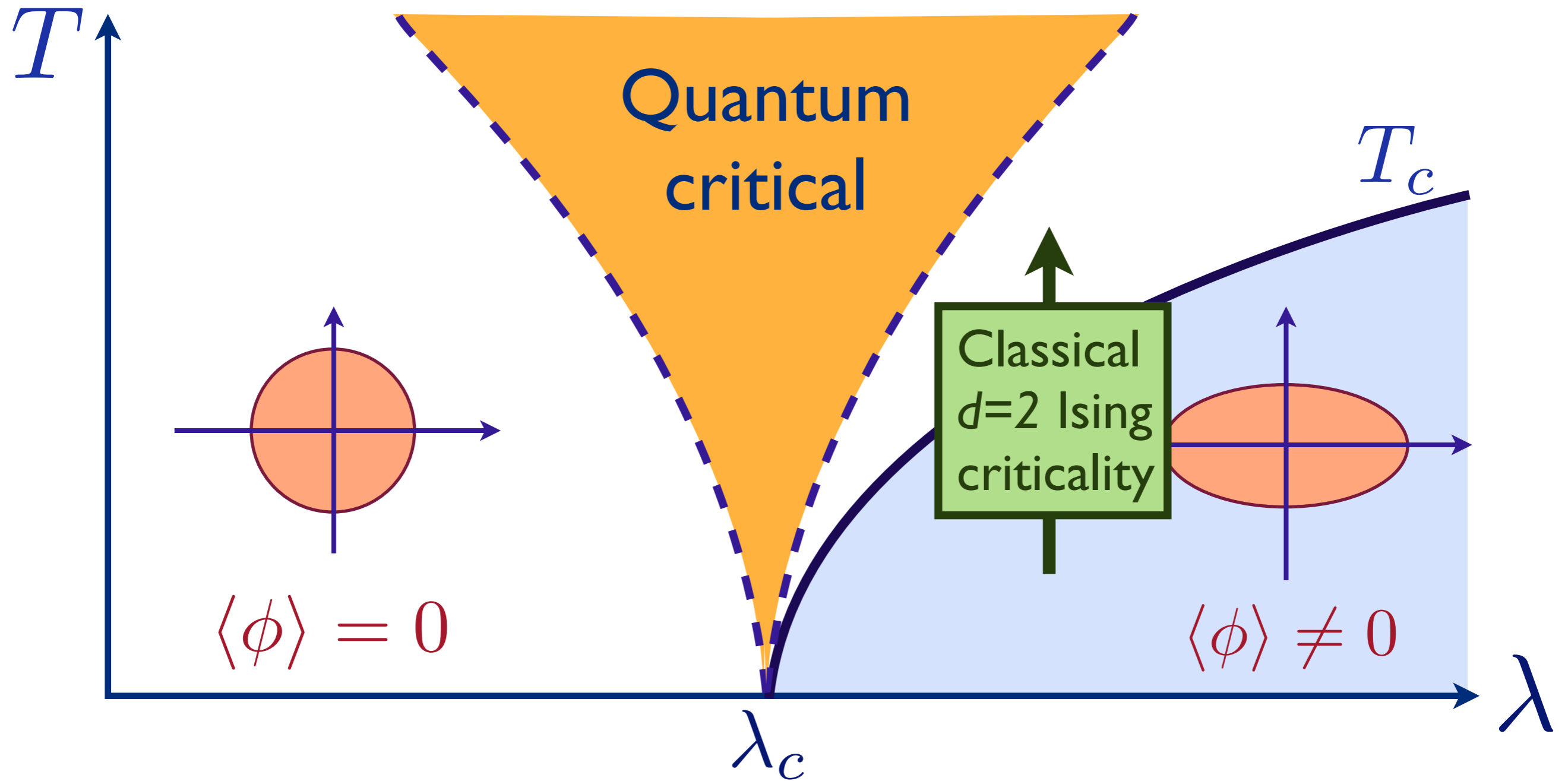
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Pomeranchuk instability



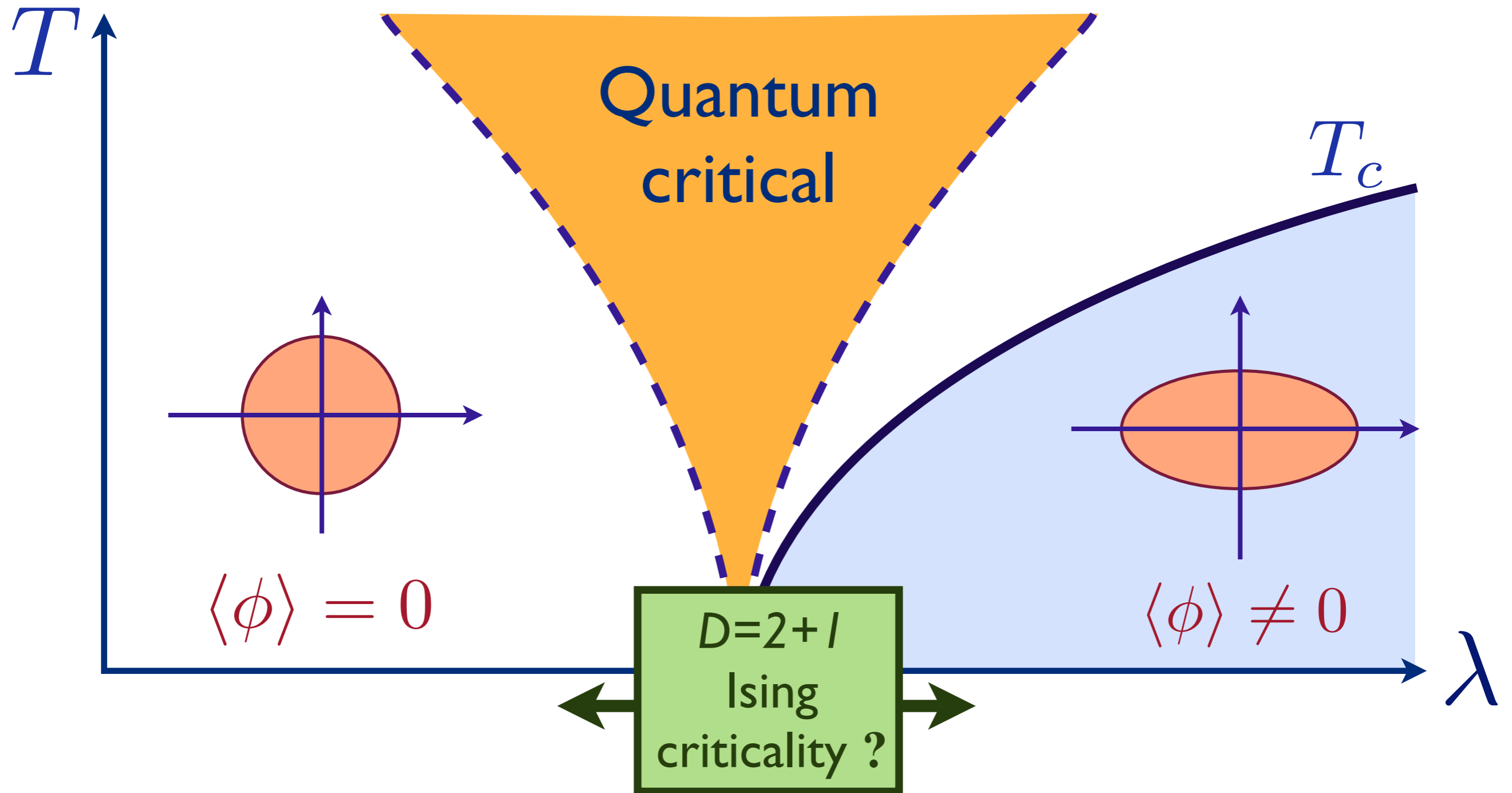
Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

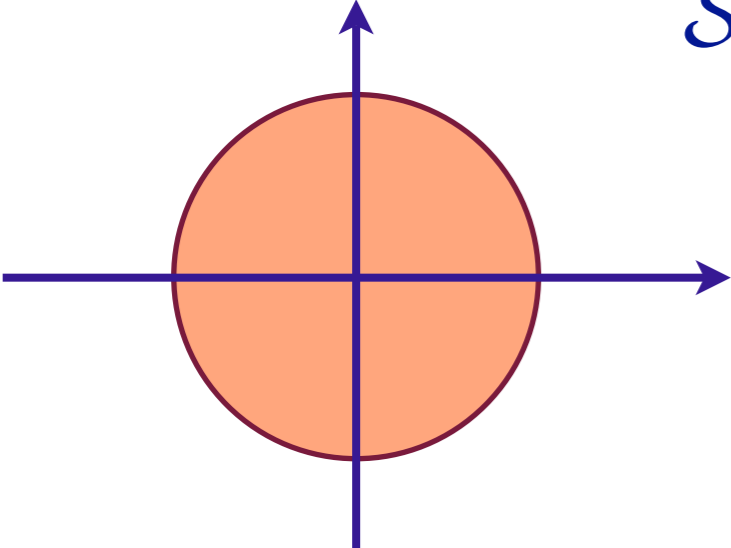
$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right]$$

Effective action for electrons:

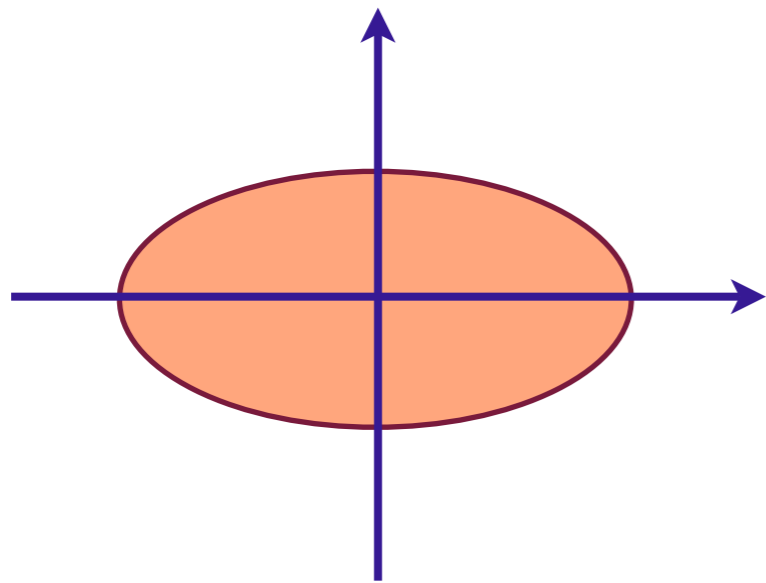

$$\begin{aligned} \mathcal{S}_c &= \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \right] \\ &\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha} \end{aligned}$$

Quantum criticality of Pomeranchuk instability

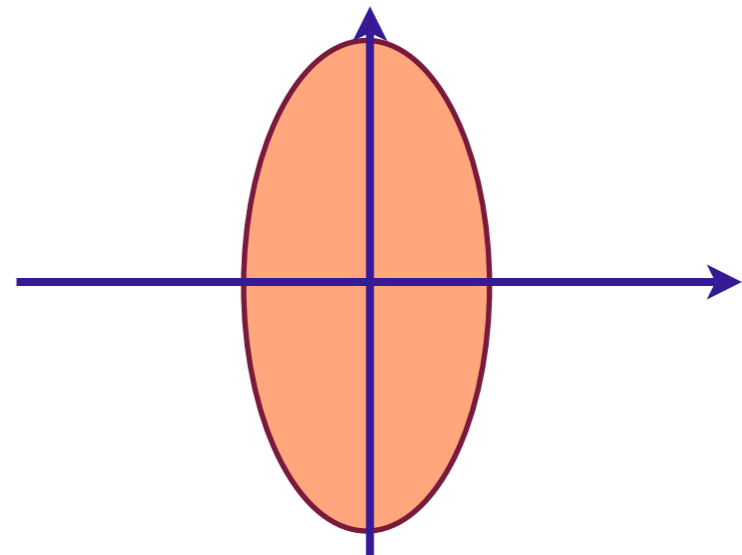
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \phi \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

for spatially independent ϕ



$$\langle \phi \rangle > 0$$



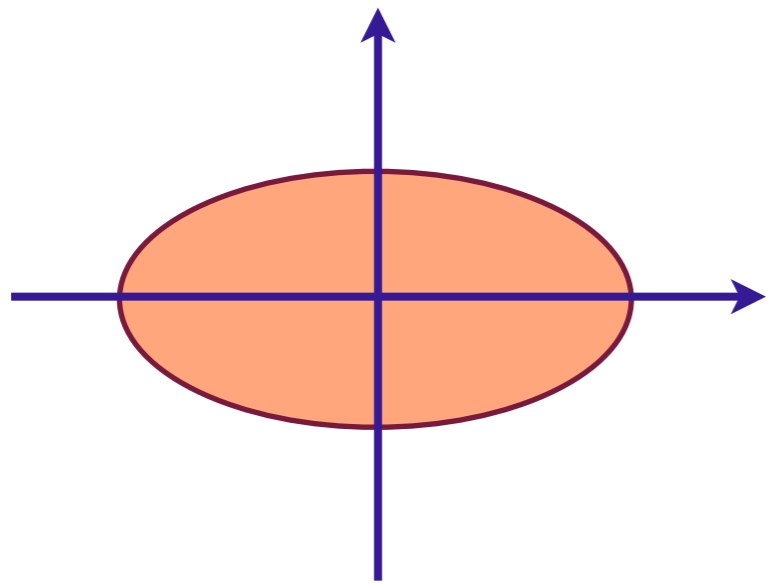
$$\langle \phi \rangle < 0$$

Quantum criticality of Pomeranchuk instability

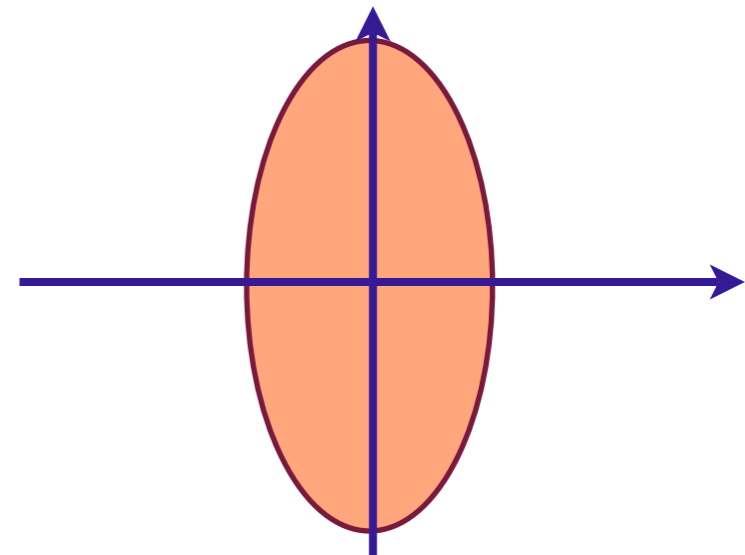
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

for spatially dependent ϕ



$$\langle \phi \rangle > 0$$



$$\langle \phi \rangle < 0$$

Quantum criticality of Pomeranchuk instability

$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

Quantum critical field theory

$$\mathcal{Z} = \int \mathcal{D}\phi \mathcal{D}c_{i\alpha} \exp(-\mathcal{S}_\phi - \mathcal{S}_c - \mathcal{S}_{\phi c})$$

Quantum criticality of Pomeranchuk instability

Hertz theory

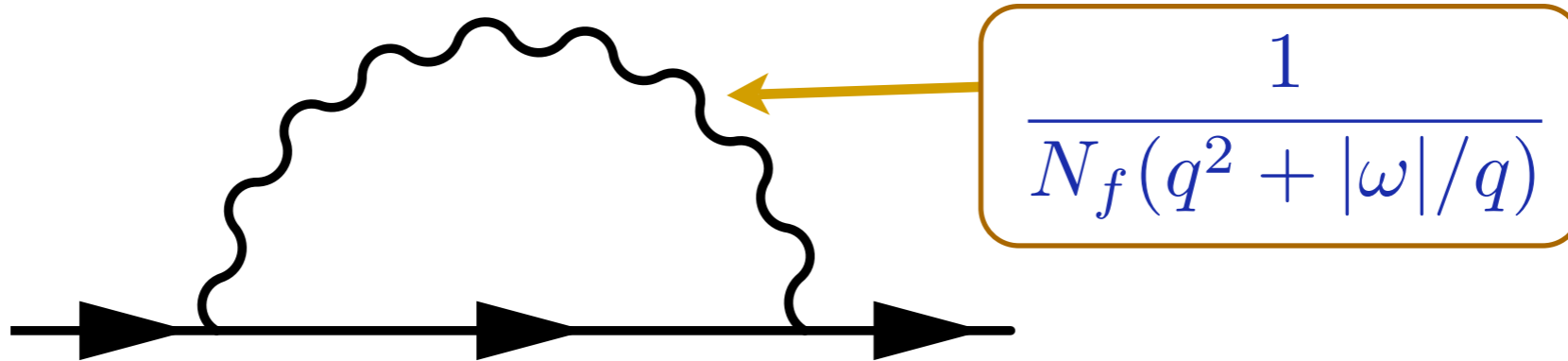
Integrate out c_α fermions and obtain non-local corrections to ϕ action

$$\delta\mathcal{S}_\phi \sim N_f \gamma^2 \int \frac{d^2q}{4\pi^2} \int \frac{d\omega}{2\pi} |\phi(\mathbf{q}, \omega)|^2 \left[\frac{|\omega|}{q} + q^2 \right] + \dots$$

This leads to a critical point with dynamic critical exponent $z = 3$ and quantum criticality controlled by the Gaussian fixed point.

Quantum criticality of Pomeranchuk instability

Hertz theory



Self energy of c_α fermions to order $1/N_f$

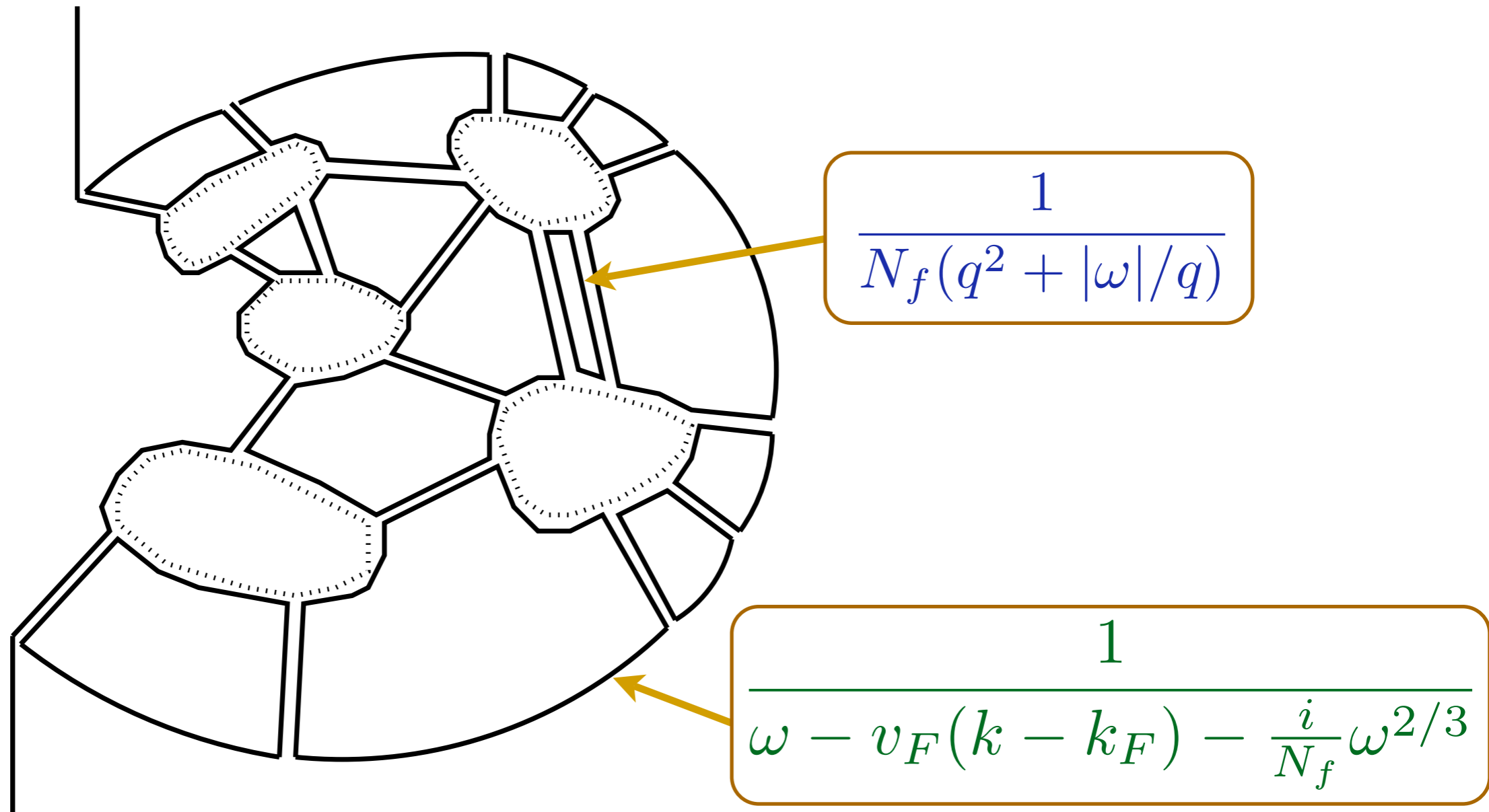
$$\Sigma_c(k, \omega) \sim \frac{i}{N_f} \omega^{2/3}$$

This leads to the Green's function

$$G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - \frac{i}{N_f} \omega^{2/3}}$$

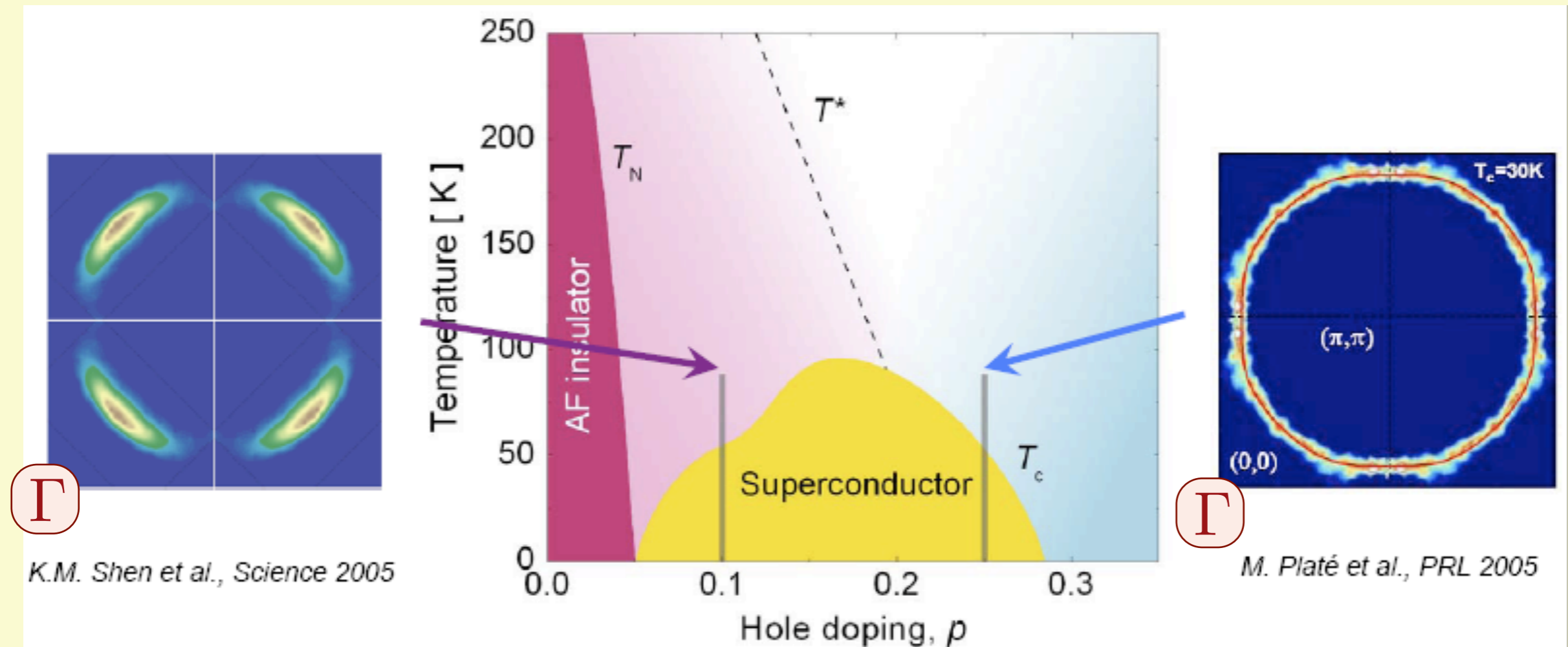
Note that the order $1/N_f$ term is more singular in the infrared than the bare term; this leads to problems in the bare $1/N_f$ expansion in terms that are dominated by low frequency fermions.

Quantum criticality of Pomeranchuk instability



The infrared singularities of fermion particle-hole pairs are most severe on planar graphs: these all contribute at leading order in $1/N_f$.

