Condensed matter applications of gauge-gravity duality

STRINGS, Uppsala, June 30, 2011

PHYSICS

VE RI TAS

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Rob Myers

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Markus Mueller

Liza Huijse

Max Metlitski

Sean Hartnoll

Reviews

Gubser: 1012.5312

Hartnoll: 1106.4324

Herzog: 0904.1975

Horowitz: 1002.1722