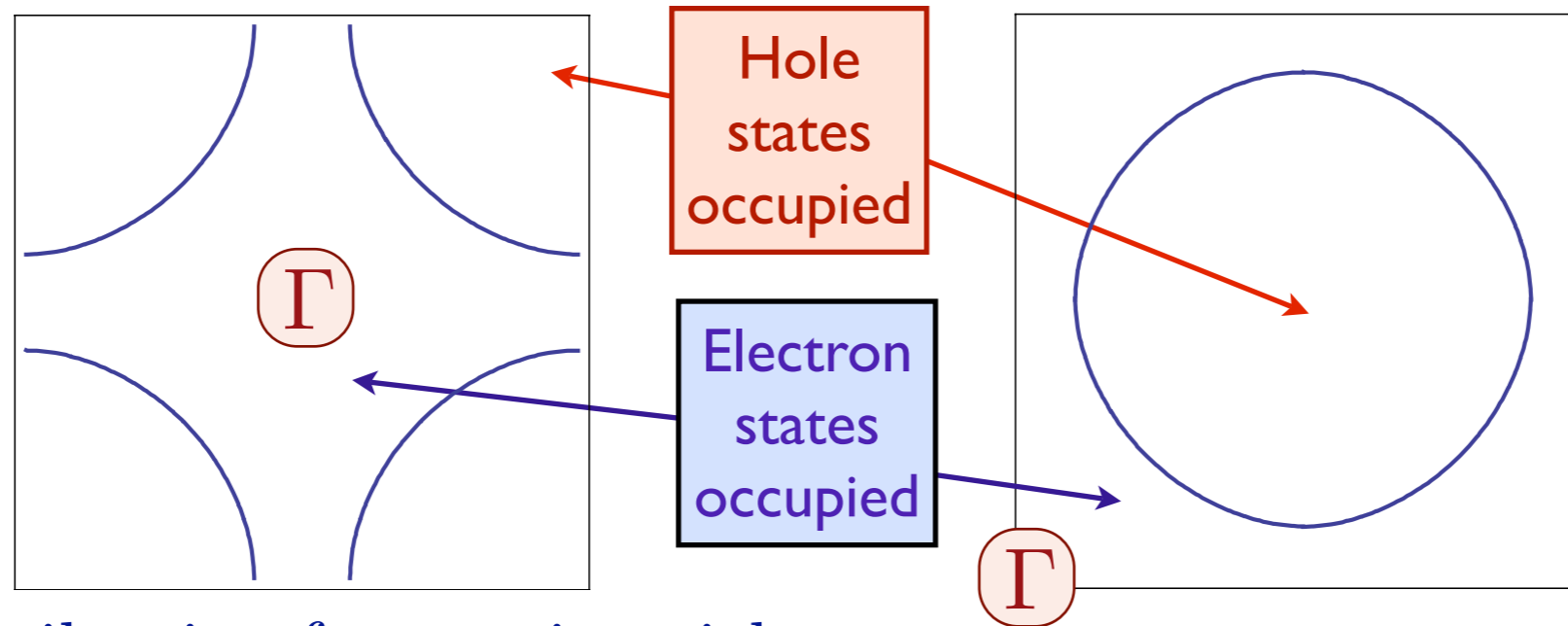
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole
Fermi-pockets

Large hole
Fermi surface

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, \mathcal{A}_e , from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1 - p) & \text{for hole-doping } p \\ 2\pi^2(1 + x) & \text{for electron-doping } x \end{cases}$$

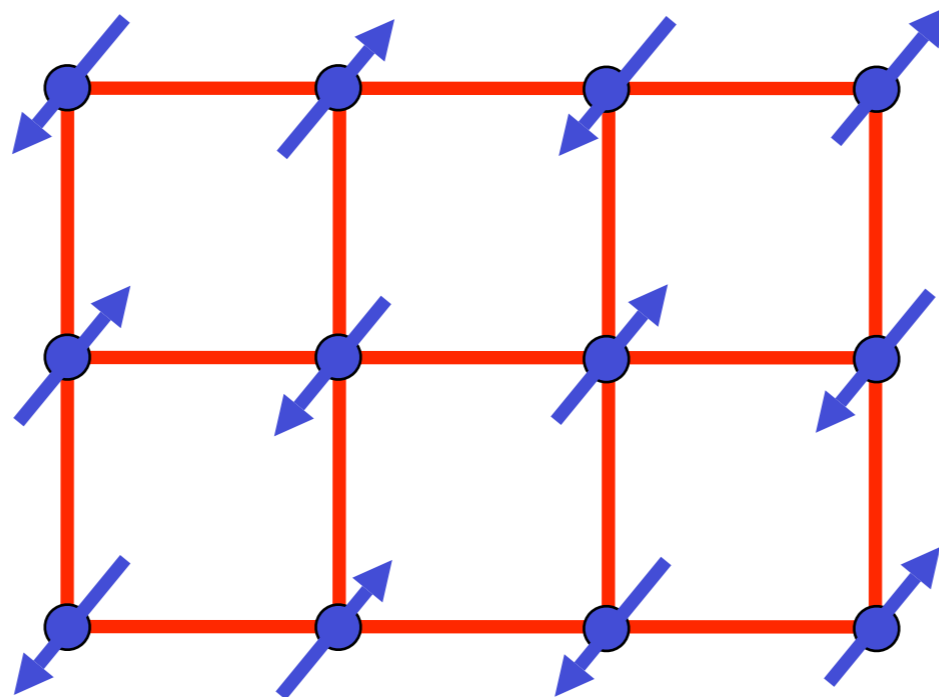
The area of the occupied hole states, \mathcal{A}_h , which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

A spin density wave (SDW) is the spontaneous appearance of an oscillatory spin polarization. The electron spin polarization is written as

$$\vec{S}(\mathbf{r}, \tau) = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the SDW order parameter, and \mathbf{K} is a fixed ordering wavevector. For simplicity we will consider the case of $\mathbf{K} = (\pi, \pi)$, but our treatment applies to general \mathbf{K} .



Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by \mathbf{K}

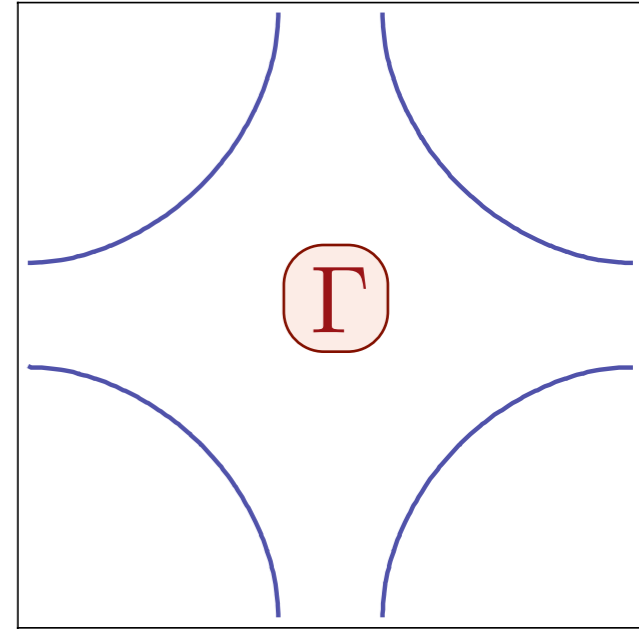
$$H_{\text{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k}, \alpha, \beta} c_{\mathbf{k}, \alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}, \beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\text{sdw}}$ for $\vec{\varphi} \propto (0, 0, 1)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \varphi^2}$$

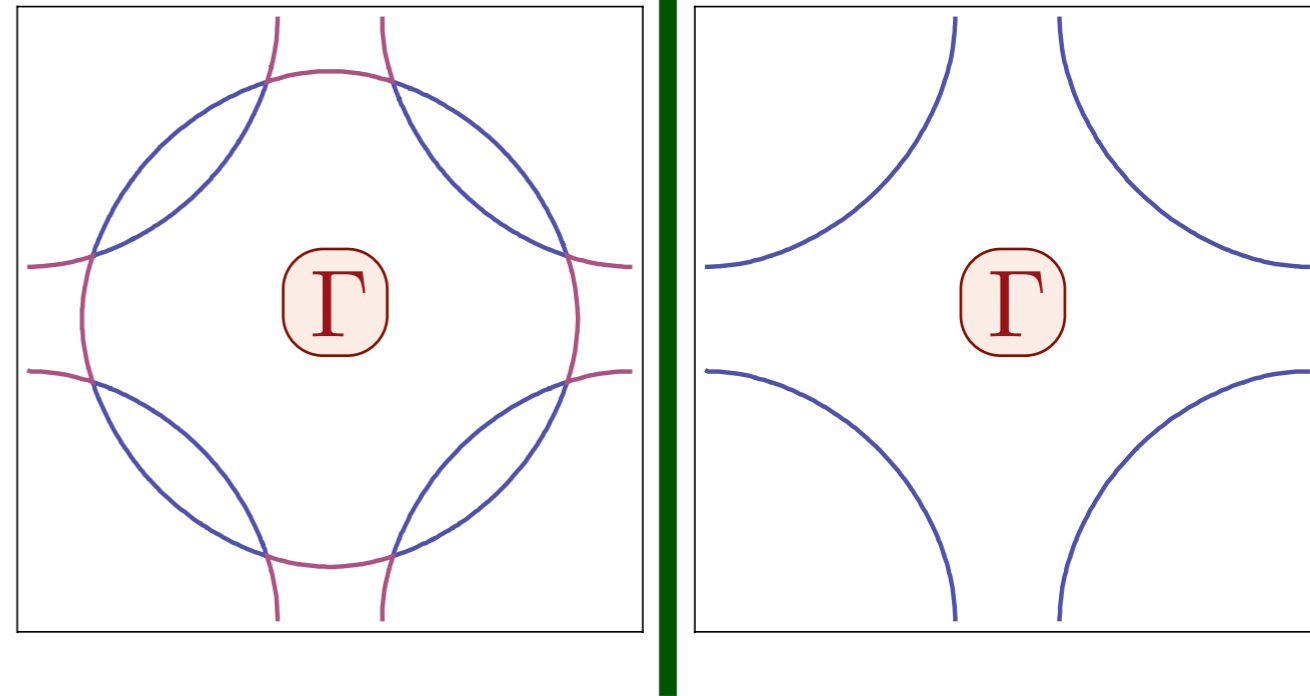
This leads to the Fermi surfaces shown in the following slides for electron and hole doping.

Spin density wave theory in hole-doped cuprates



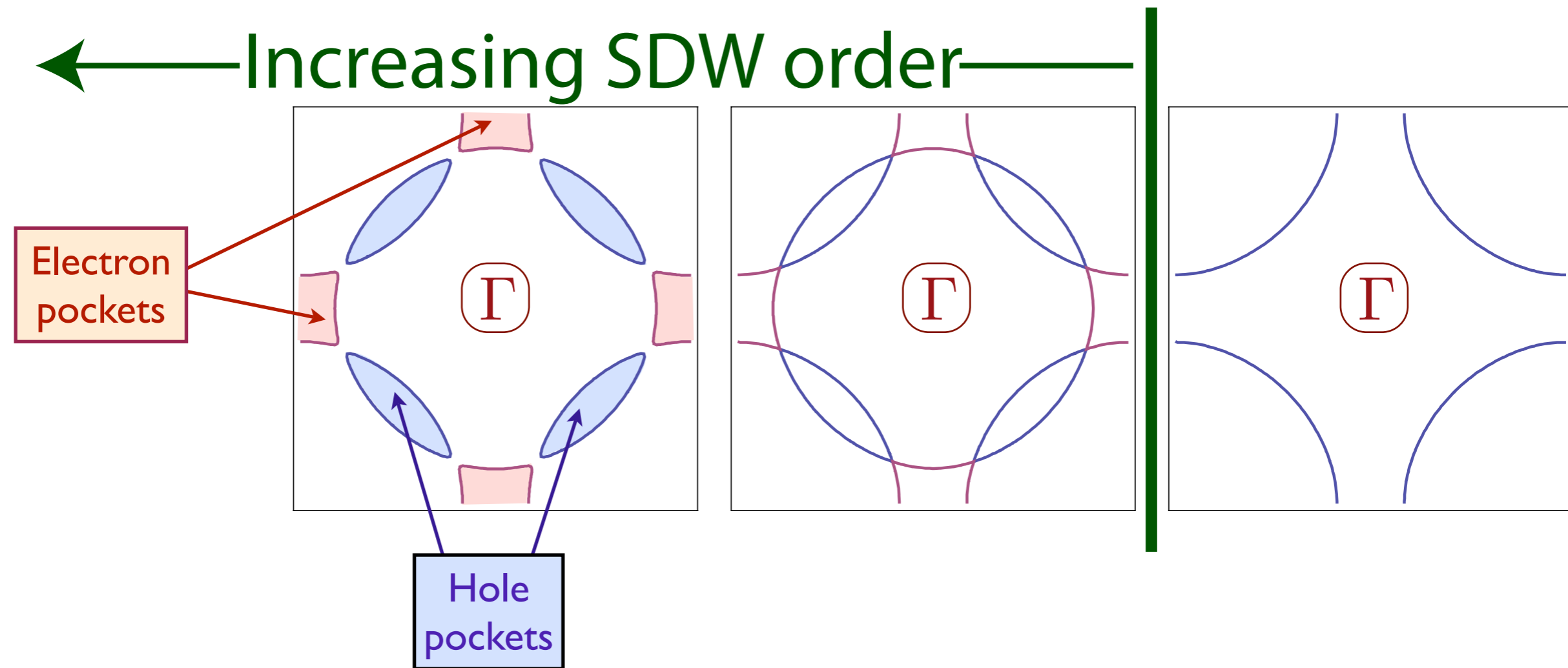
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

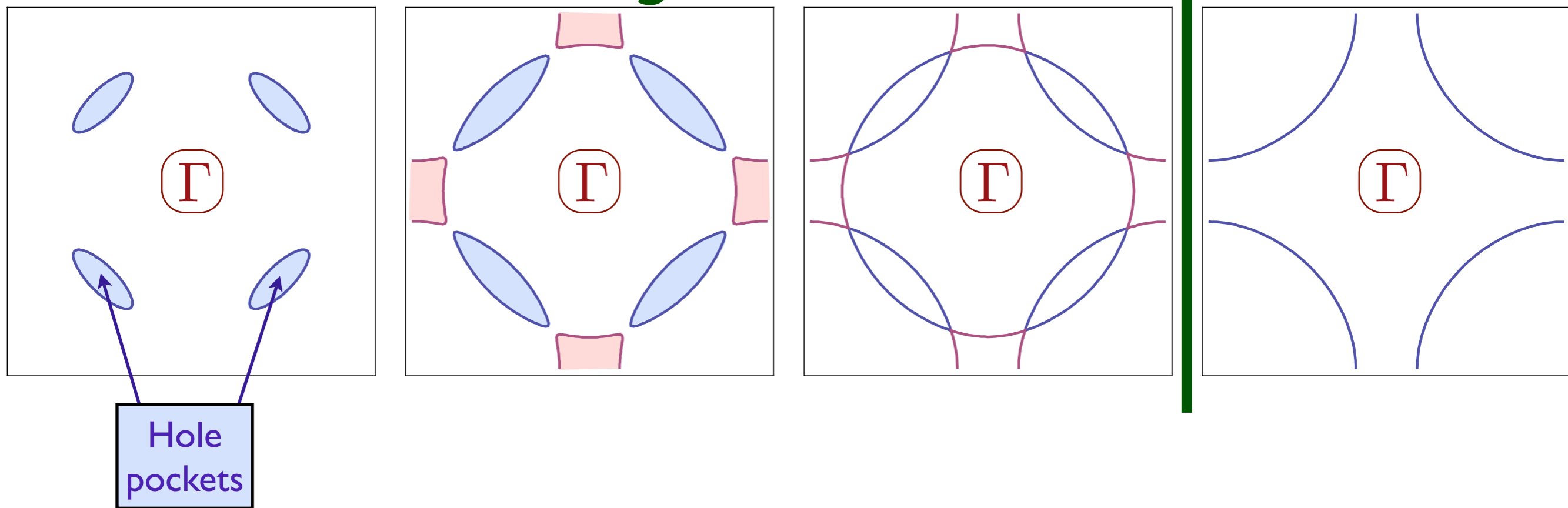
Spin density wave theory in hole-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates

← Increasing SDW order →



SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).

A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by \mathbf{K}

$$H_{\text{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k}, \alpha, \beta} c_{\mathbf{k}, \alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{\mathbf{k} + \mathbf{K}, \beta}$$

where $\vec{\sigma}$ are the Pauli matrices. At the quantum critical point for the onset of SDW order, we integrate out the fermions and derive an effective action functional for $\vec{\varphi}$.

Spin density wave theory

This functional has the form

$$\mathcal{S} = \int \frac{d^2 q}{4\pi^2} \int \frac{d\omega}{2\pi} |\vec{\varphi}(\mathbf{q}, \omega)|^2 \left[r + q^2 + \chi(\mathbf{K}, \omega) \right] + u \int d^2 x d\tau (\vec{\varphi}^2(x, \tau))^2 + \dots$$

The susceptibility, χ , has a non-analytic dependence on ω because of Landau damping:

$$\chi(\mathbf{K}, \omega) = \chi_0 + \chi_1 |\omega| + \dots$$

This leads to a critical point with dynamic critical exponent $z = 2$, and upper-critical dimension $d = 2$.

Spin density wave theory

This functional has the form

$$\mathcal{S} = \int \frac{d^2 q}{4\pi^2} \int \frac{d\omega}{2\pi} |\vec{\varphi}(\mathbf{q}, \omega)|^2 \left[r + q^2 + \chi(\mathbf{K}, \omega) \right] \\ + u \int d^2 x d\tau (\vec{\varphi}^2(x, \tau))^2 + \dots$$

However, the higher order corrections require summation of all planar graphs, as in the Pomeranchuk instability.

M. Metlitski

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3. AdS₂ theory

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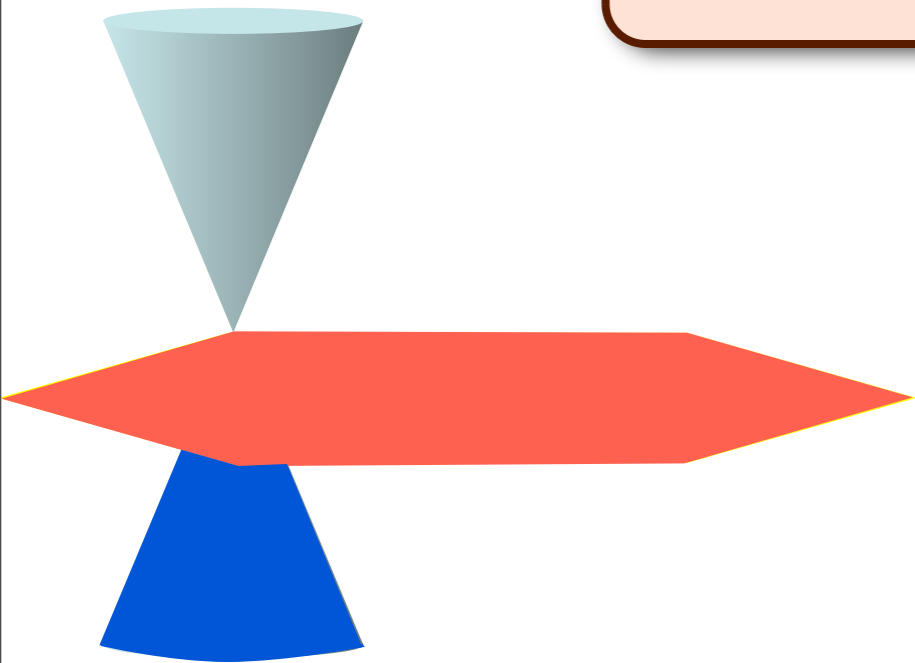
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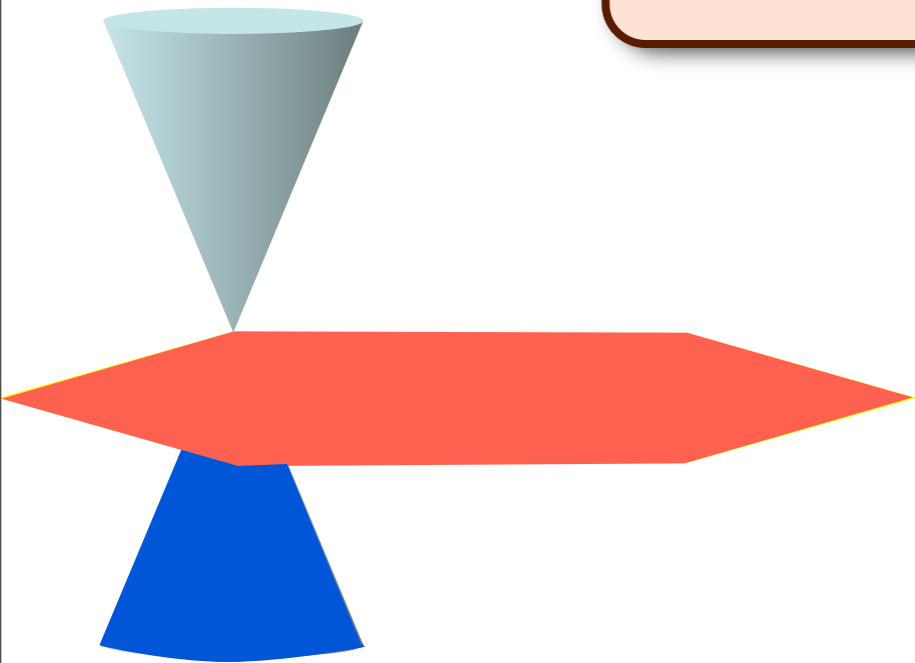
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Conformal field theory
in $2+1$ dimensions at $T = 0$

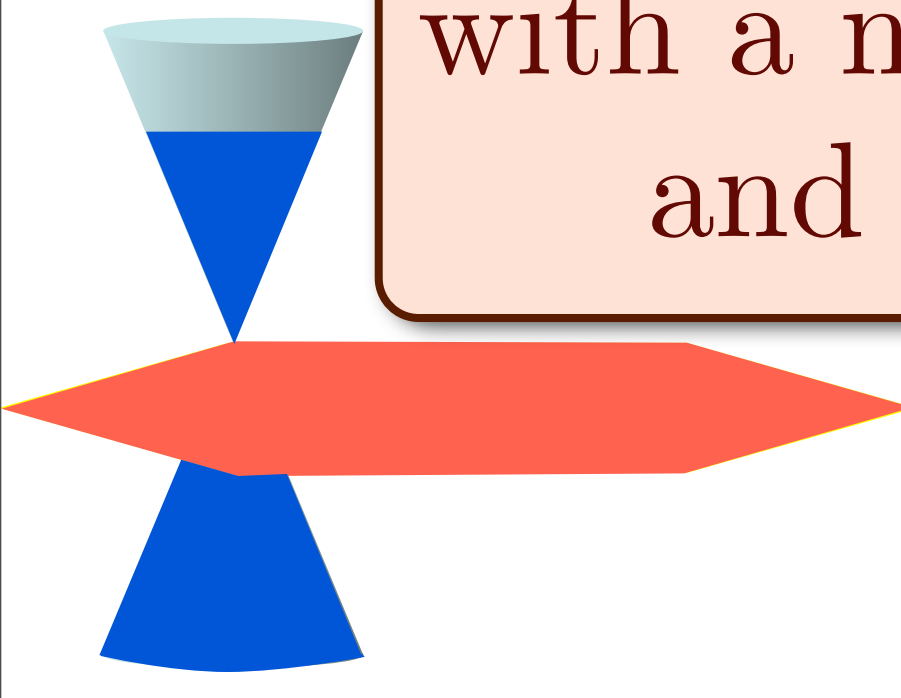


Einstein gravity
on AdS_4

Conformal field theory
in $2+1$ dimensions at $T > 0$

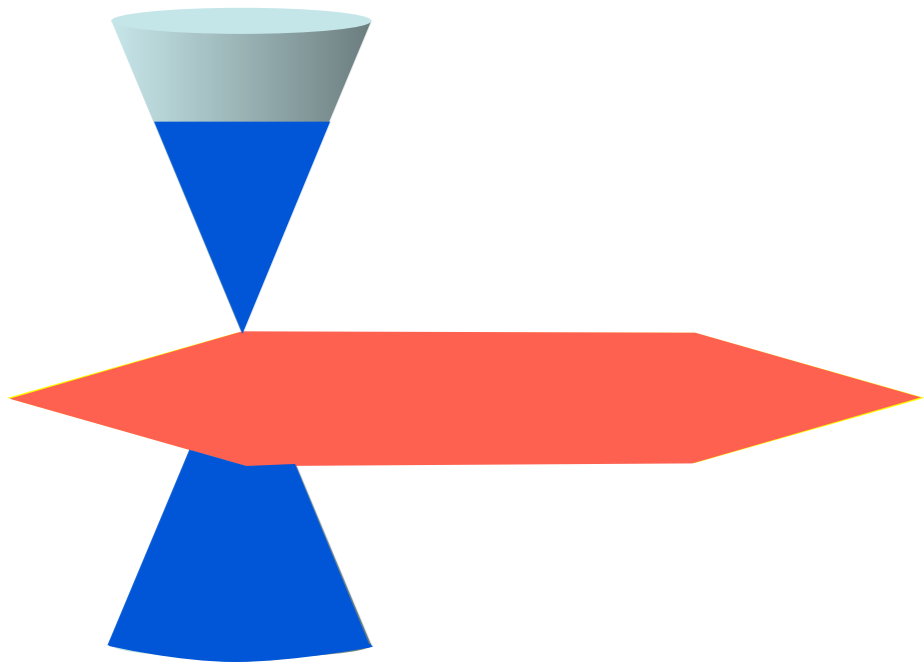


Einstein gravity on AdS_4
with a Schwarzschild
black hole

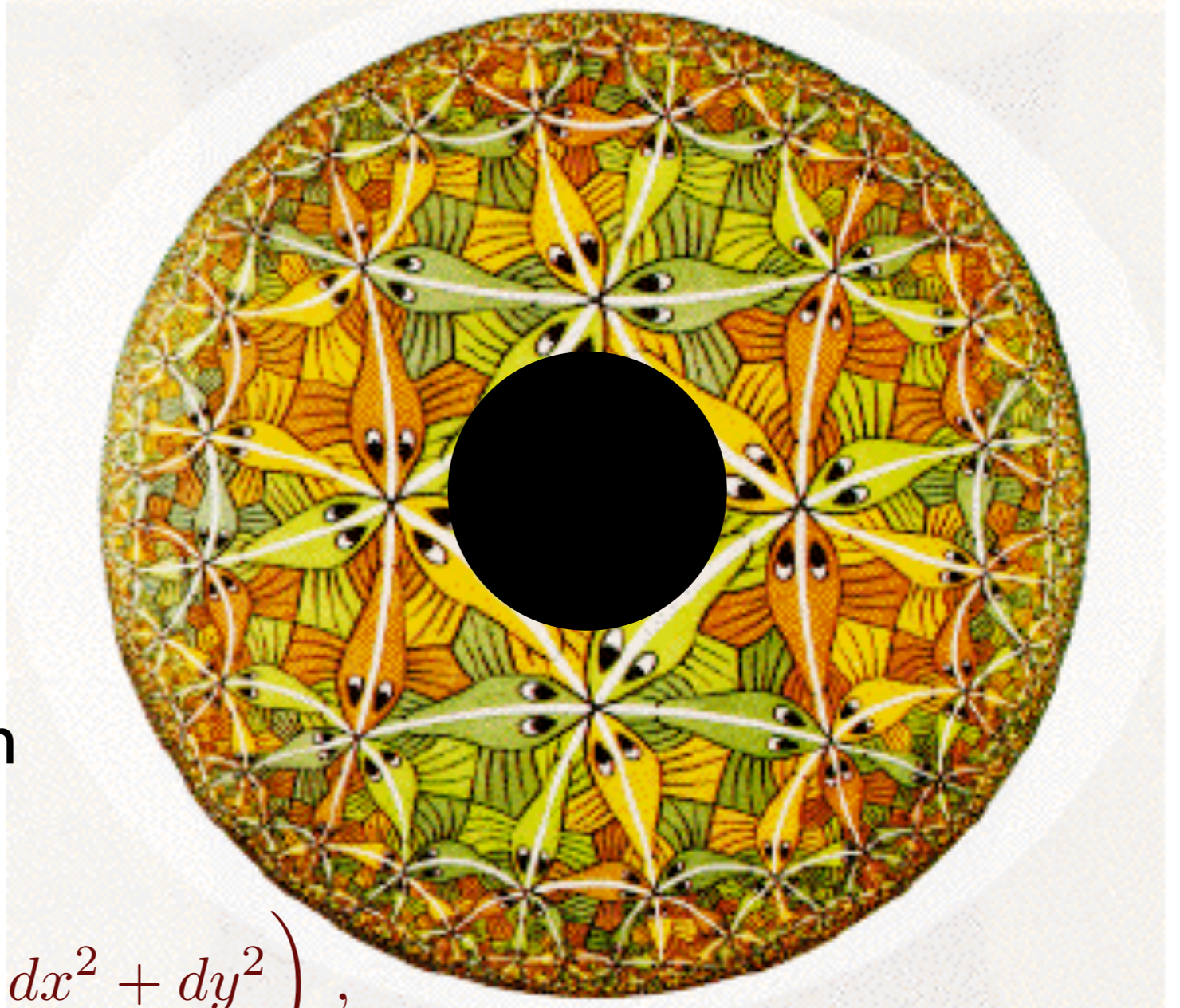


Conformal field theory
in $2+1$ dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B

Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges



AdS₄-Reissner-Nordstrom black hole

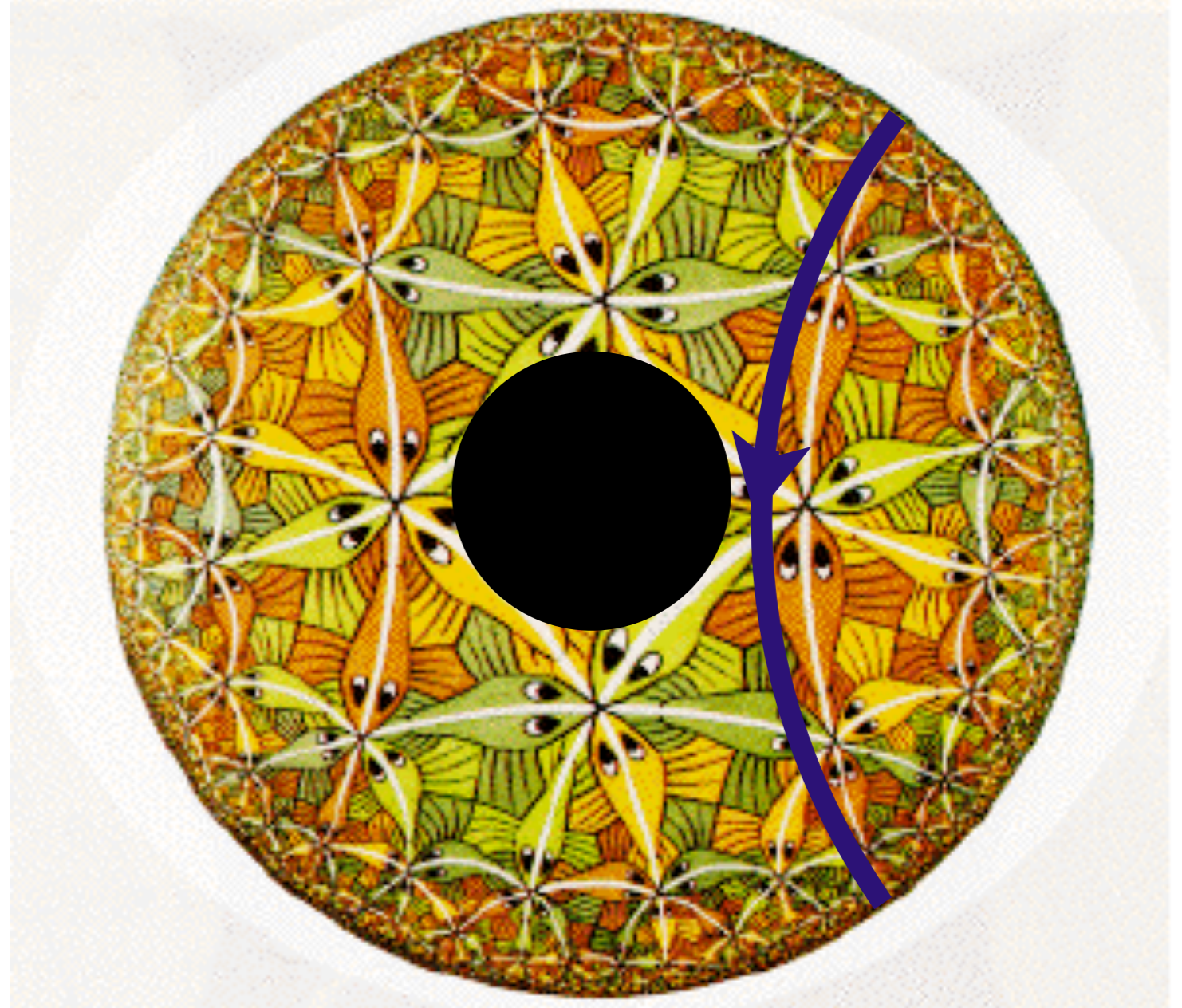
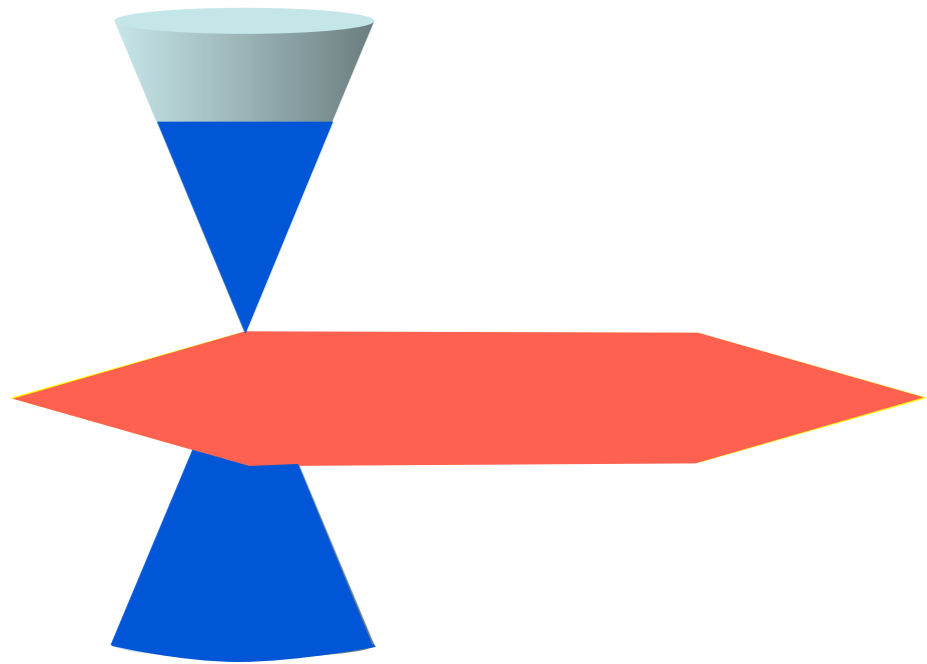


$$ds^2 = \frac{L^2}{r^2} \left(f(r) d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2 \right),$$

$$f(r) = 1 - \left(1 + \frac{(r_+^2 \mu^2 + r_+^4 B^2)}{\gamma^2} \right) \left(\frac{r}{r_+} \right)^3 + \frac{(r_+^2 \mu^2 + r_+^4 B^2)}{\gamma^2} \left(\frac{r}{r_+} \right)^4,$$

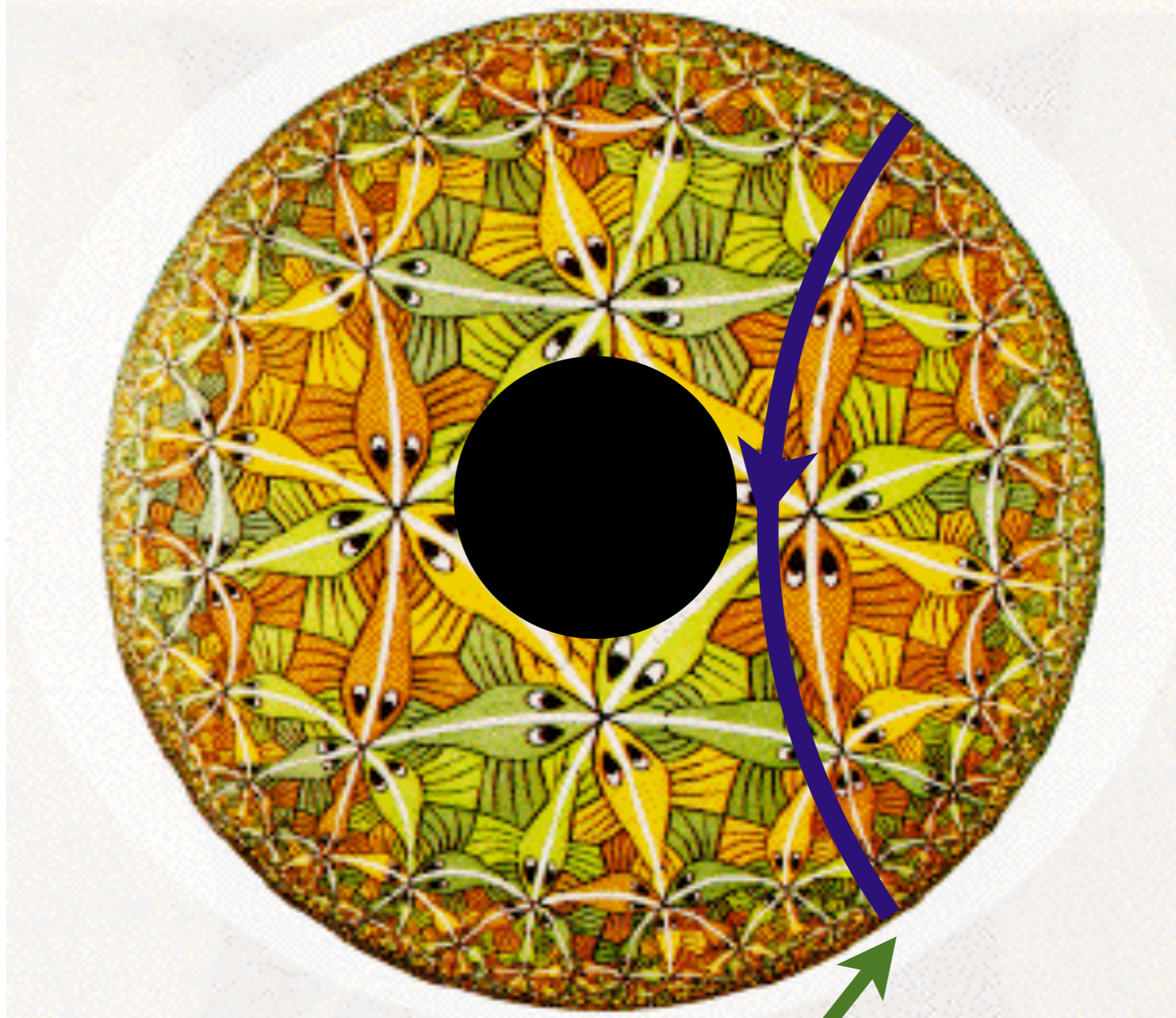
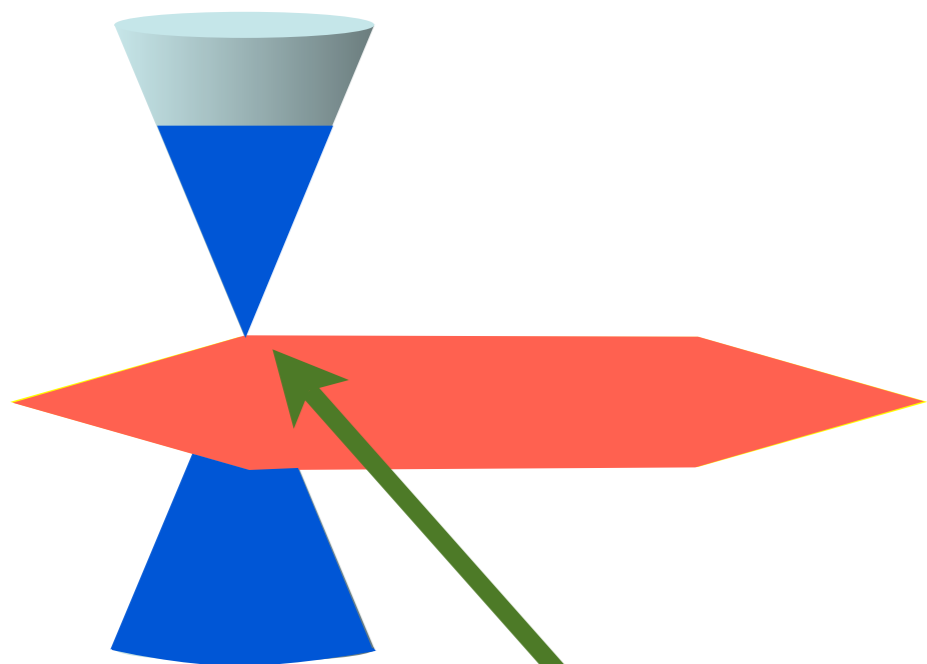
$$A = i\mu \left[1 - \frac{r}{r_+} \right] d\tau + Bx dy.$$

$$T = \frac{1}{4\pi r_+} \left(3 - \frac{r_+^2 \mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2} \right).$$



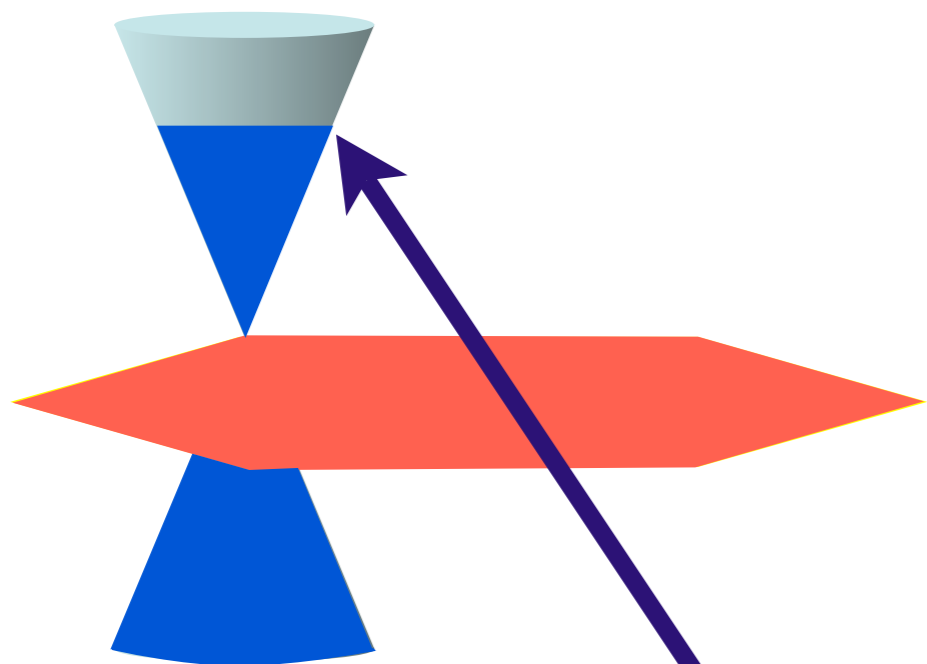
Examine free energy and Green's function
of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



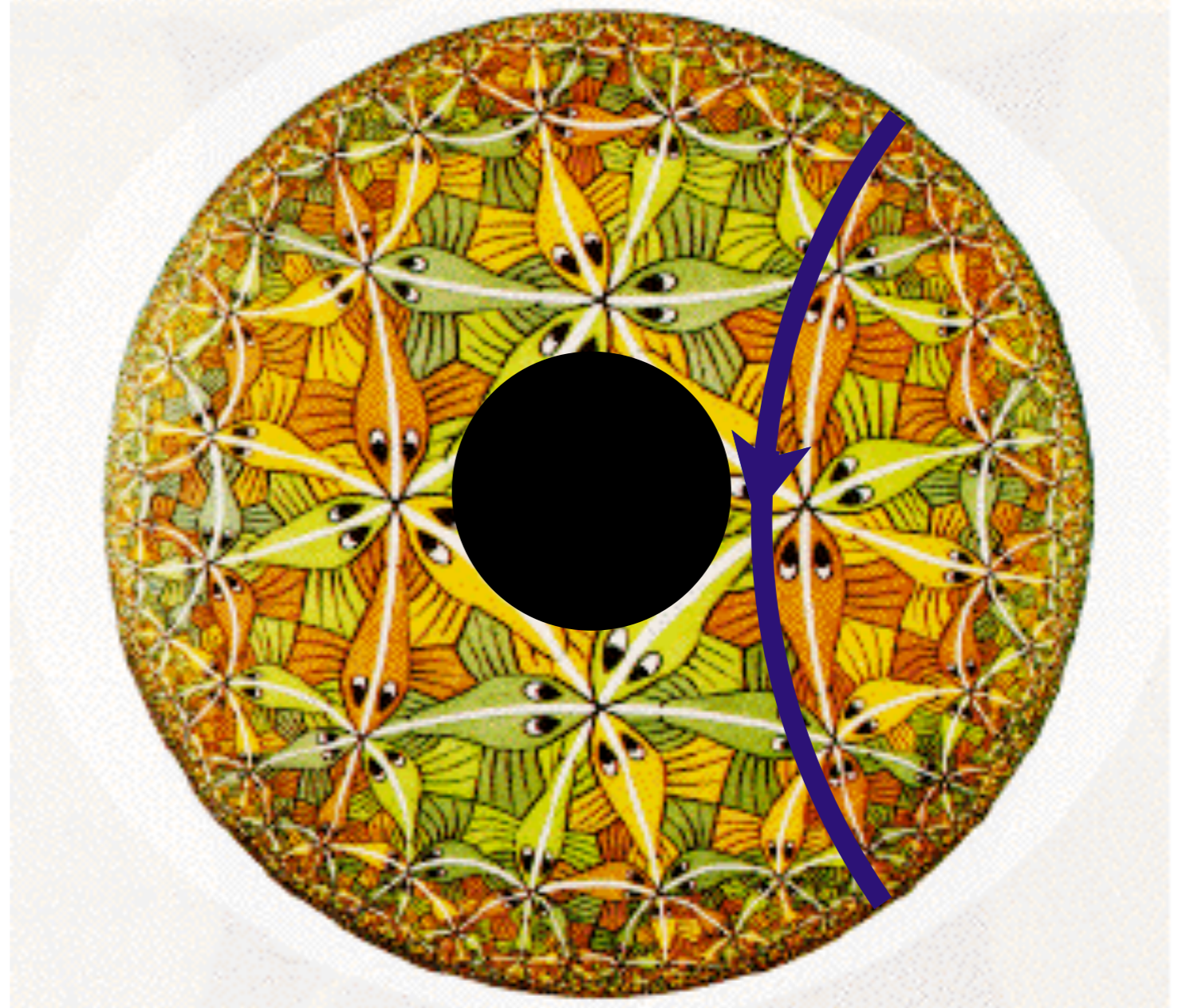
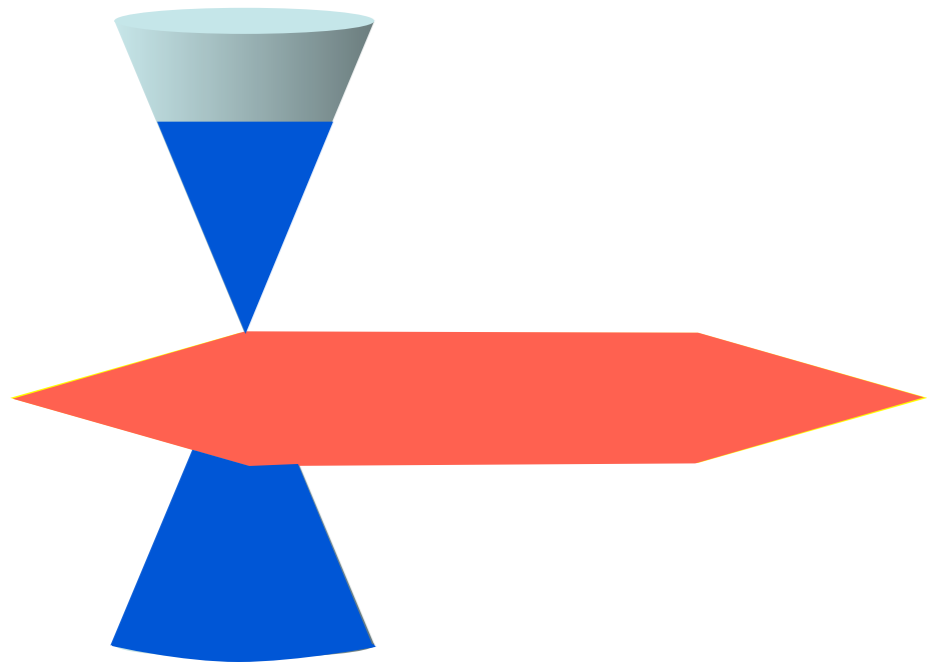
Short time behavior depends upon conformal AdS_4 geometry near boundary

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



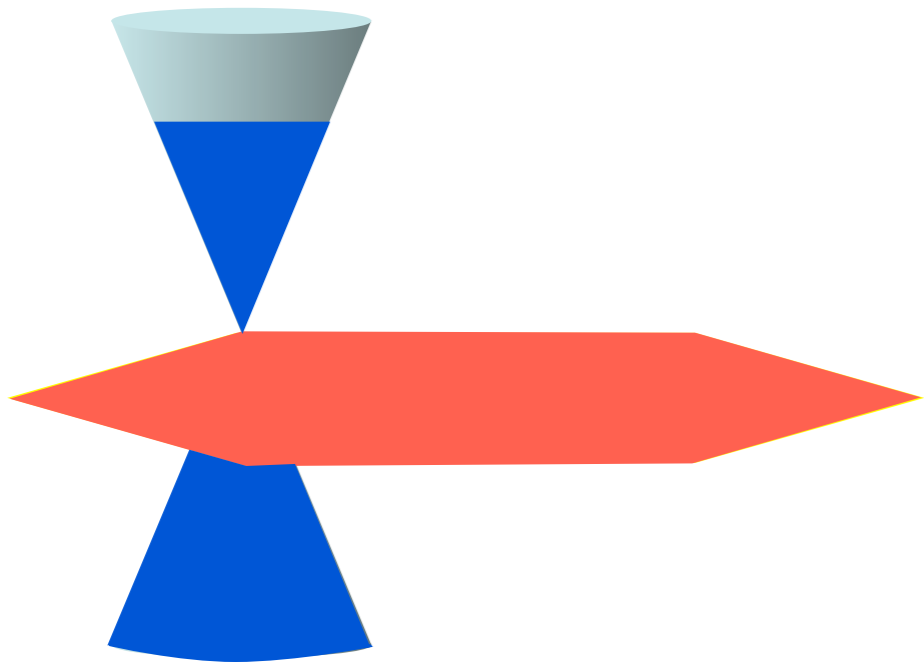
Long time behavior depends upon
near-horizon geometry of black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

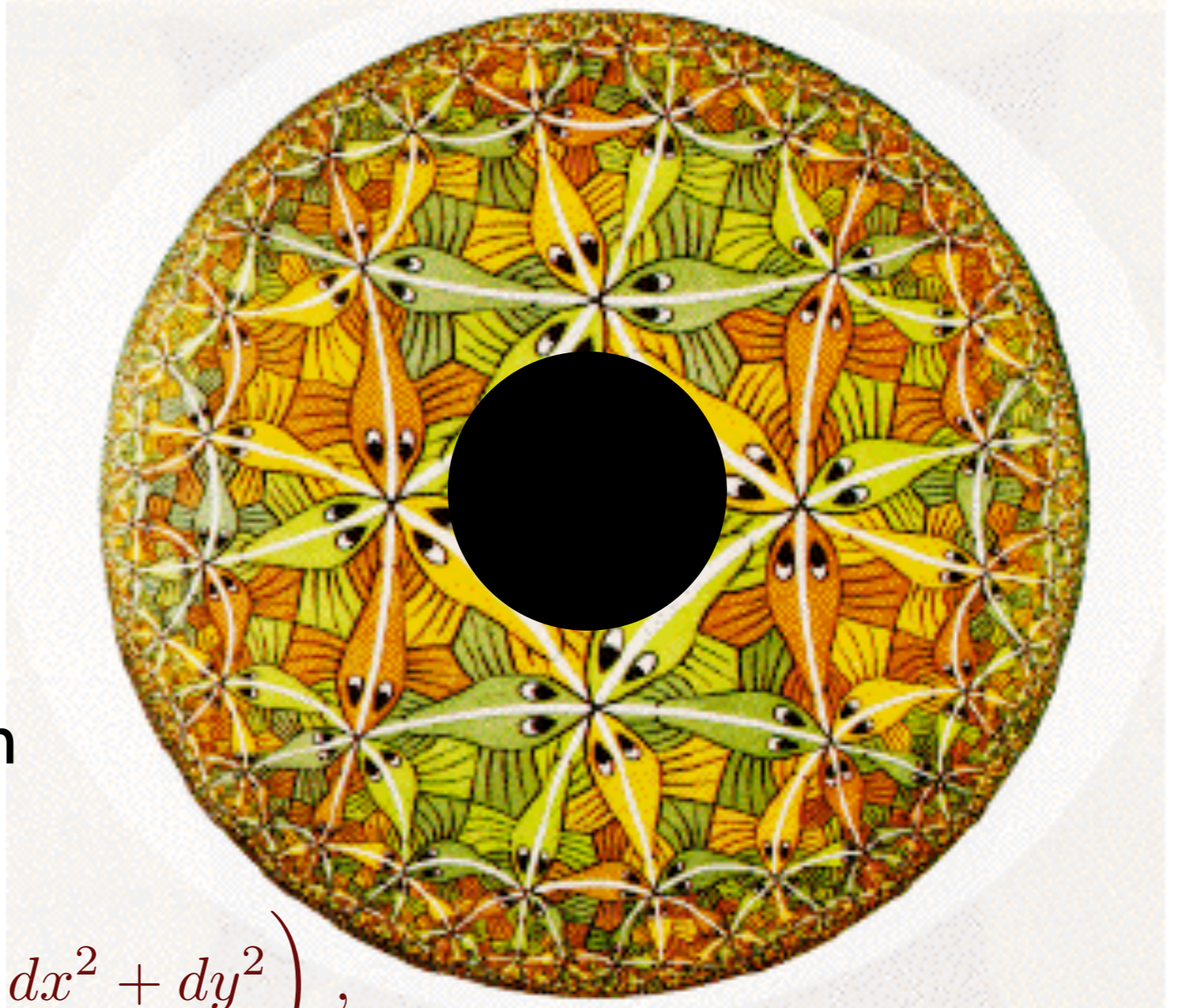


Radial direction of gravity theory is
measure of energy scale in CFT

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



AdS₄-Reissner-Nordstrom black hole

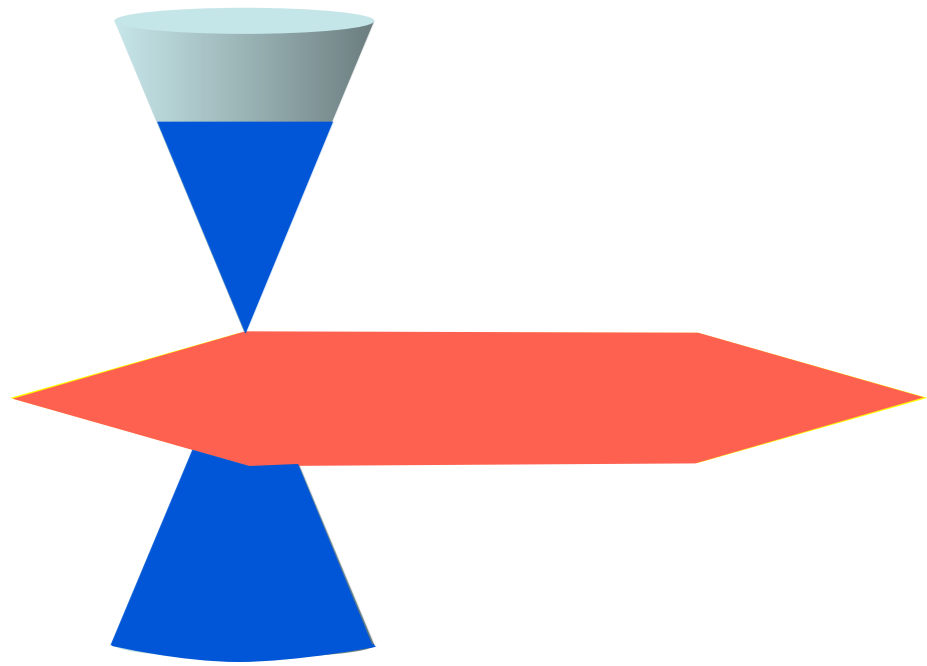


$$ds^2 = \frac{L^2}{r^2} \left(f(r) d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2 \right),$$

$$f(r) = 1 - \left(1 + \frac{(r_+^2 \mu^2 + r_+^4 B^2)}{\gamma^2} \right) \left(\frac{r}{r_+} \right)^3 + \frac{(r_+^2 \mu^2 + r_+^4 B^2)}{\gamma^2} \left(\frac{r}{r_+} \right)^4,$$

$$A = i\mu \left[1 - \frac{r}{r_+} \right] d\tau + Bx dy.$$

$$T = \frac{1}{4\pi r_+} \left(3 - \frac{r_+^2 \mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2} \right).$$

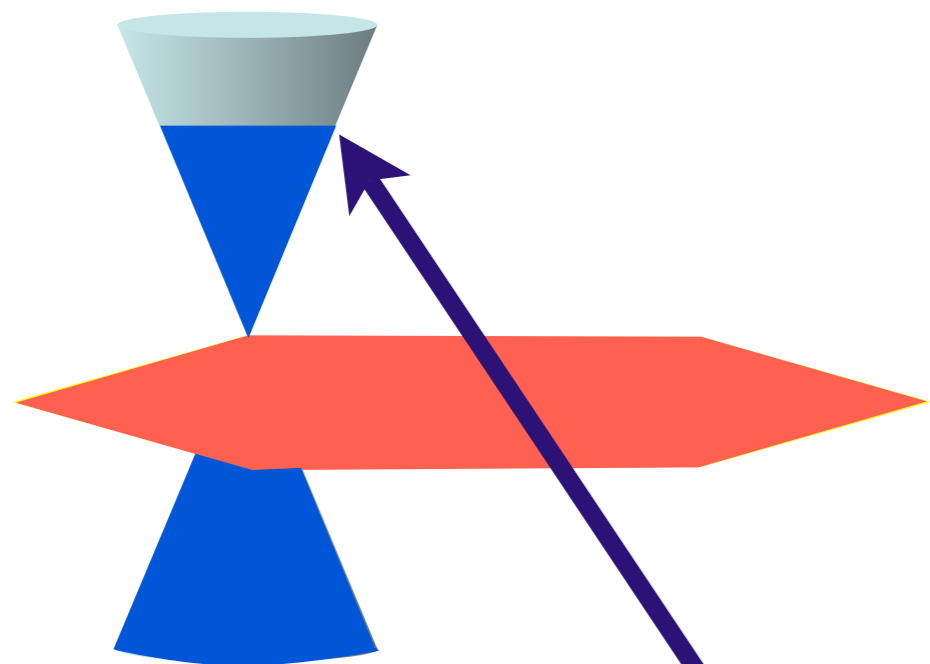


AdS₂ x R² near-horizon
geometry



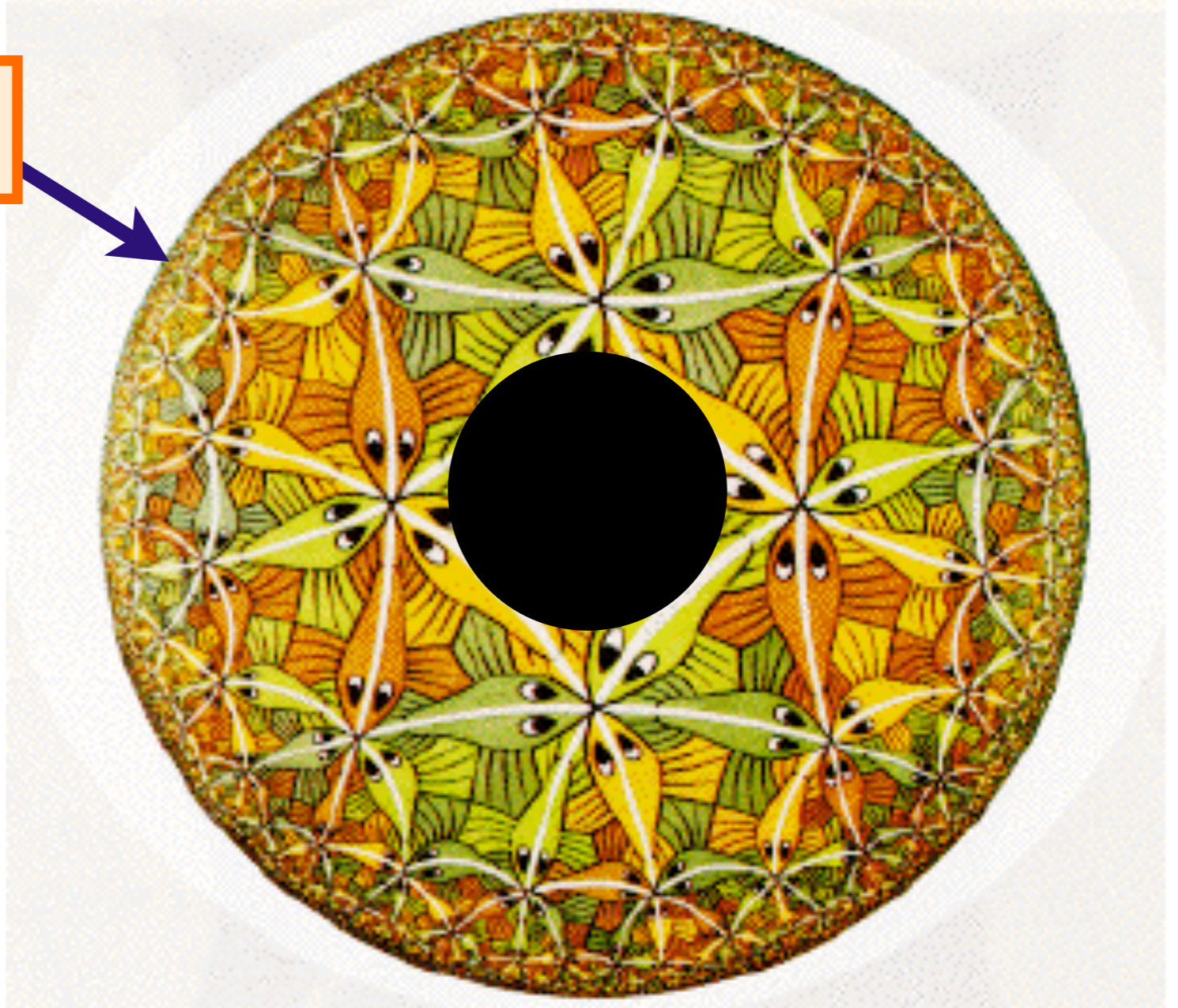
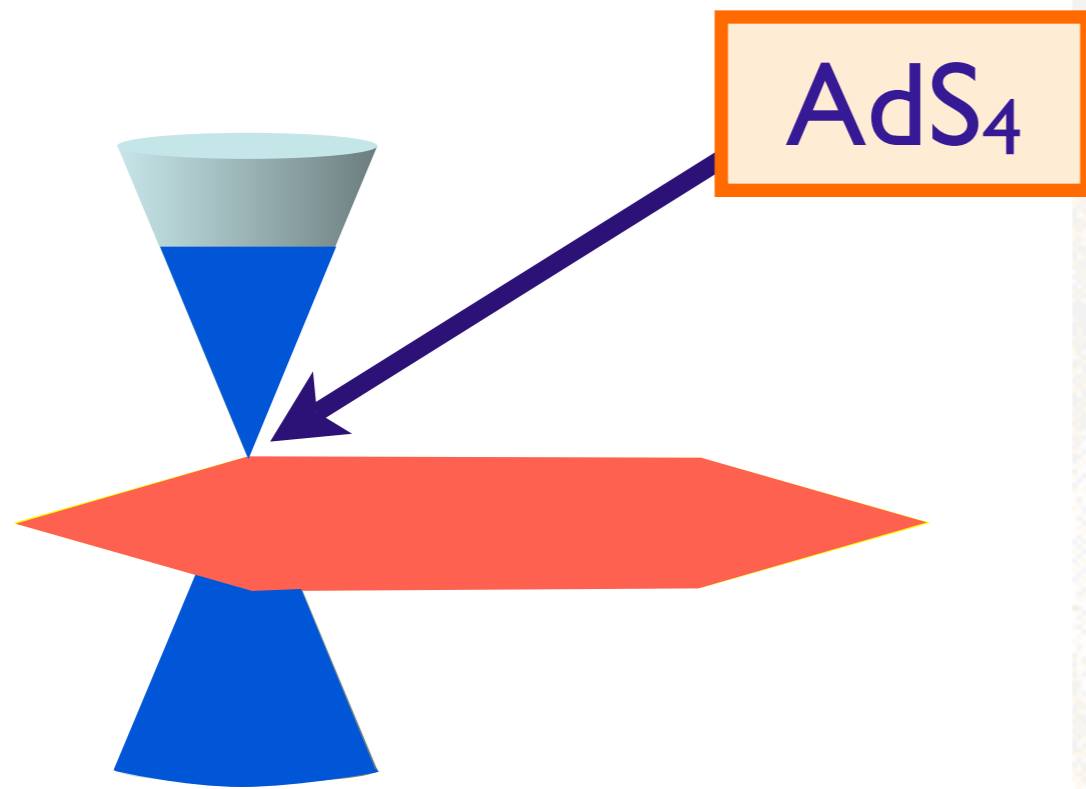
$$r - r_+ \sim \frac{1}{\zeta}$$

$$ds^2 = \frac{R^2}{\zeta^2} (-d\tau^2 + d\zeta^2) + \frac{r_+^2}{R^2} (dx^2 + dy^2)$$



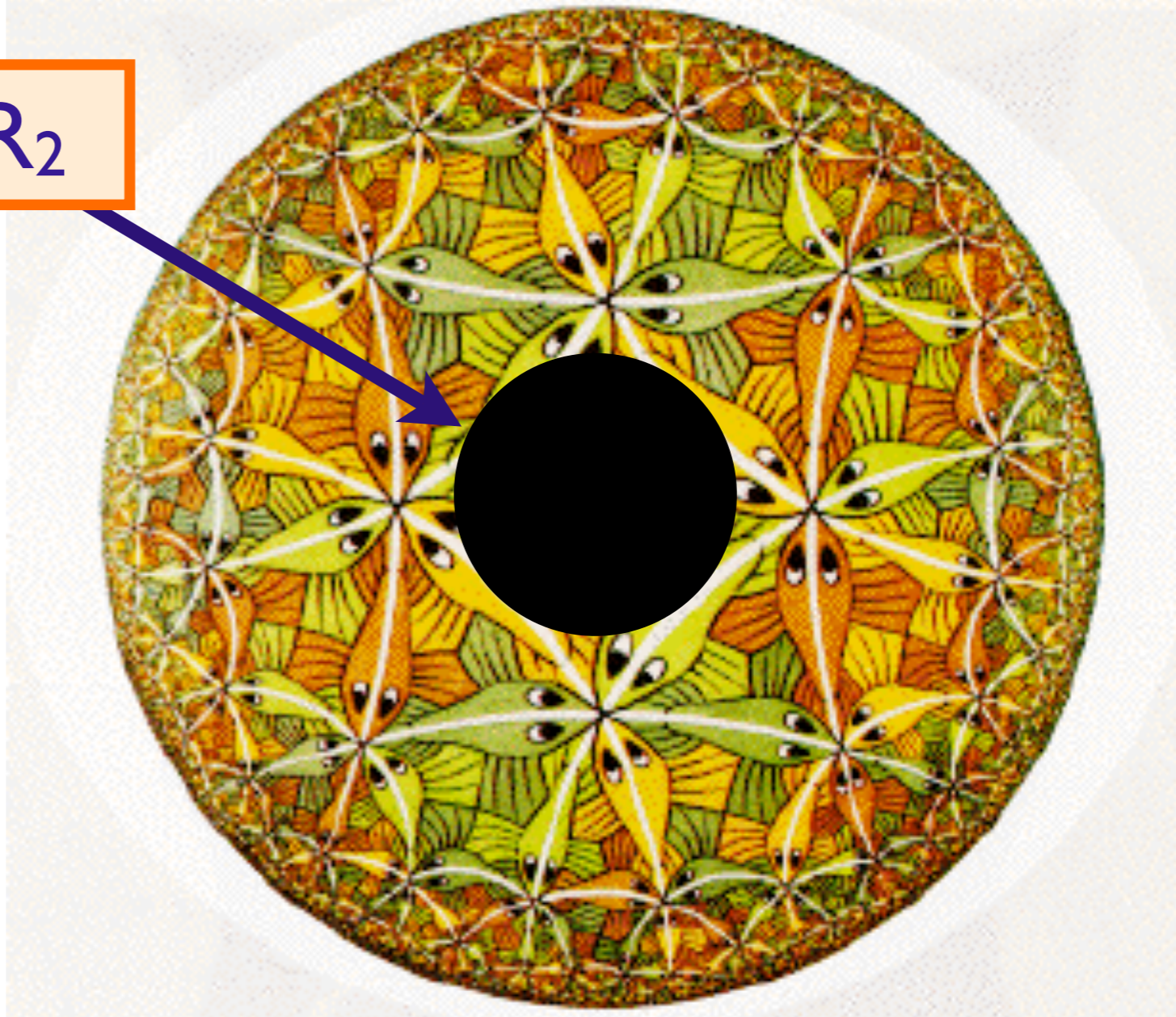
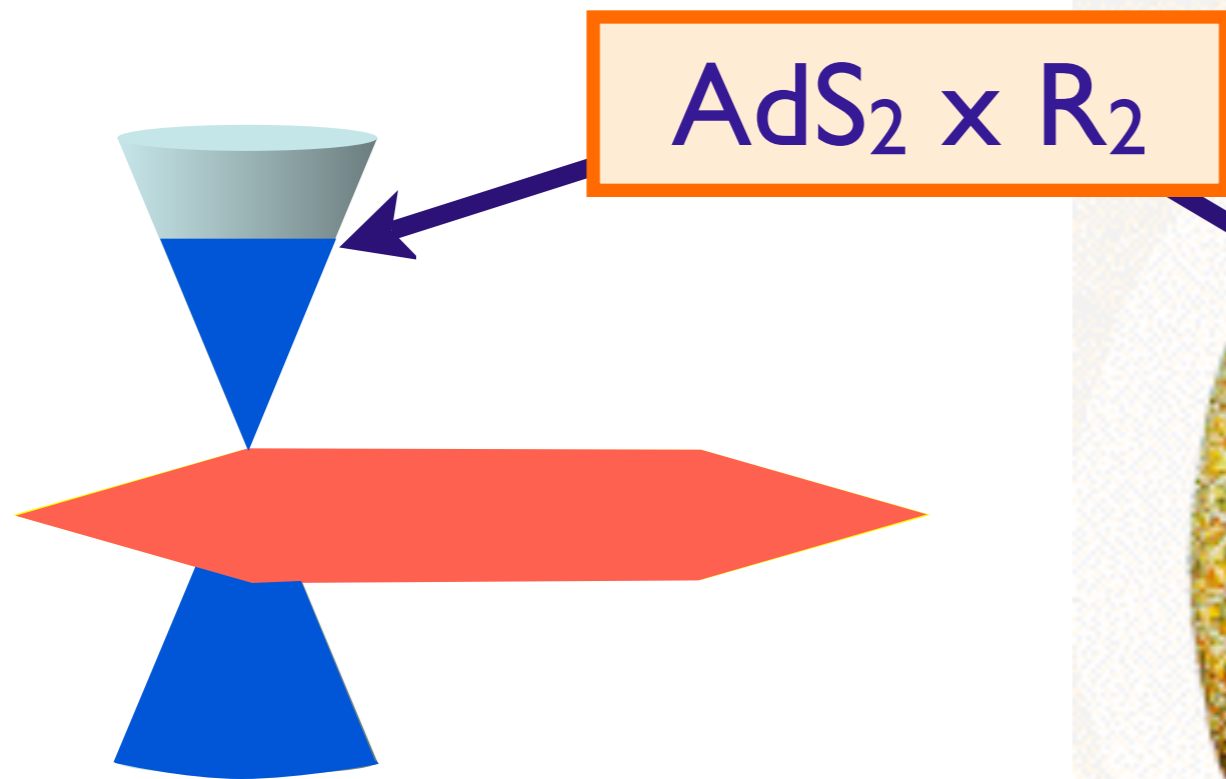
Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694



Geometric interpretation of RG flow

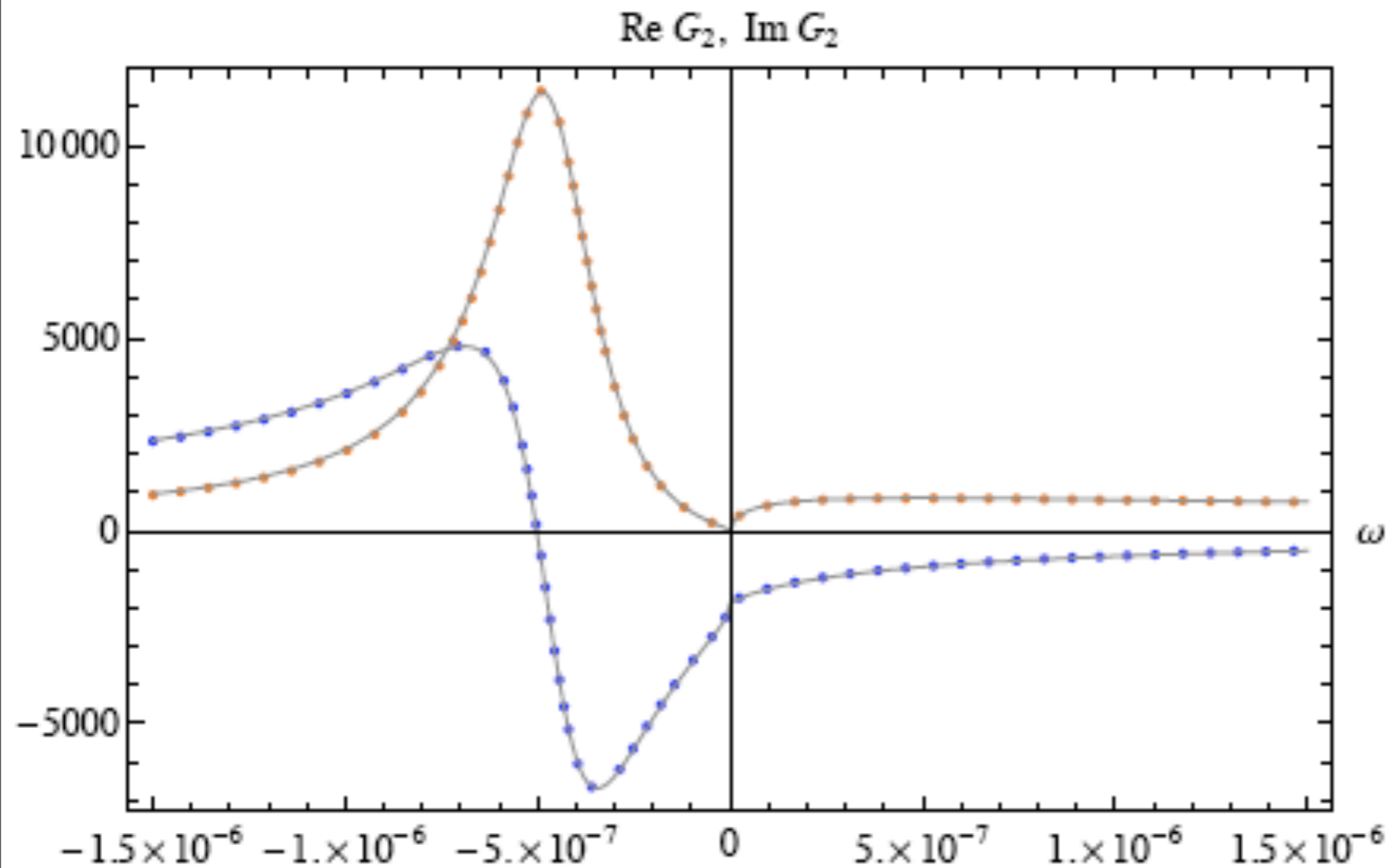
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694



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Green's function of a fermion

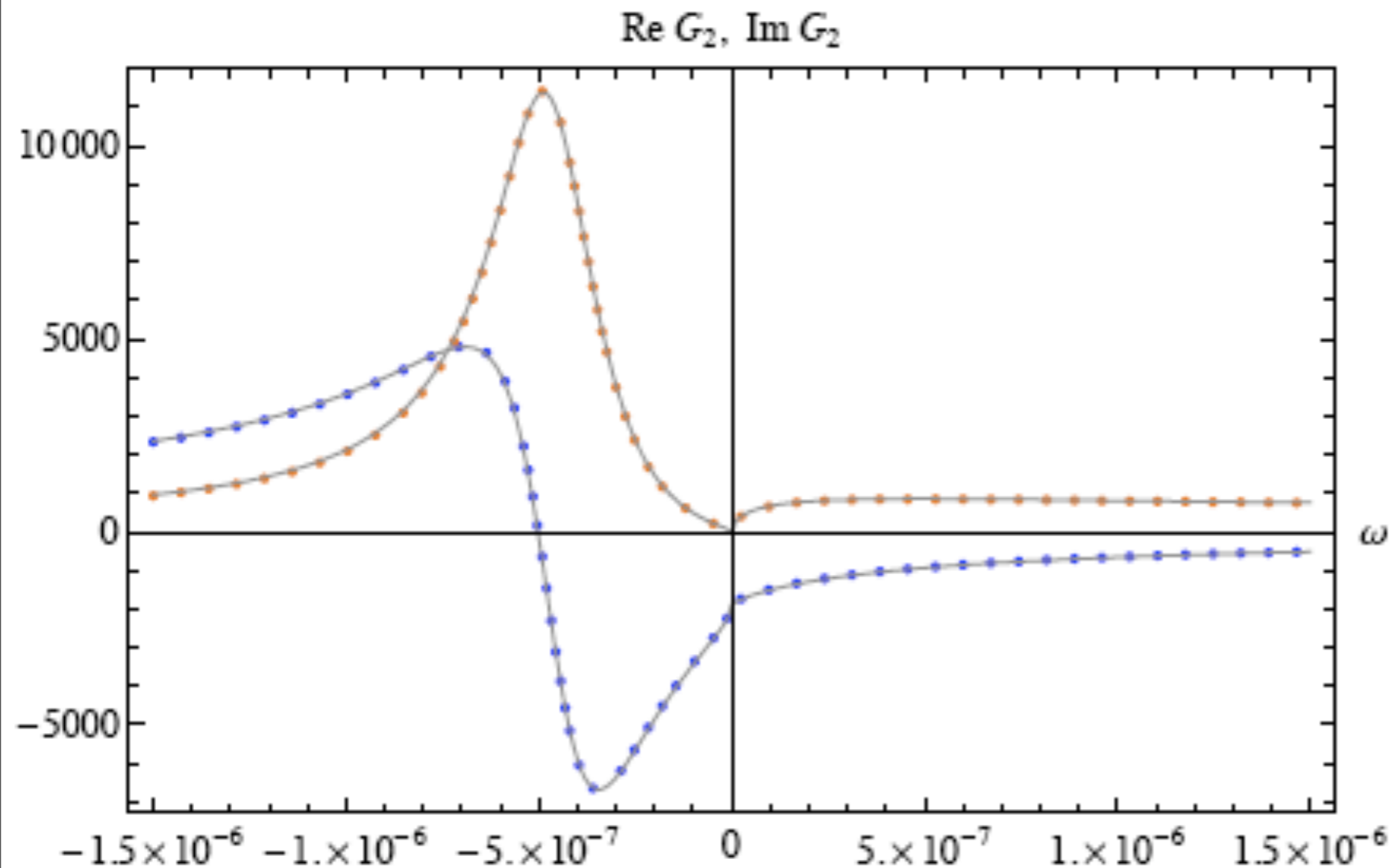


T. Faulkner, H. Liu,
J. McGreevy, and
D. Vegh,
arXiv:0907.2694

$$G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega\theta(k)}$$

See also M. Cubrovic, J Zaanen, and K. Schalm, arXiv:0904.1993

Green's function of a fermion



T. Faulkner, H. Liu,
J. McGreevy, and
D. Vegh,
arXiv:0907.2694

$$G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

Similar to non-Fermi liquid theories of Fermi surfaces coupled to gauge fields, and at quantum critical points

Free energy from gravity theory

The free energy is expressed as a sum over the “quasinormal frequencies”, z_ℓ , of the black hole. Here ℓ represents any set of quantum numbers:

$$\mathcal{F}_{\text{boson}} = -T \sum_{\ell} \ln \left(\frac{|z_\ell|}{2\pi T} \left| \Gamma \left(\frac{iz_\ell}{2\pi T} \right) \right|^2 \right)$$
$$\mathcal{F}_{\text{fermion}} = T \sum_{\ell} \ln \left(\left| \Gamma \left(\frac{iz_\ell}{2\pi T} + \frac{1}{2} \right) \right|^2 \right)$$

Application of this formula shows that the fermions exhibit the dHvA quantum oscillations with expected period ($2\pi/(\text{Fermi surface area})$) in $1/B$, but with an amplitude corrected from the Fermi liquid formula of Lifshitz-Kosevich.

Outline

B. Finite density quantum matter

1. Graphene

Fermi surfaces and Fermi liquids

2. Quantum phase transitions of Fermi liquids

*Pomeranchuk instability and spin density waves;
Fermi surfaces and “non-Fermi liquids”*

3. AdS₂ theory

4. Cuprate superconductivity

Outline

B. Finite density quantum matter

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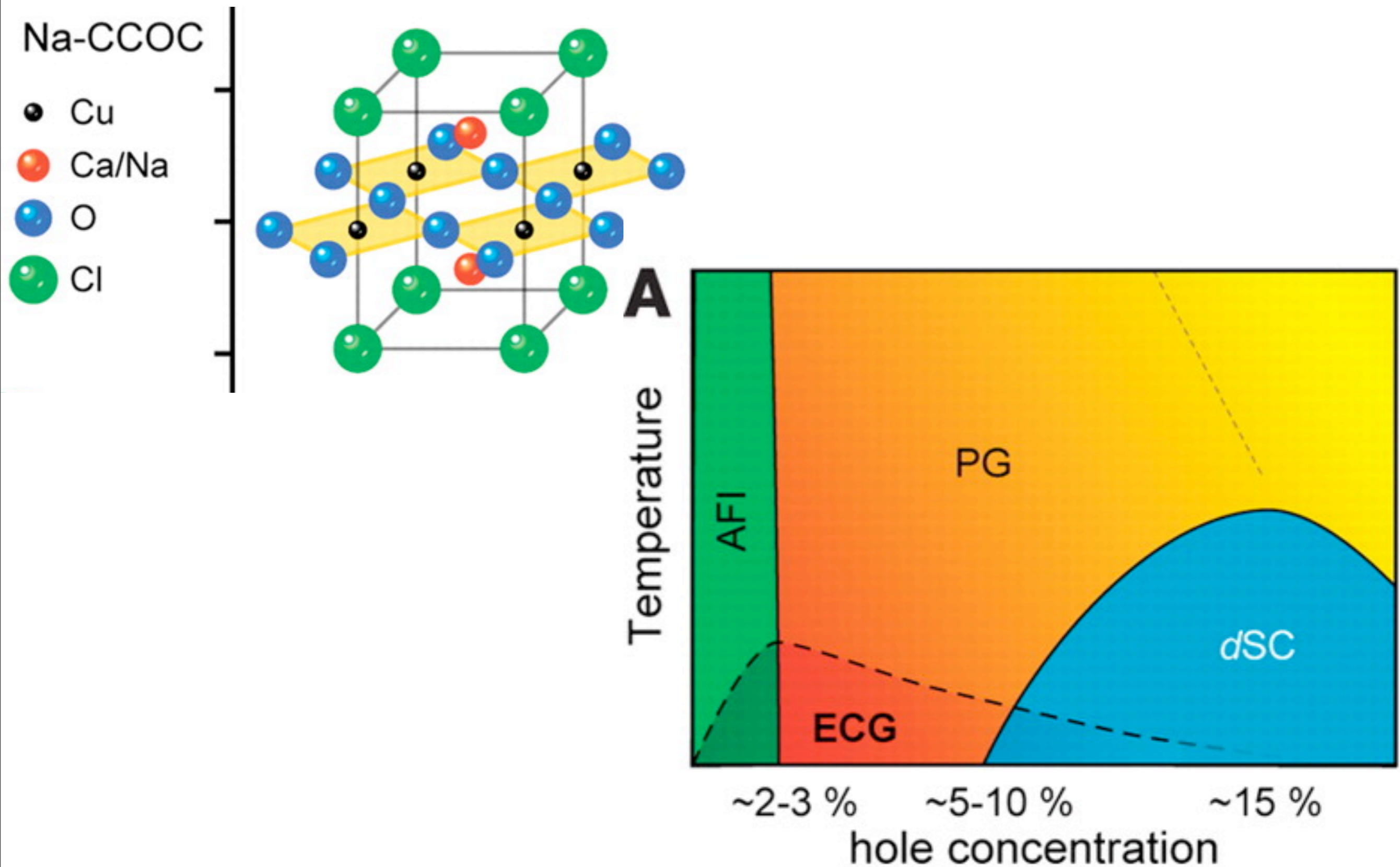
2. Quantum phase transitions of Fermi liquids

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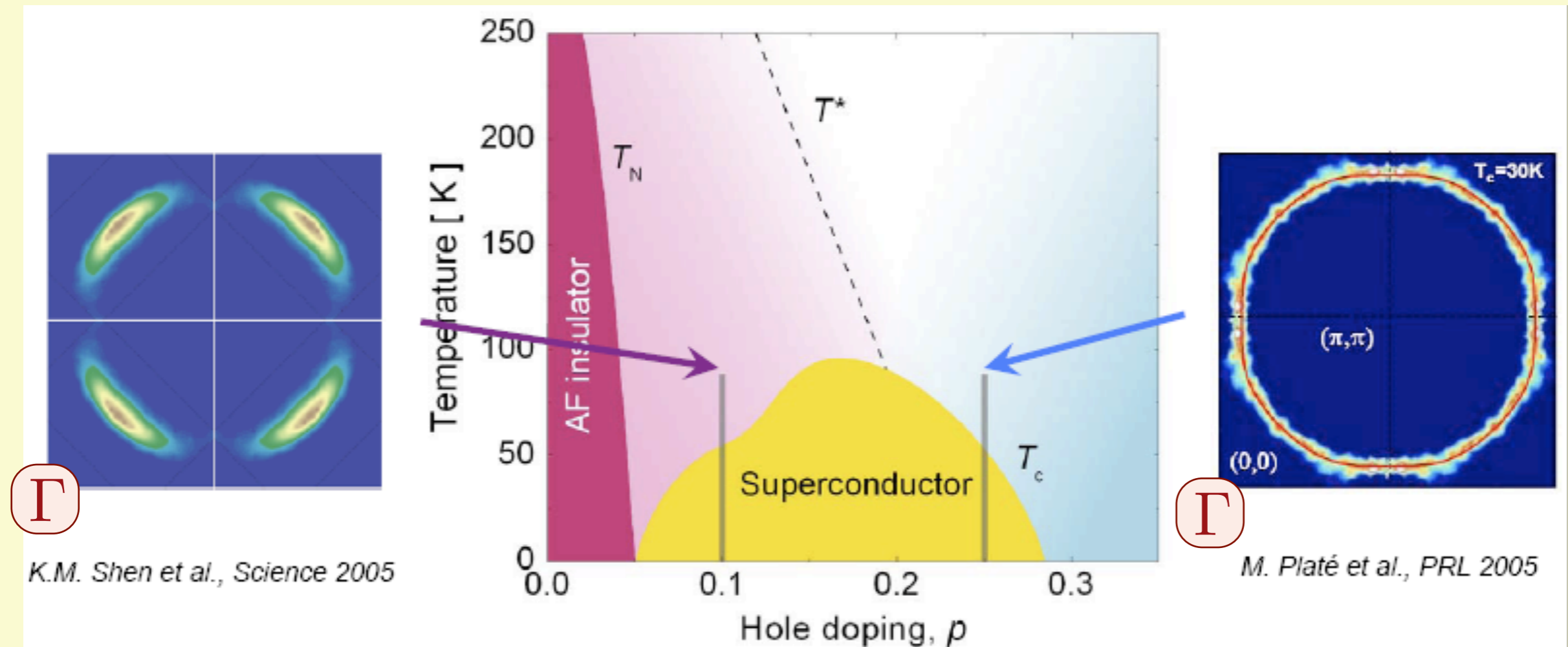
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The cuprate superconductors



Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



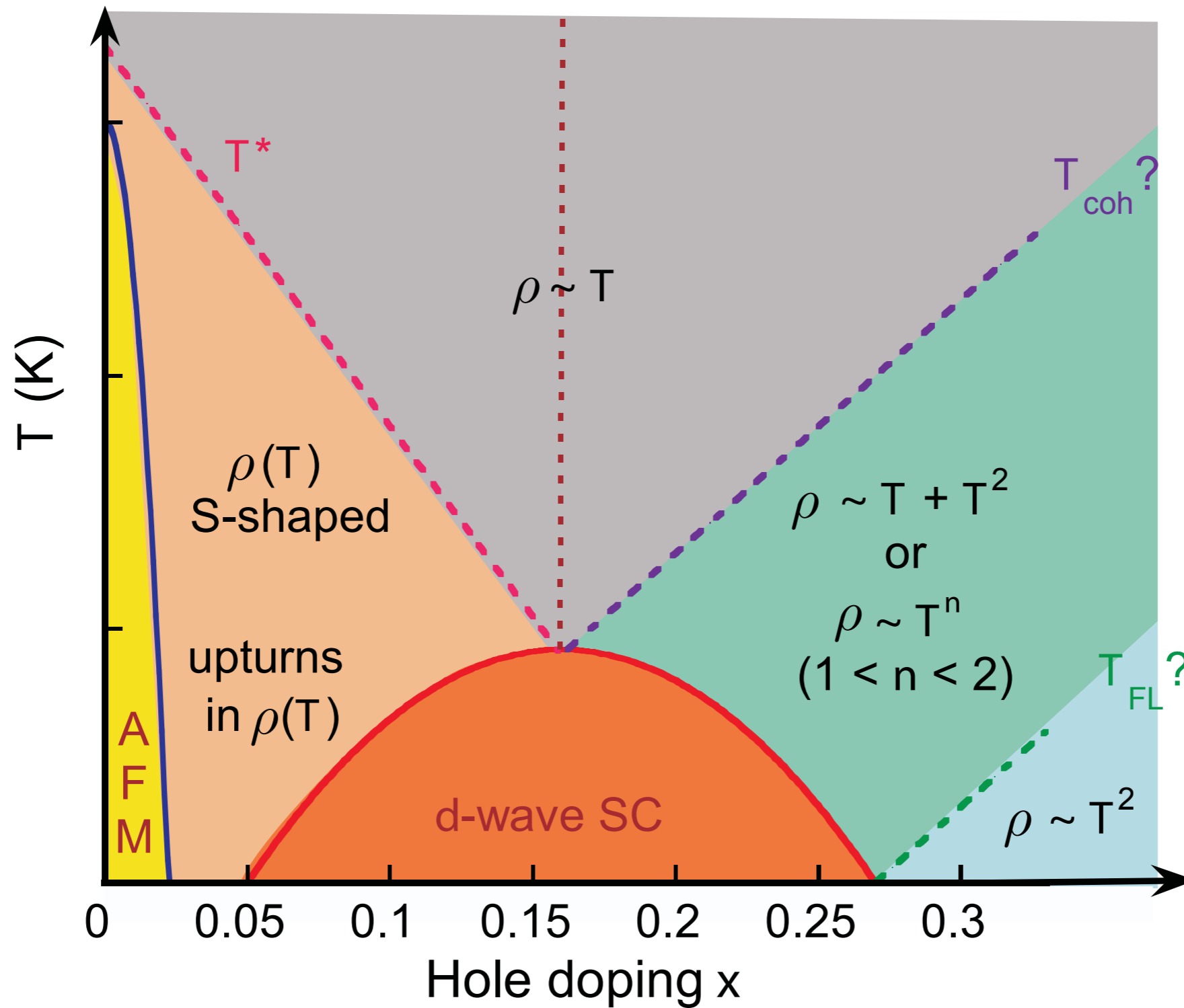
Smaller hole
Fermi-pockets

Large hole
Fermi surface

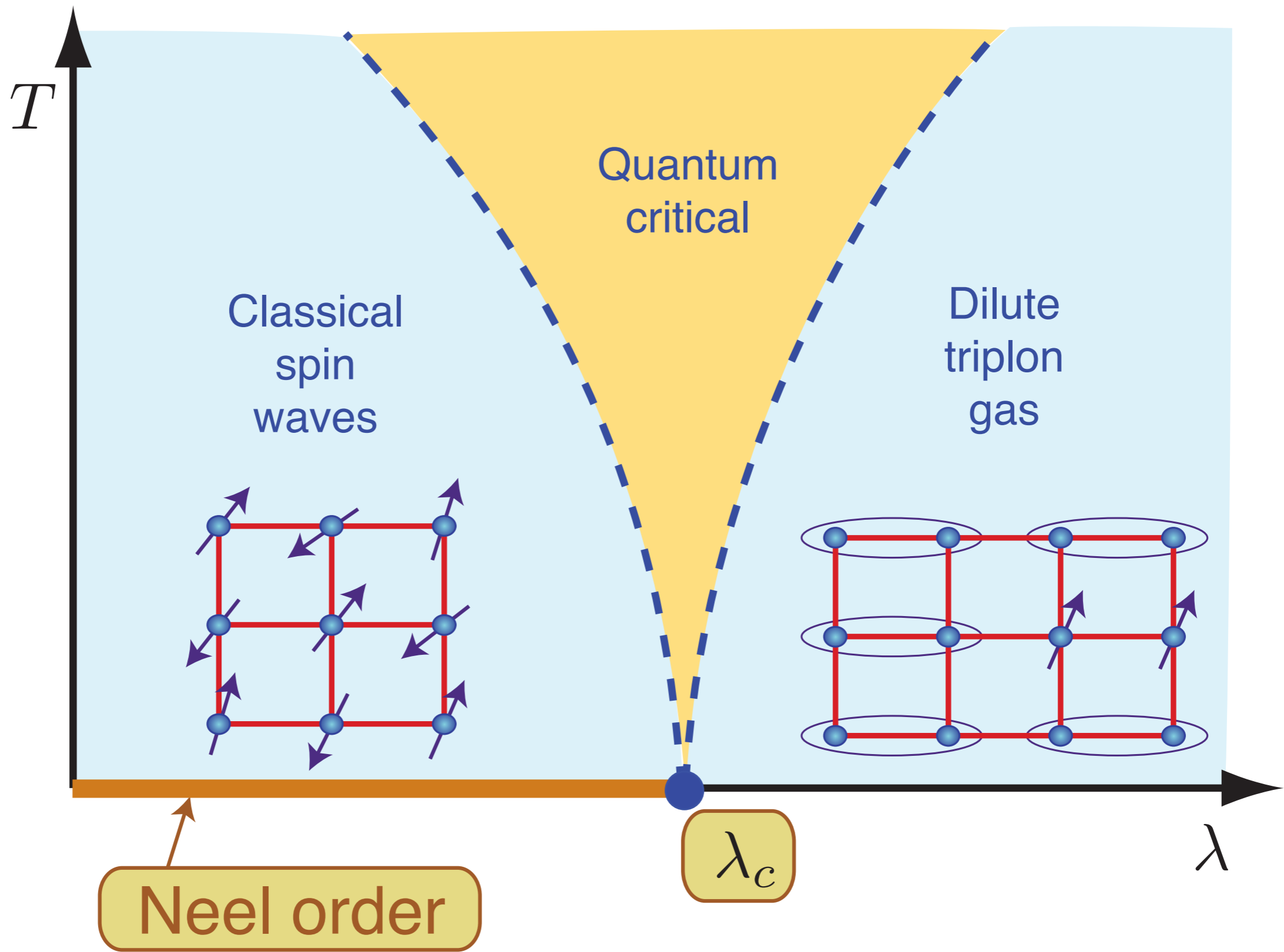
The cuprate superconductors

Multiple quantum phase transitions involving at least two order parameters (antiferromagnetism and superconductivity) and a topological change in the Fermi surface

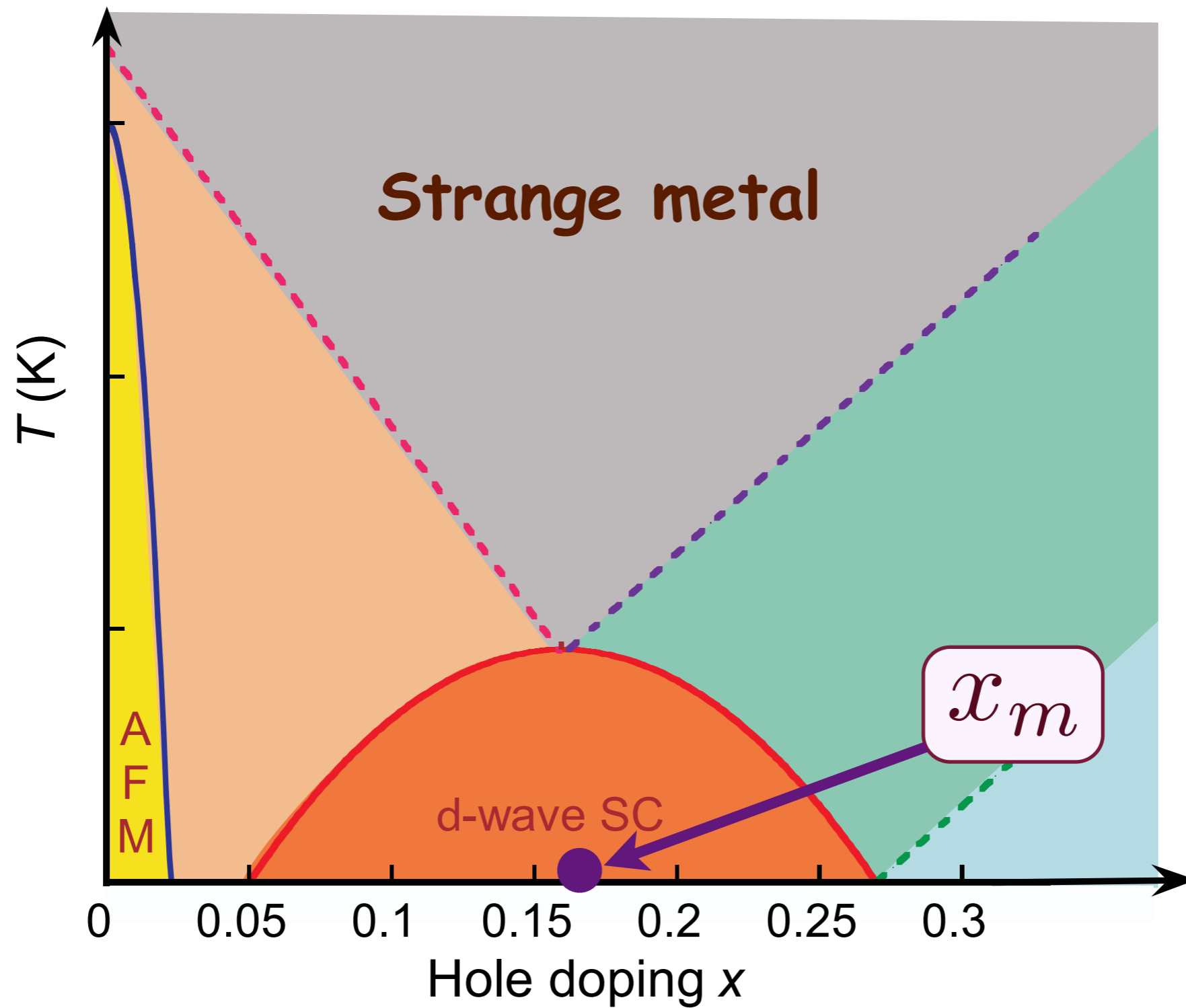
Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

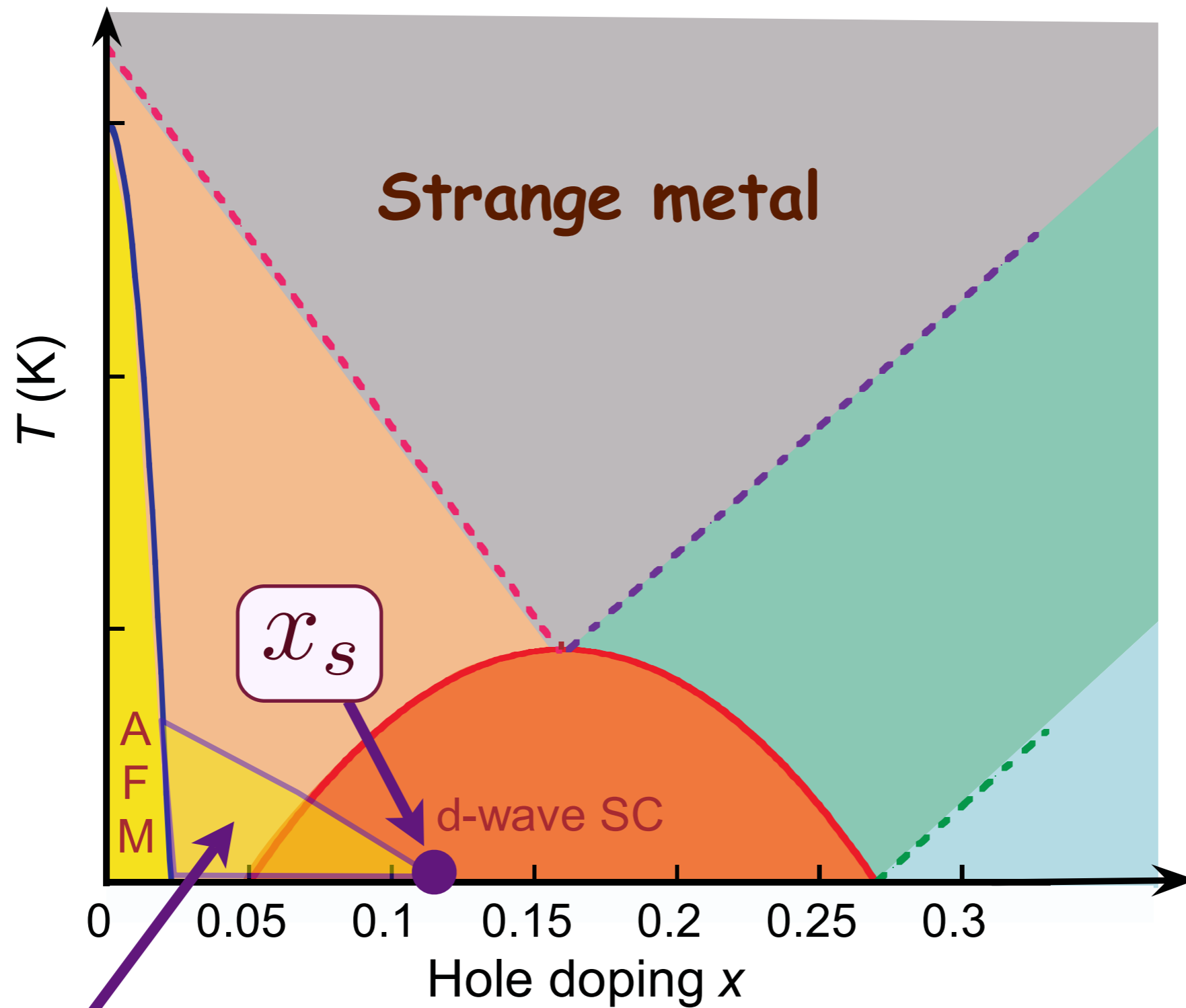


Crossovers in transport properties of hole-doped cuprates



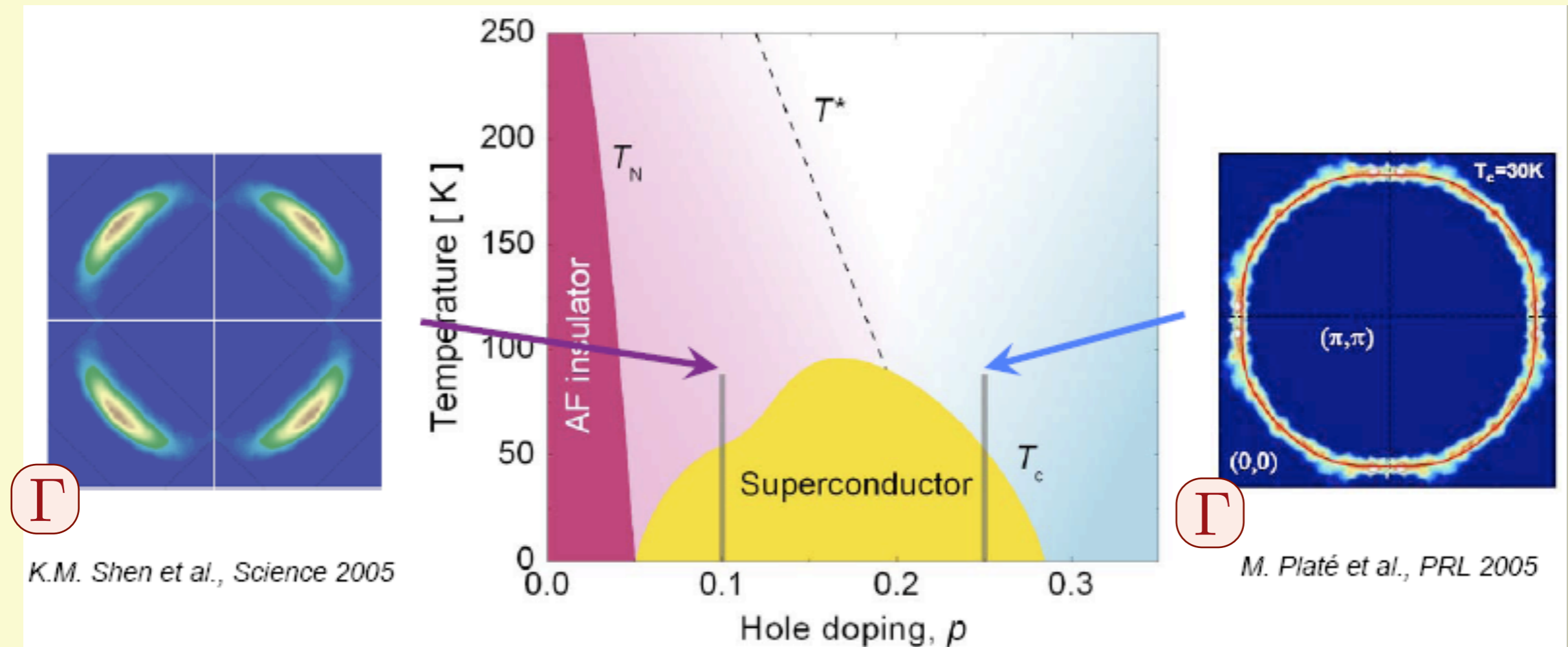
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

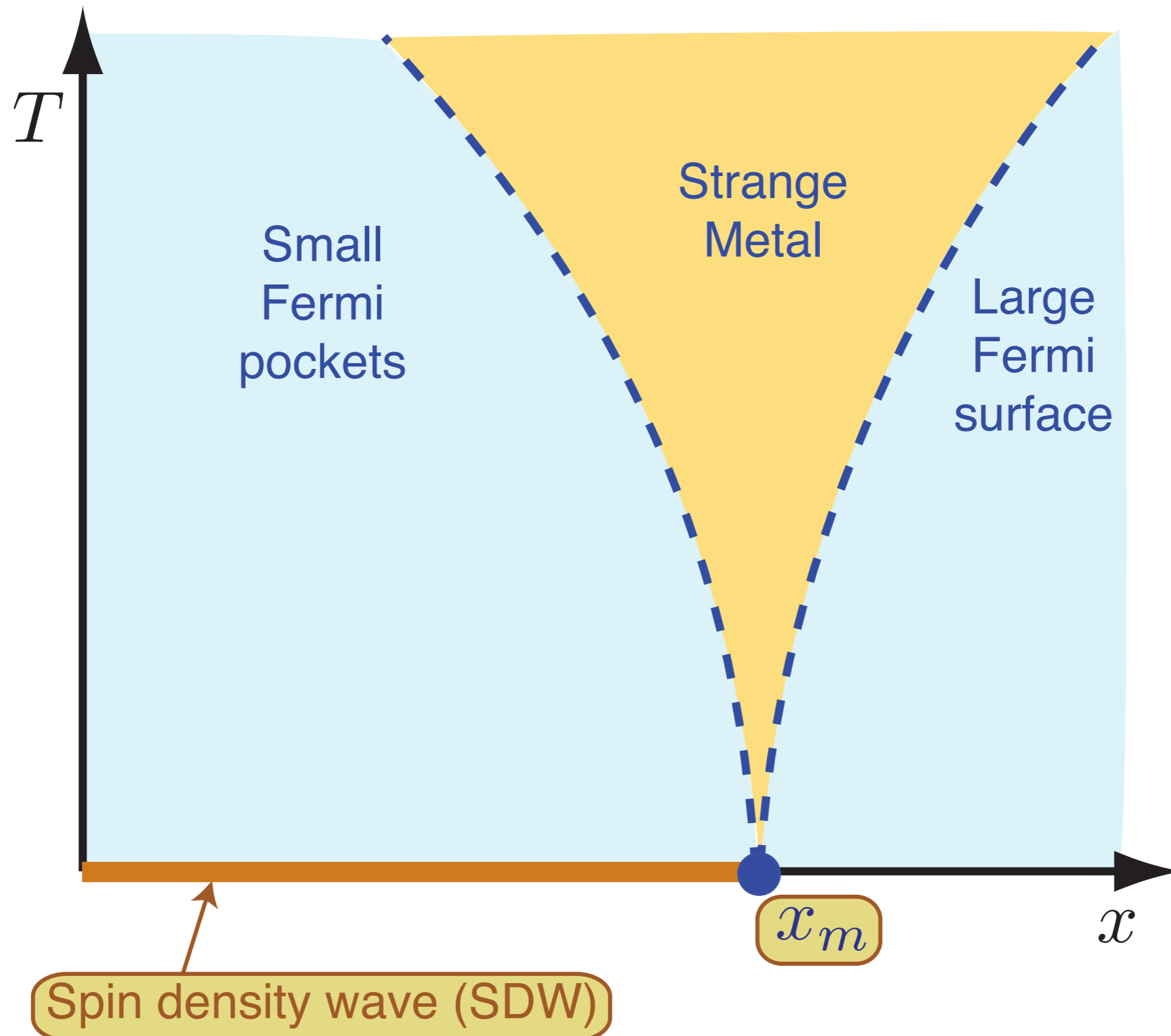
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole
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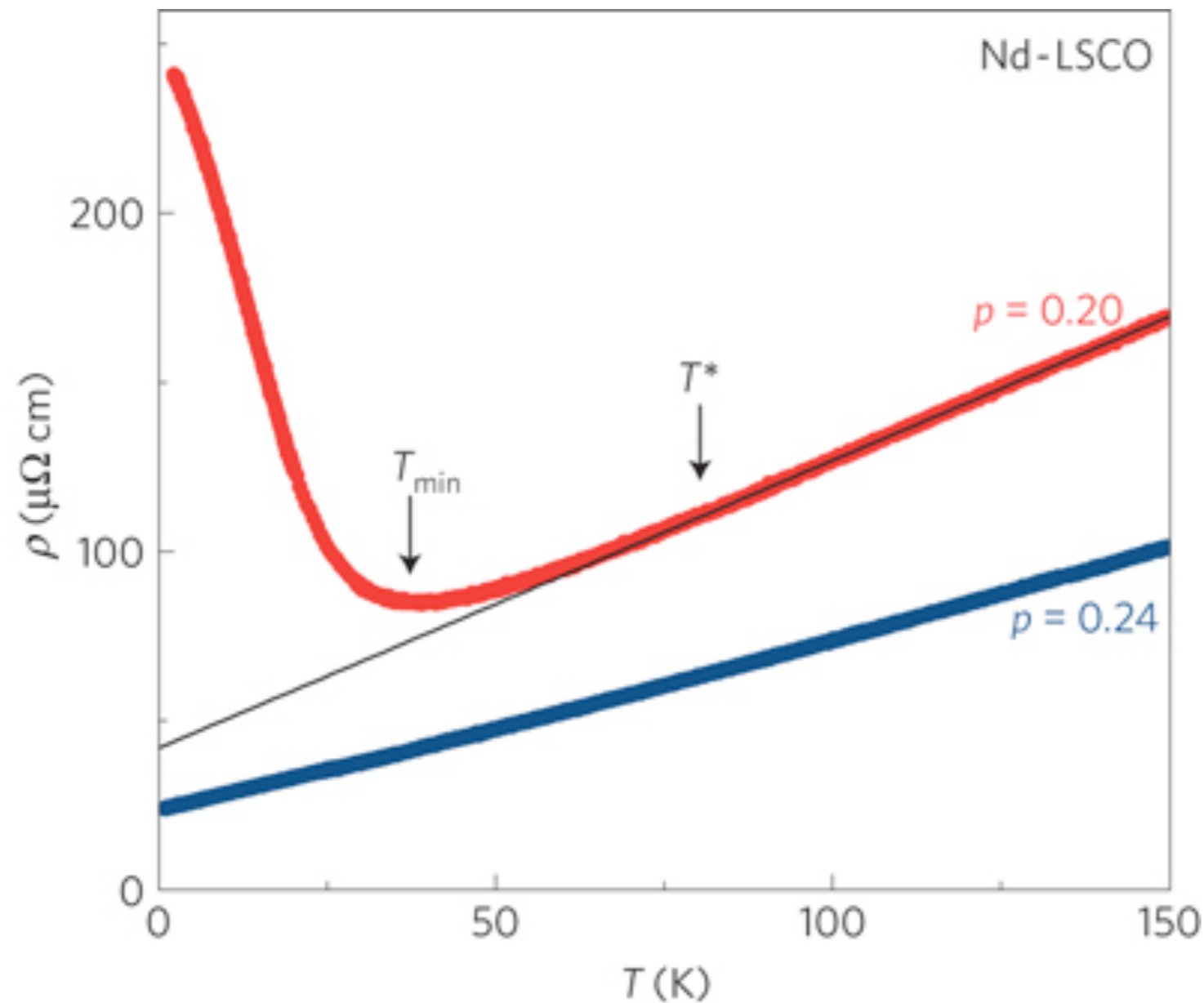
Large hole
Fermi surface

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c



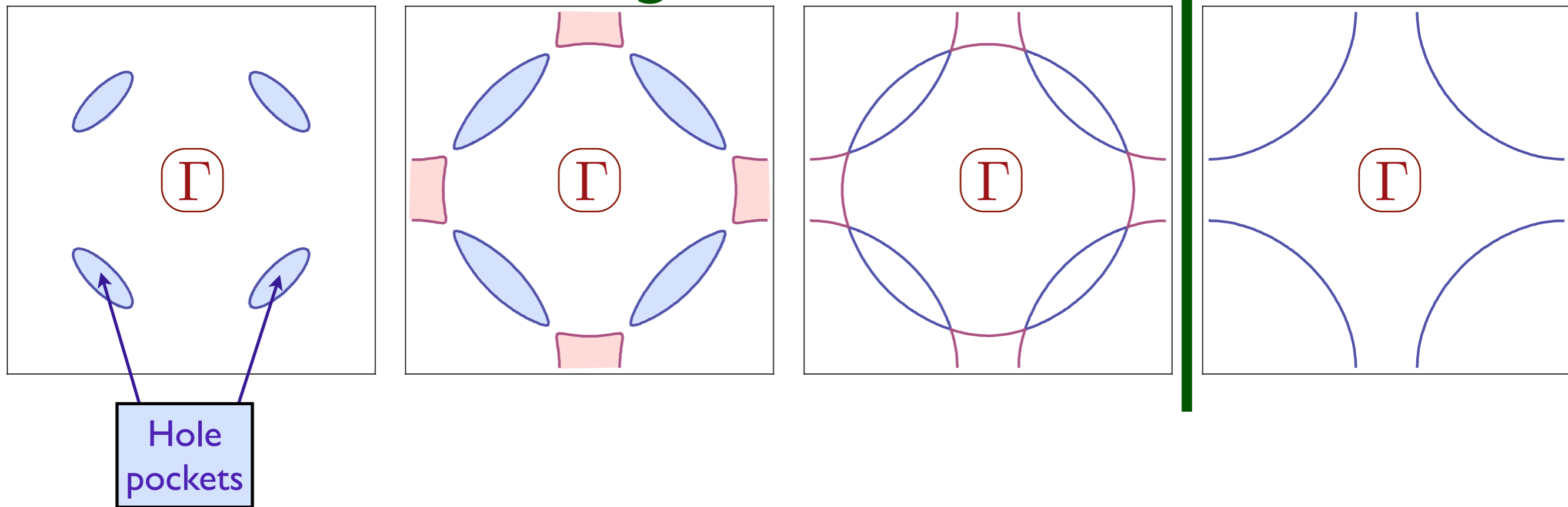
Magnetic field of
upto 35 T
used to suppress
superconductivity

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high- T_c superconductor

R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)

Spin density wave theory in hole-doped cuprates

← Increasing SDW order →

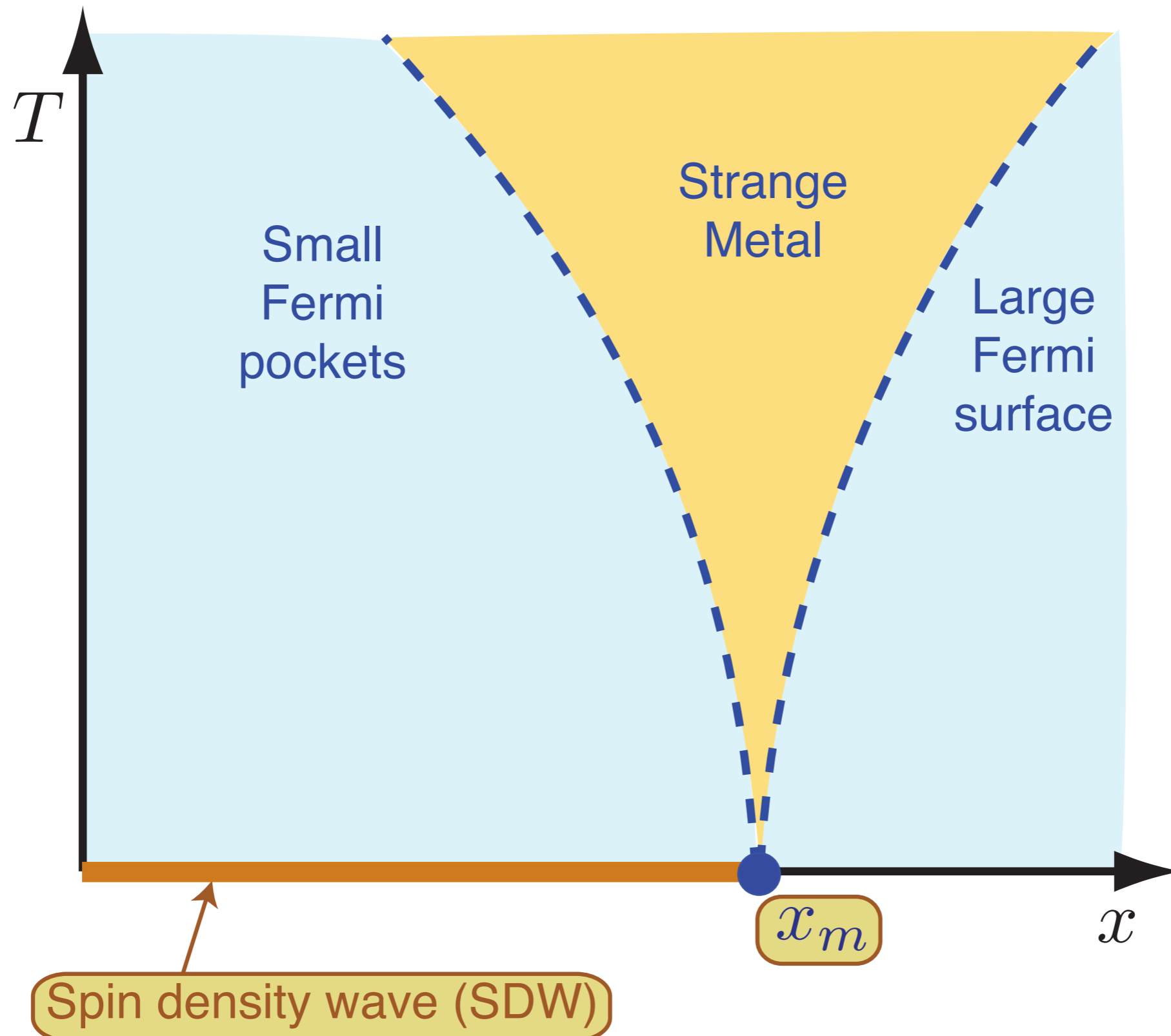


Quantum phase transition involves *both*
a SDW order parameter $\vec{\varphi}$,
and a topological change in the Fermi surface

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).

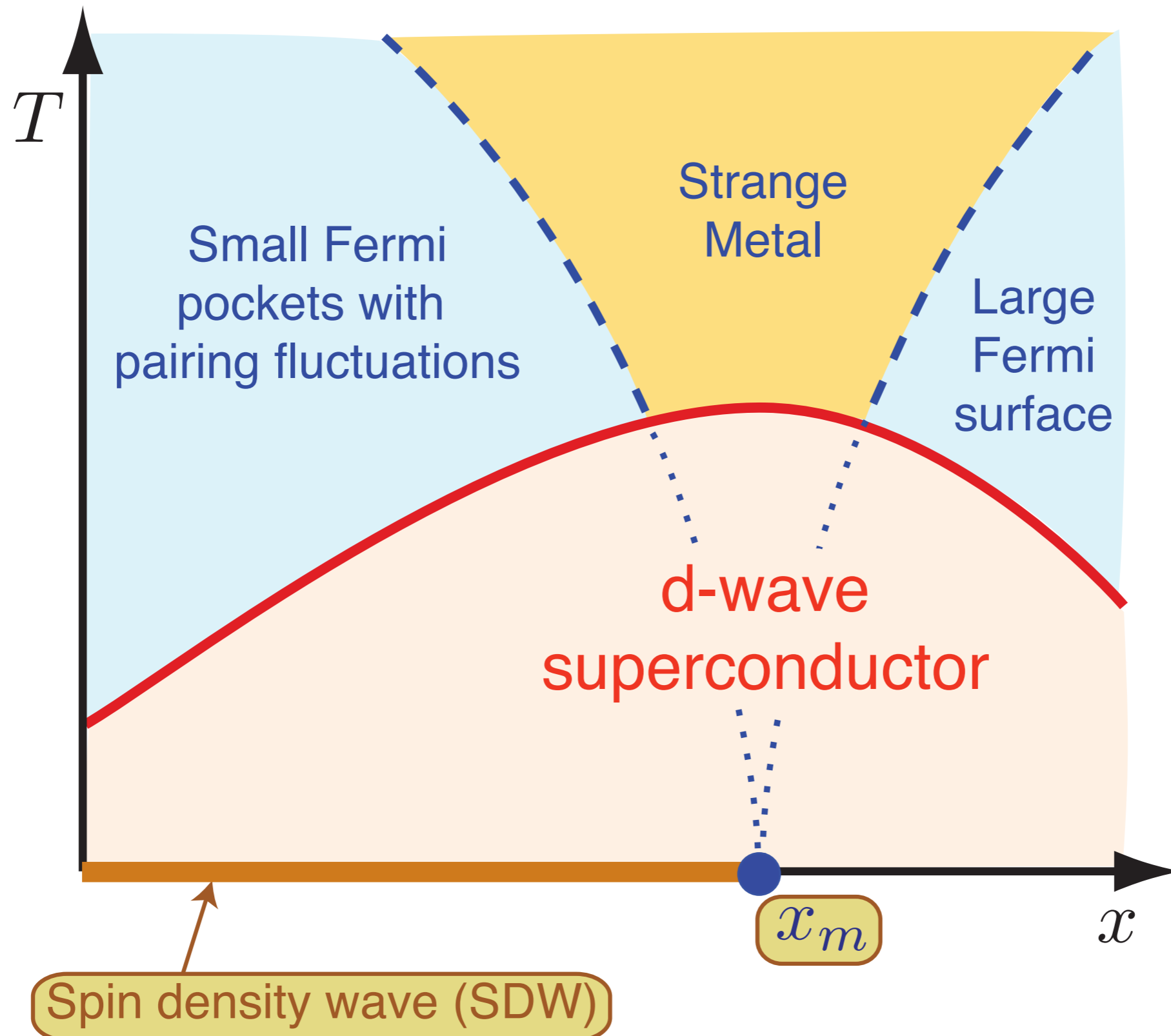
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory of quantum criticality in the cuprates



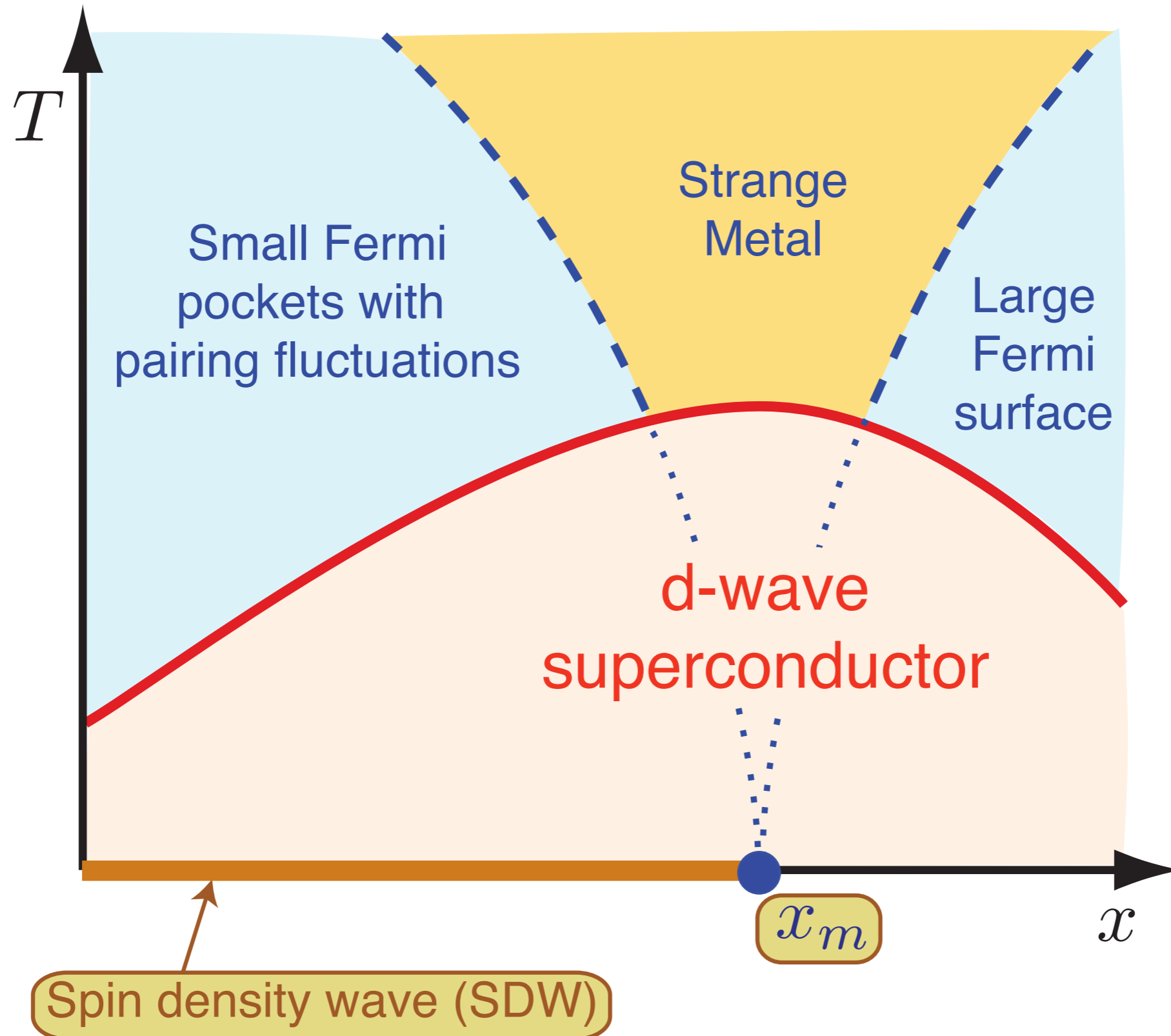
Underlying SDW ordering quantum critical point
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Theory of quantum criticality in the cuprates



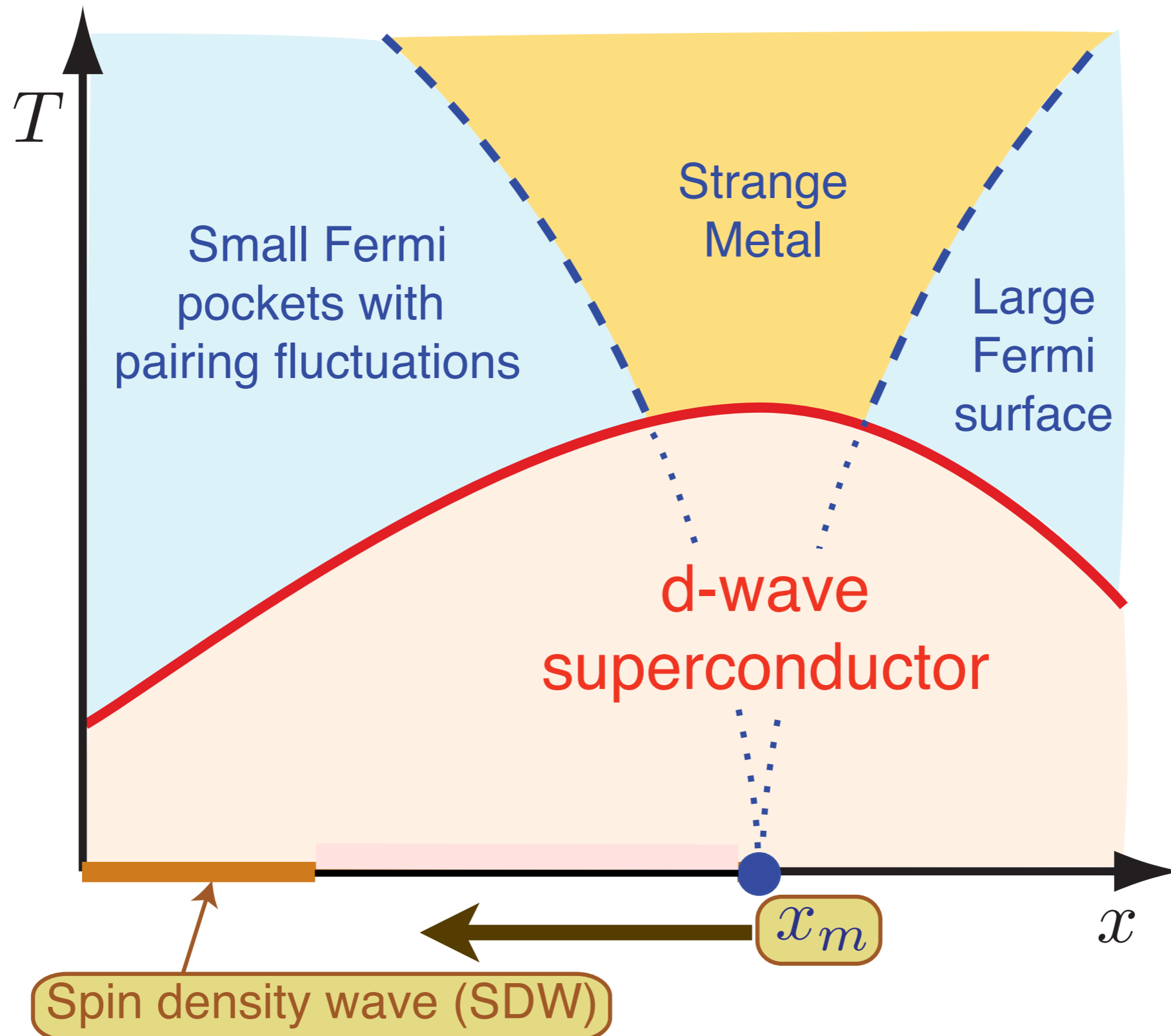
Onset of d -wave superconductivity
hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



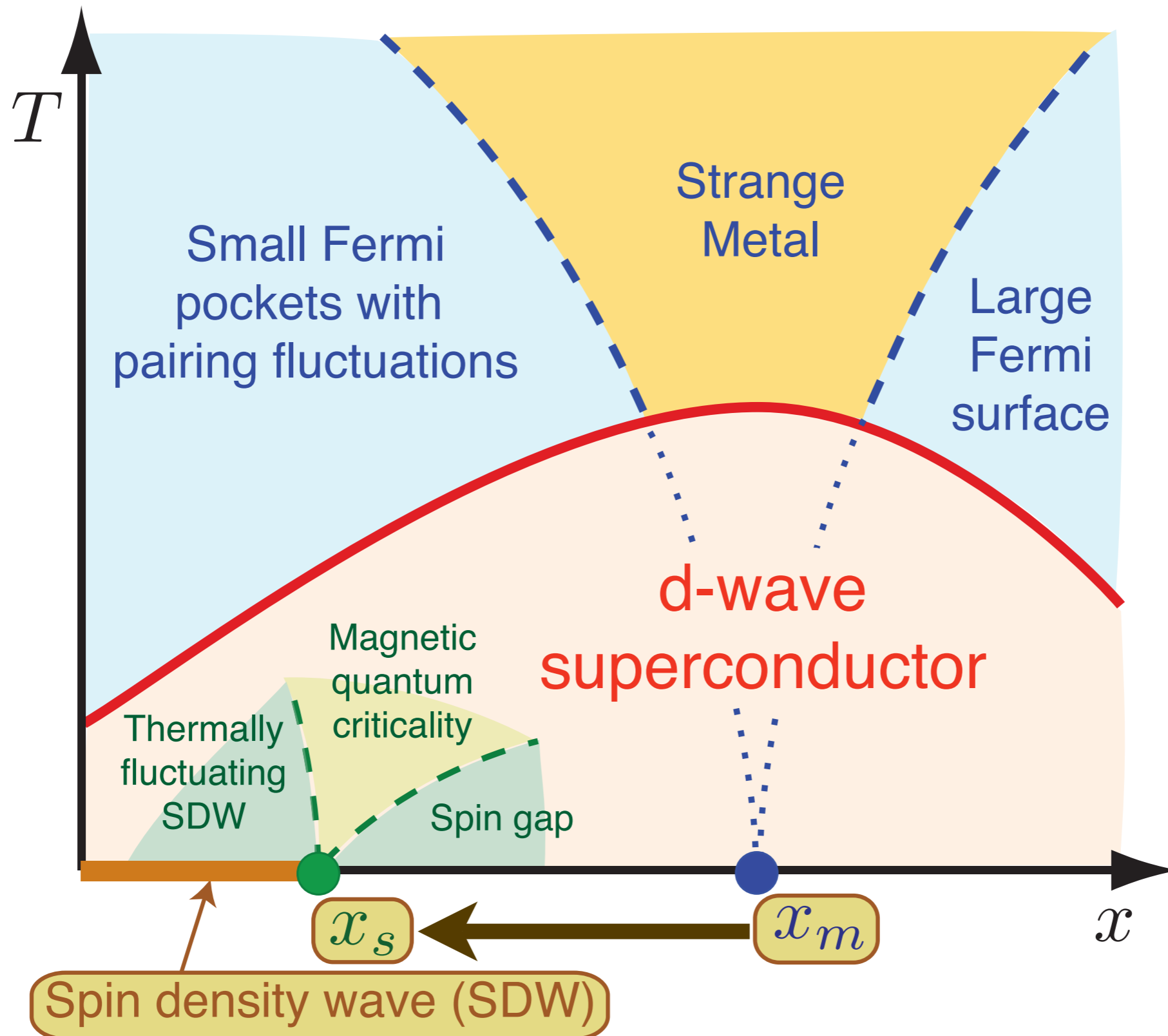
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



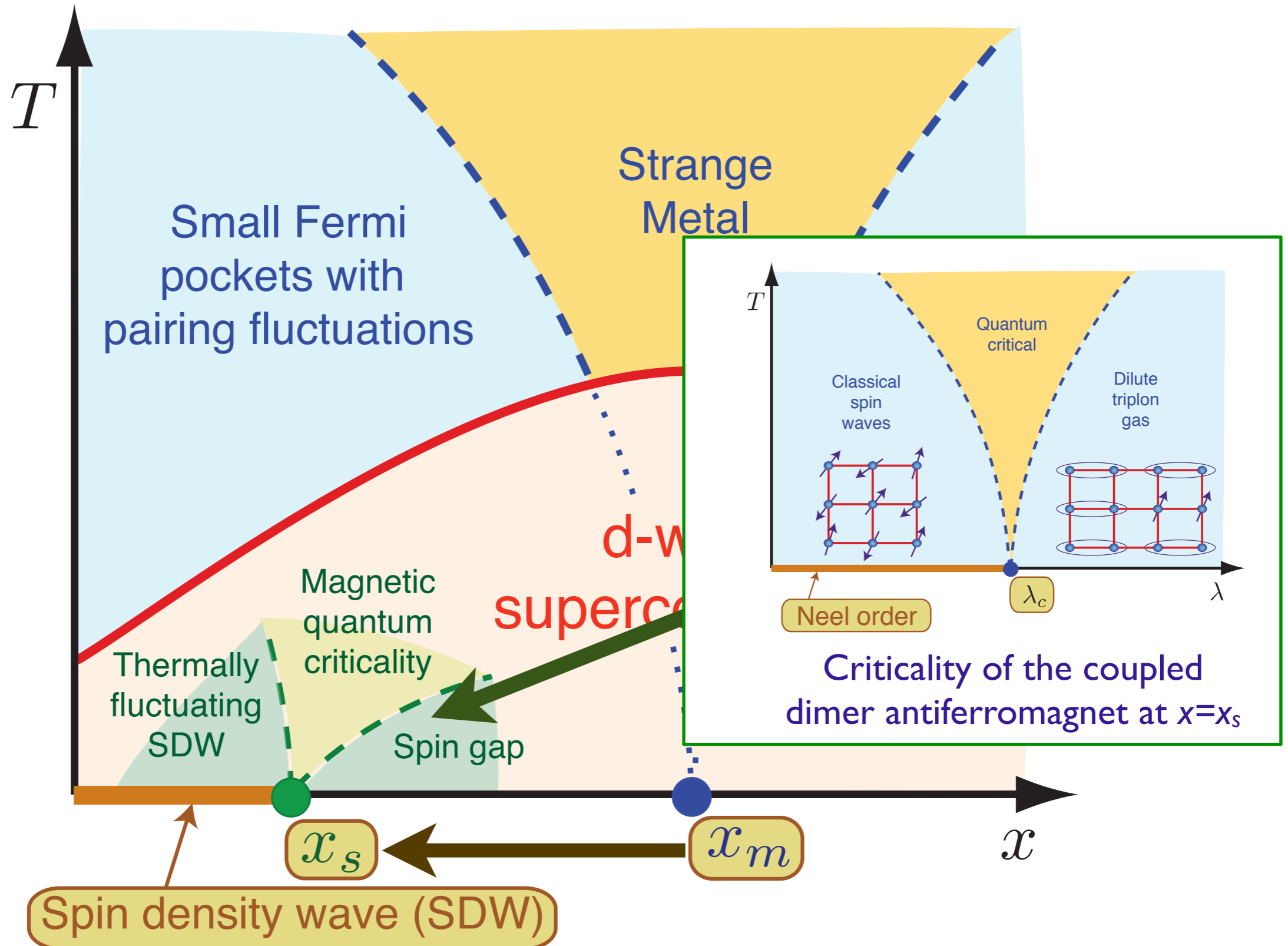
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Theory of quantum criticality in the cuprates



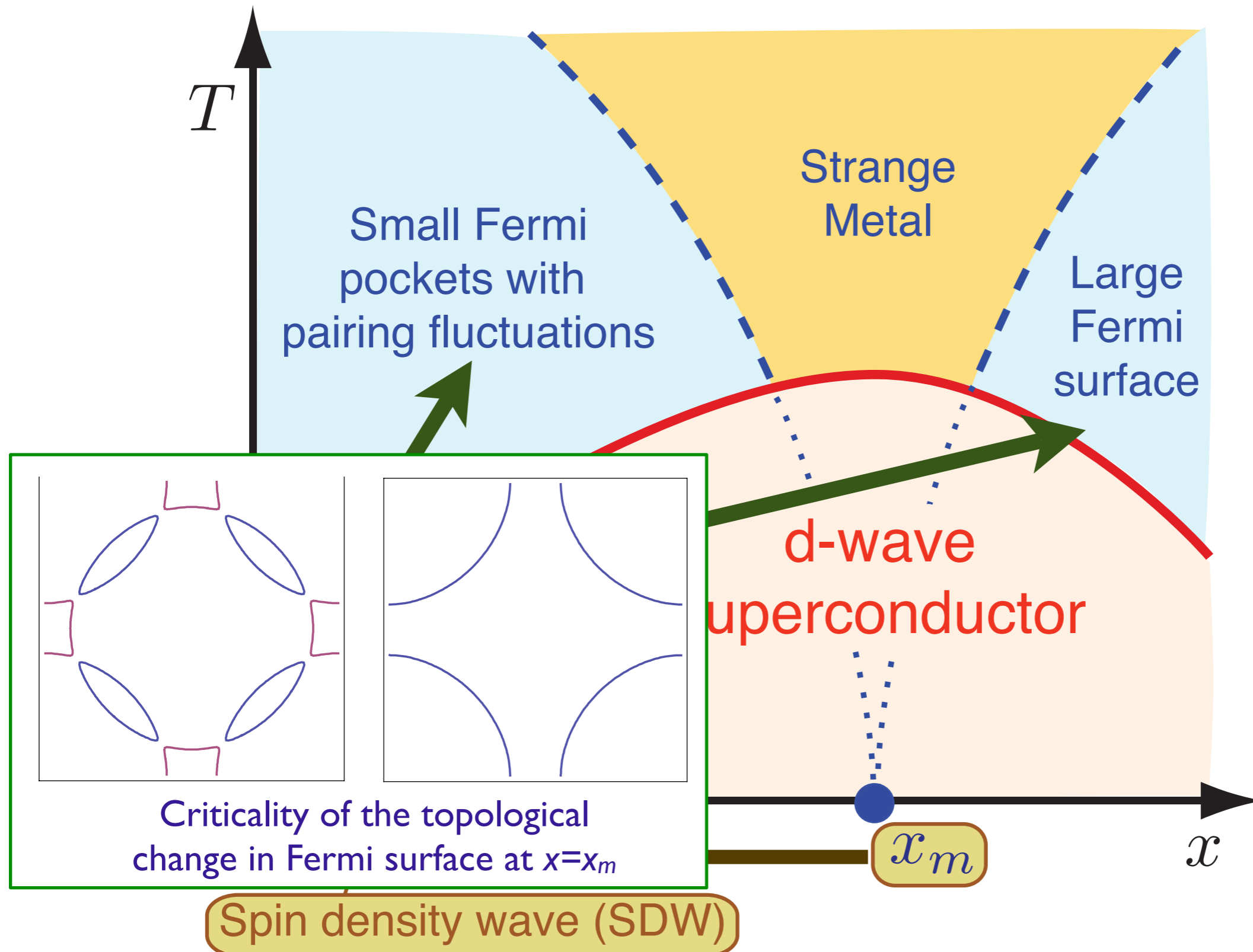
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Theory of quantum criticality in the cuprates

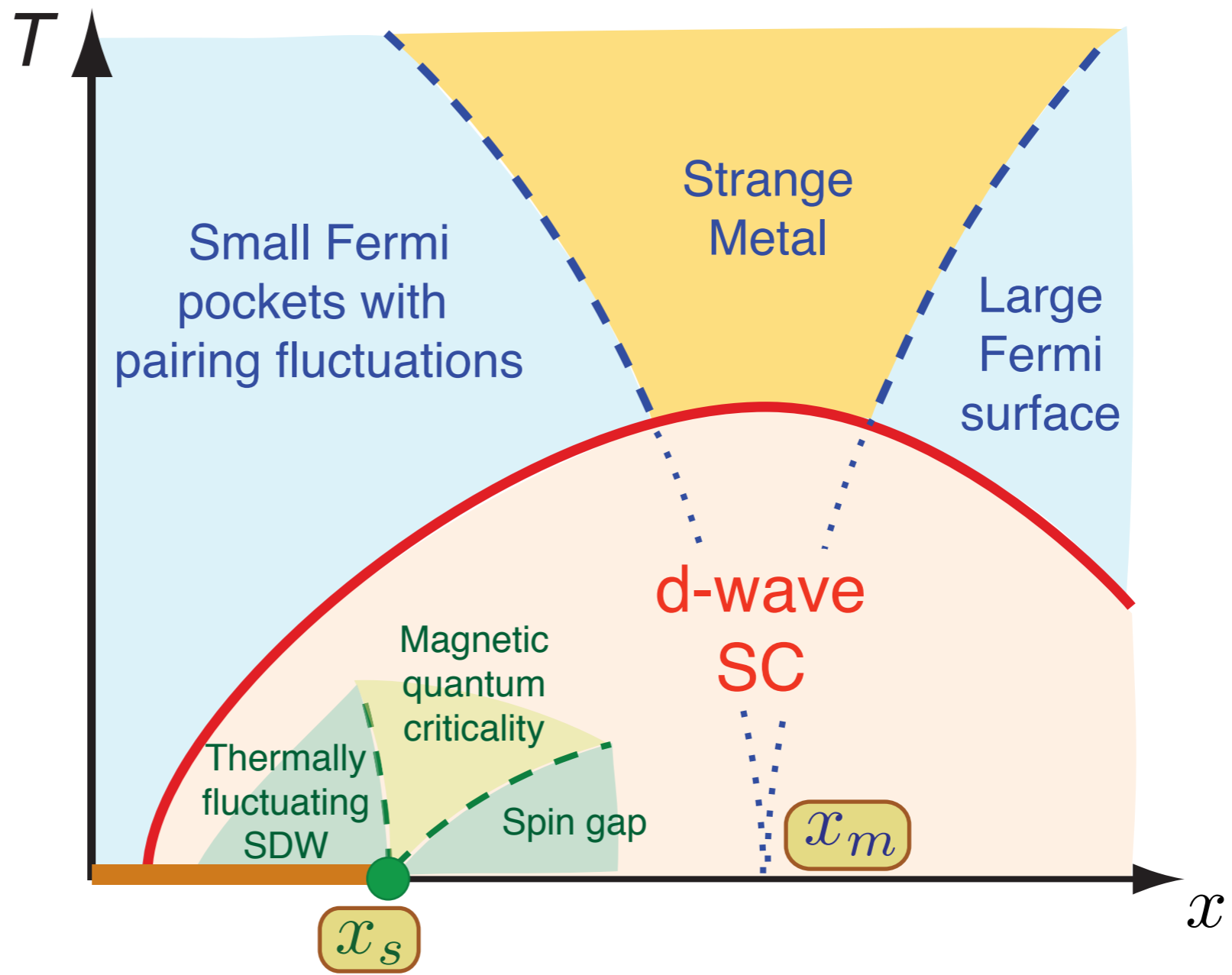


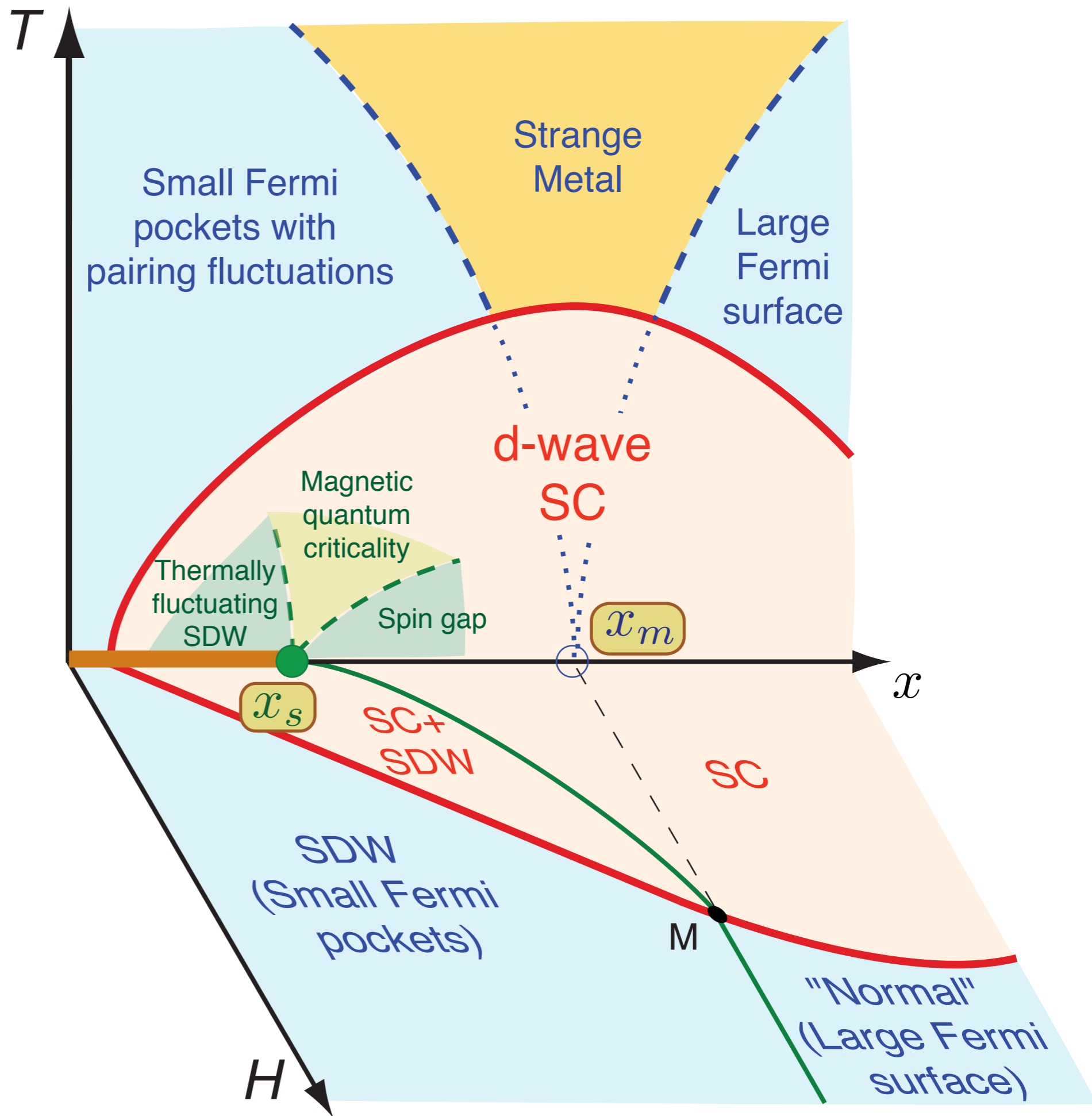
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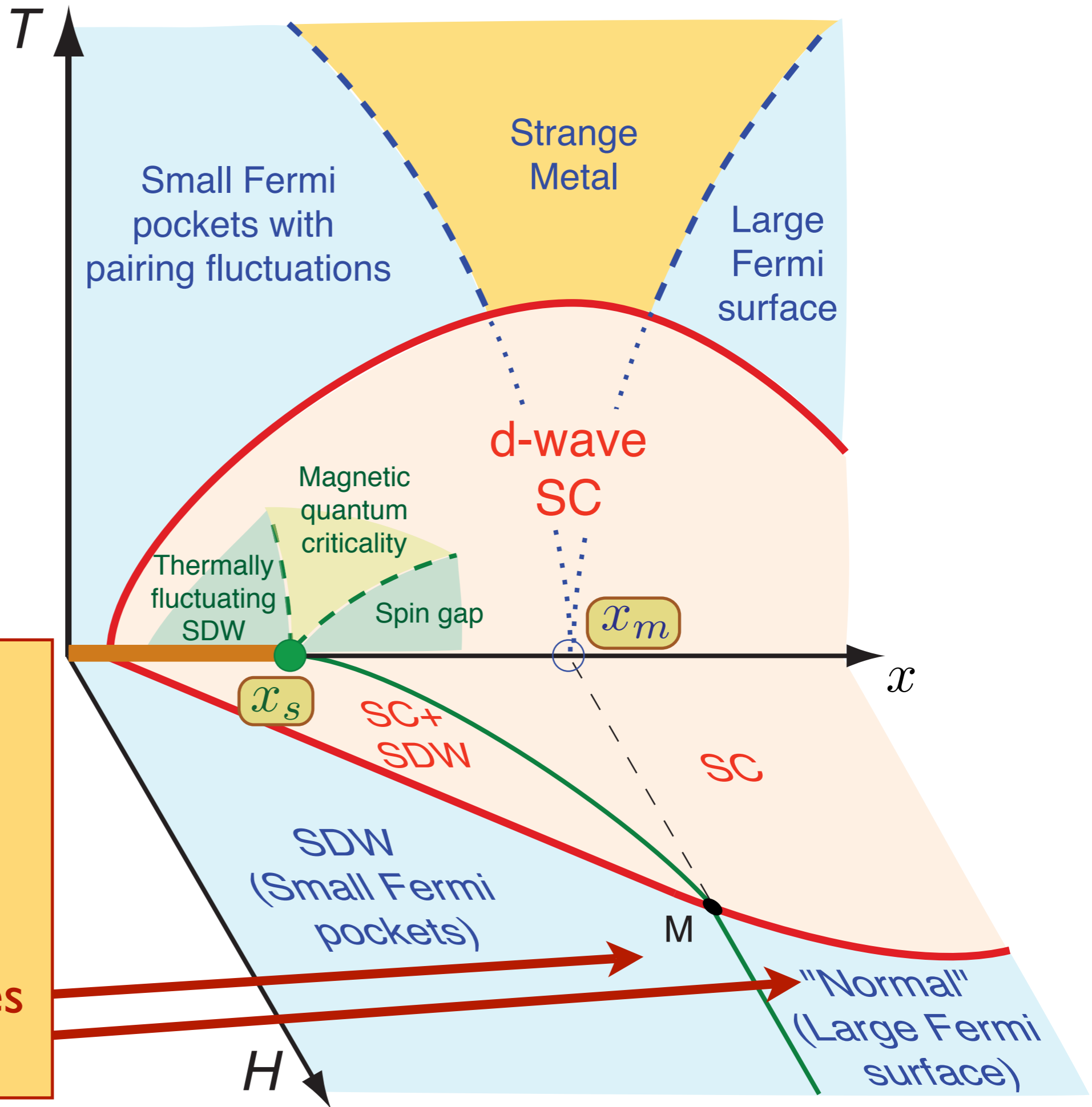
Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

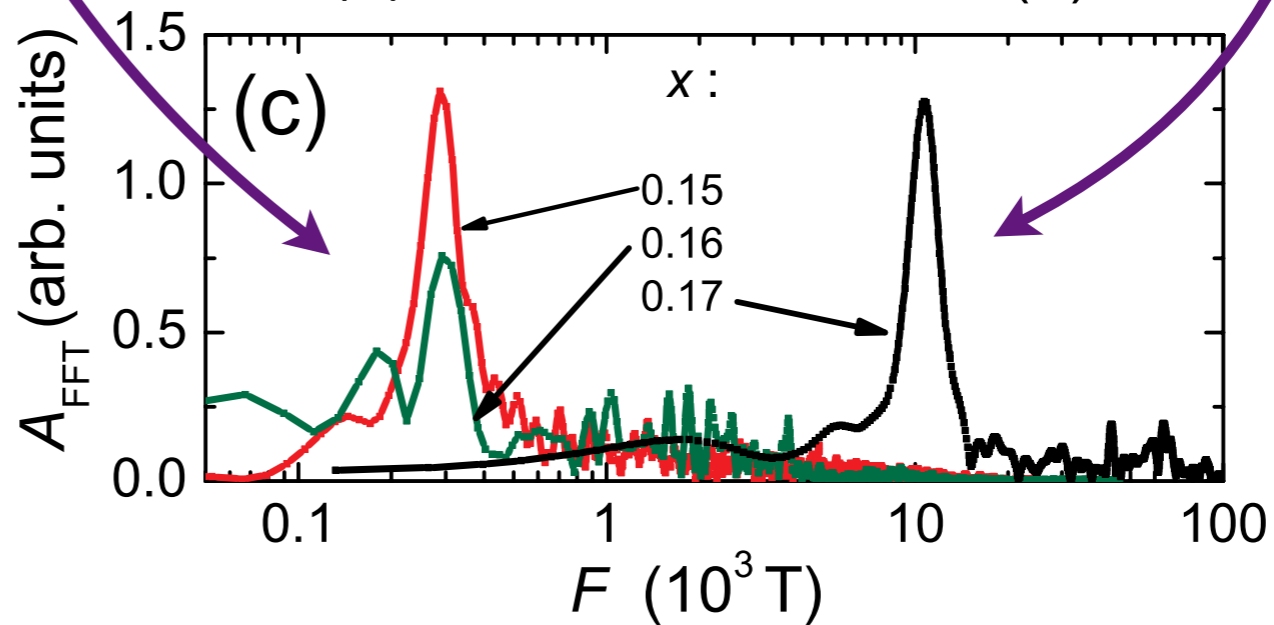
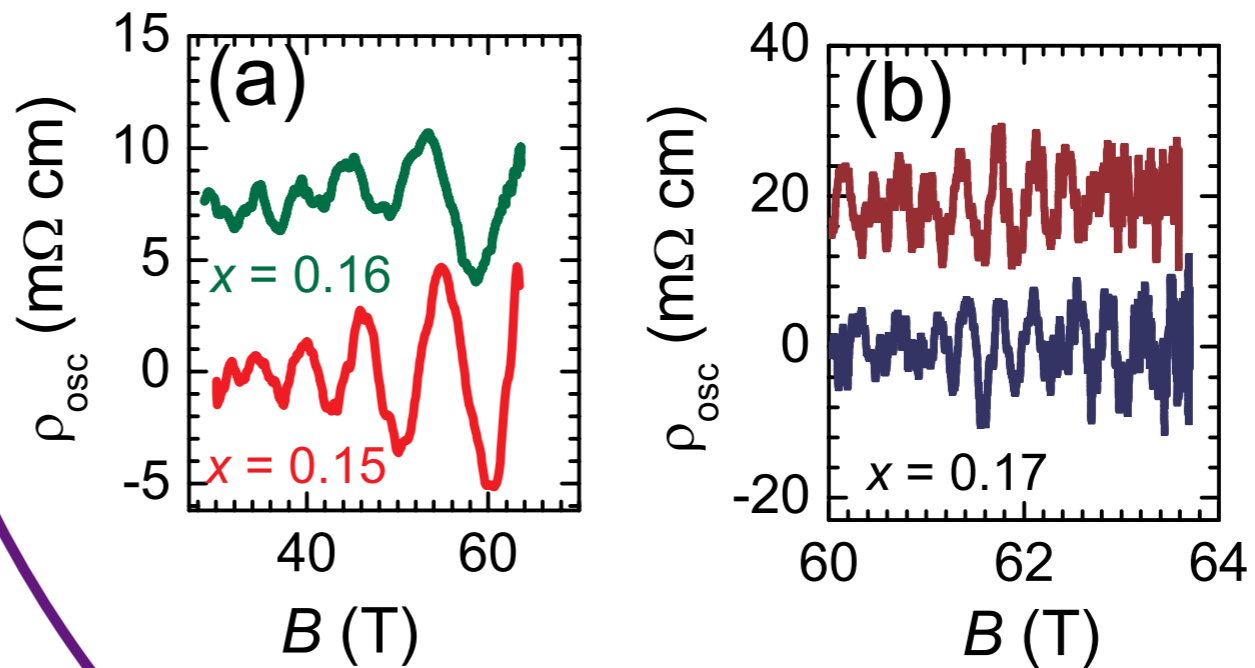
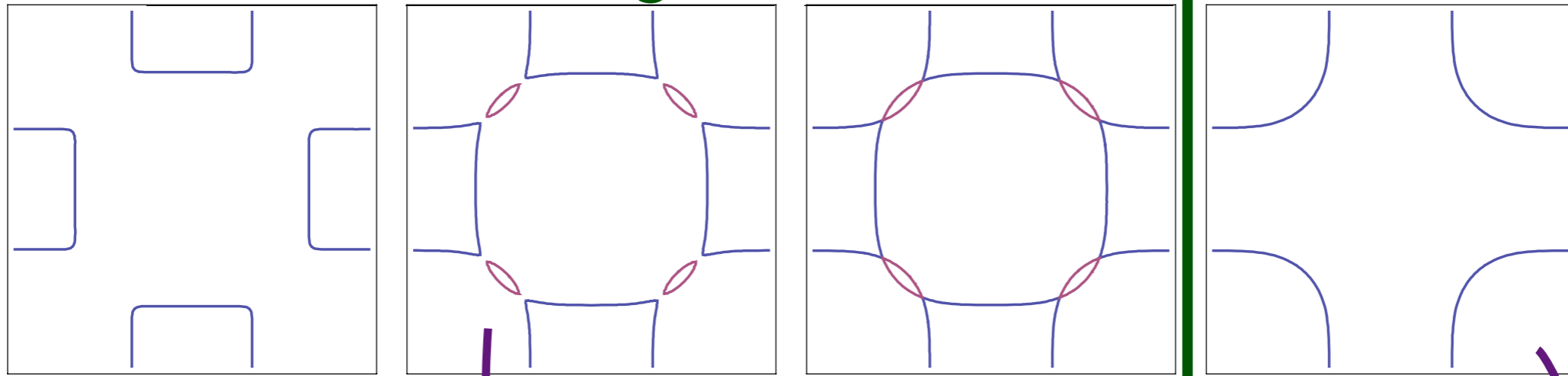






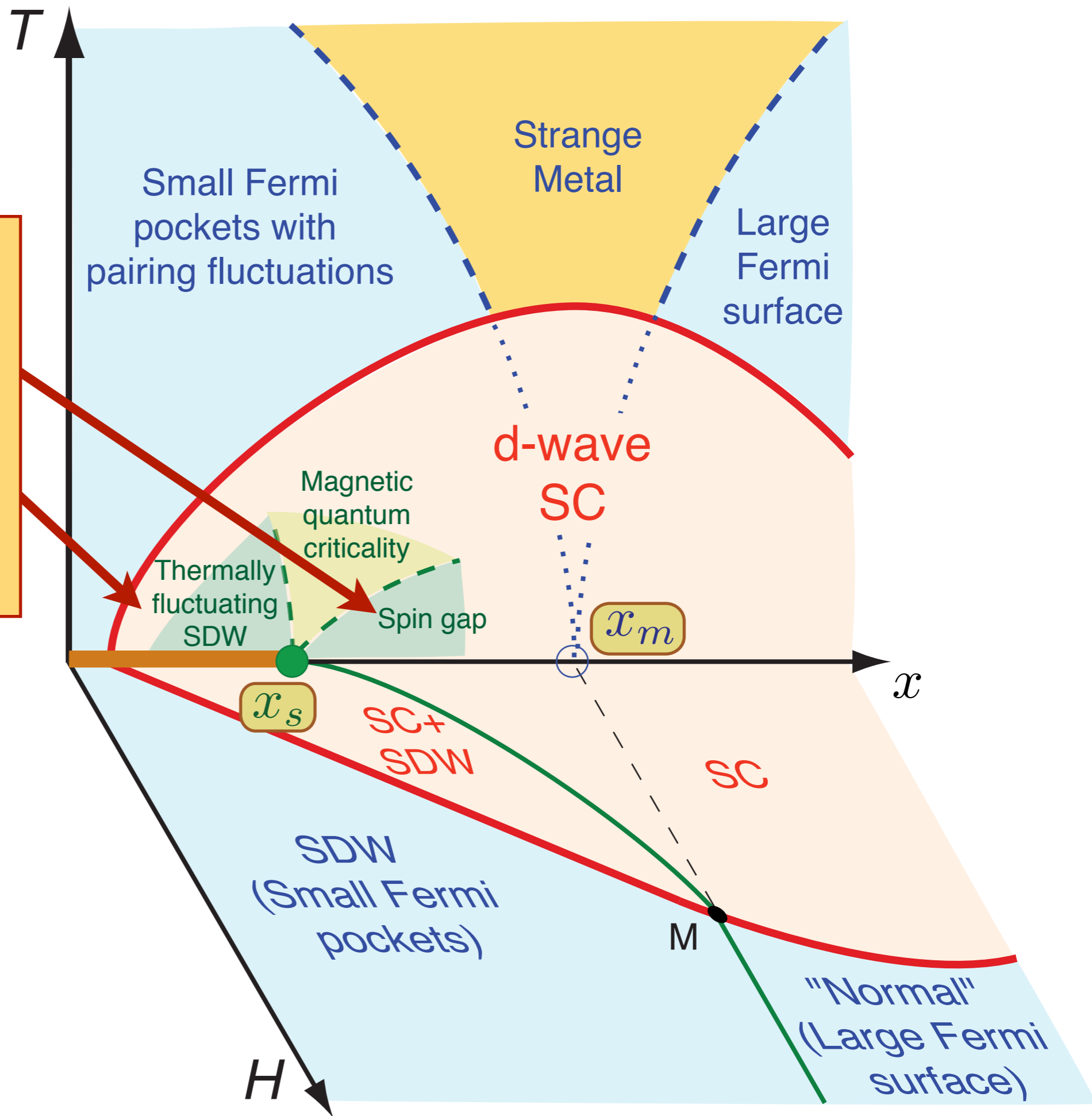
Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$

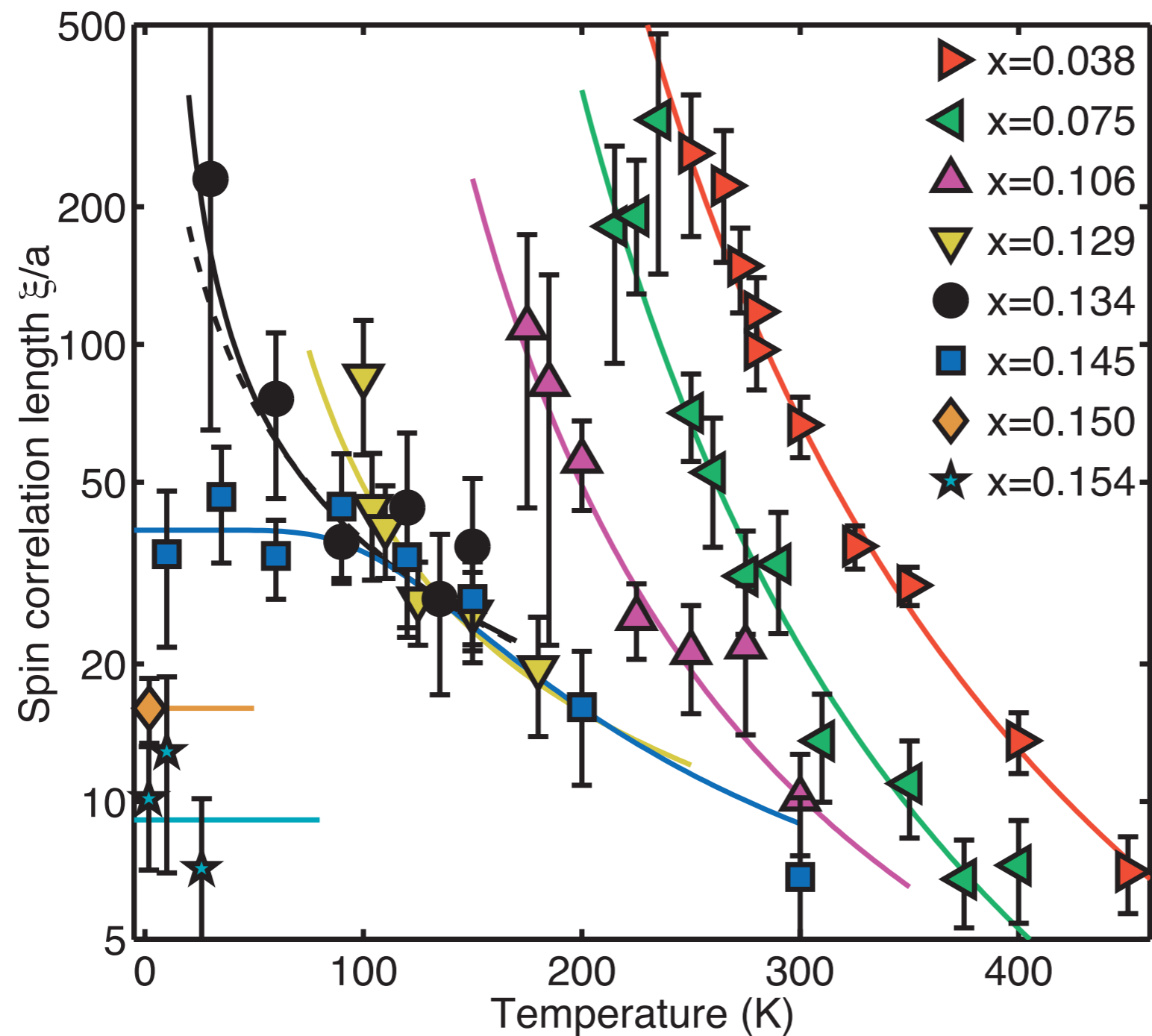
← Increasing SDW order →



T. Helm, M.V. Kartsovni,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
R. Gross, arXiv:0906.1431

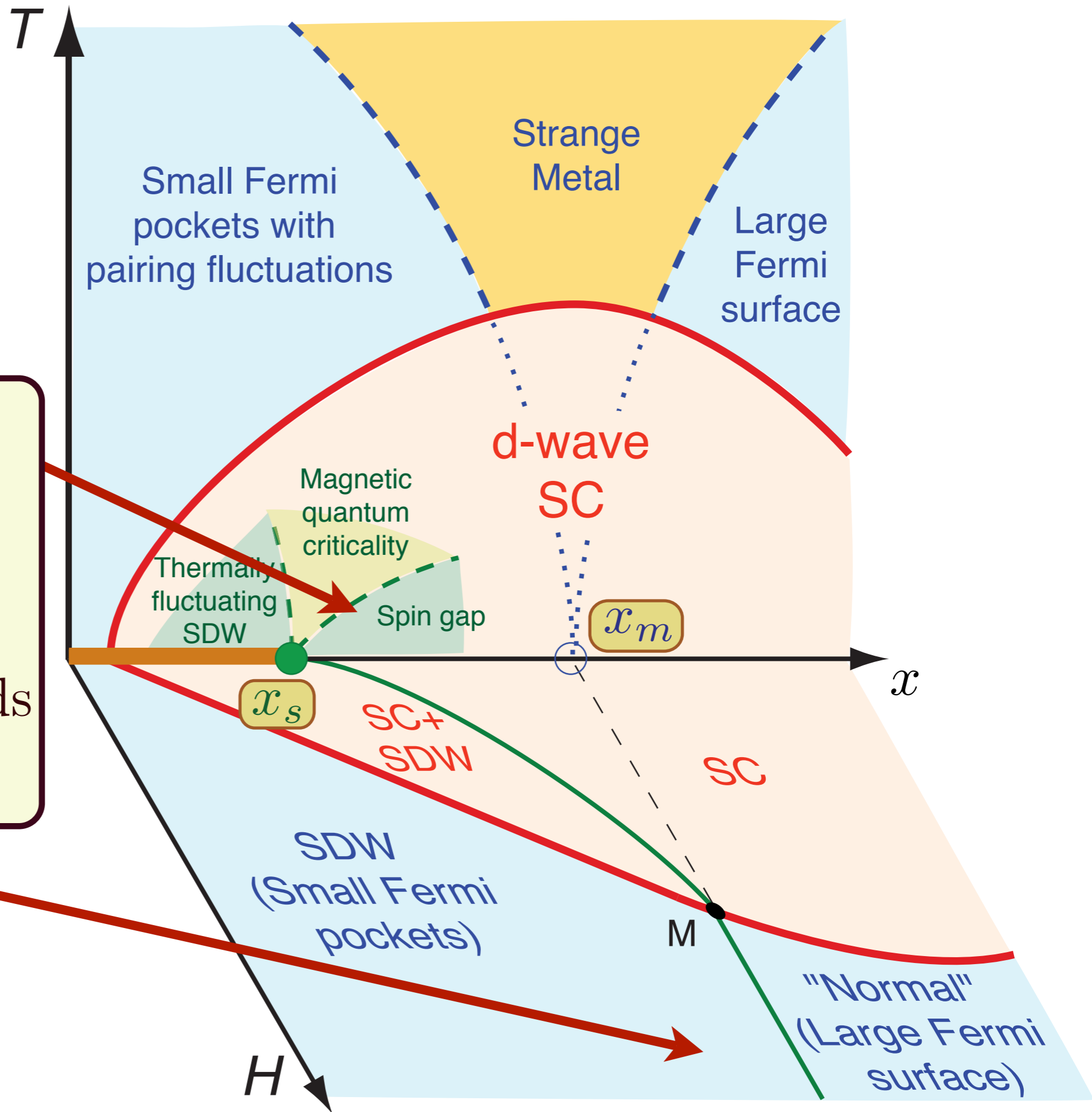
Neutron scattering at $H=0$ in **same** material identifies $x_s = 0.14 < x_m$





E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven,
Nature **445**, 186 (2007).

Experiments on $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ show that at low fields $x_s = 0.14$, while at high fields $x_m = 0.165$.



Conclusions

General theory of finite temperature dynamics and transport near quantum critical points, with applications to antiferromagnets, graphene, and superconductors

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been “hiding in plain sight”.

It is shifted to lower doping by the onset of superconductivity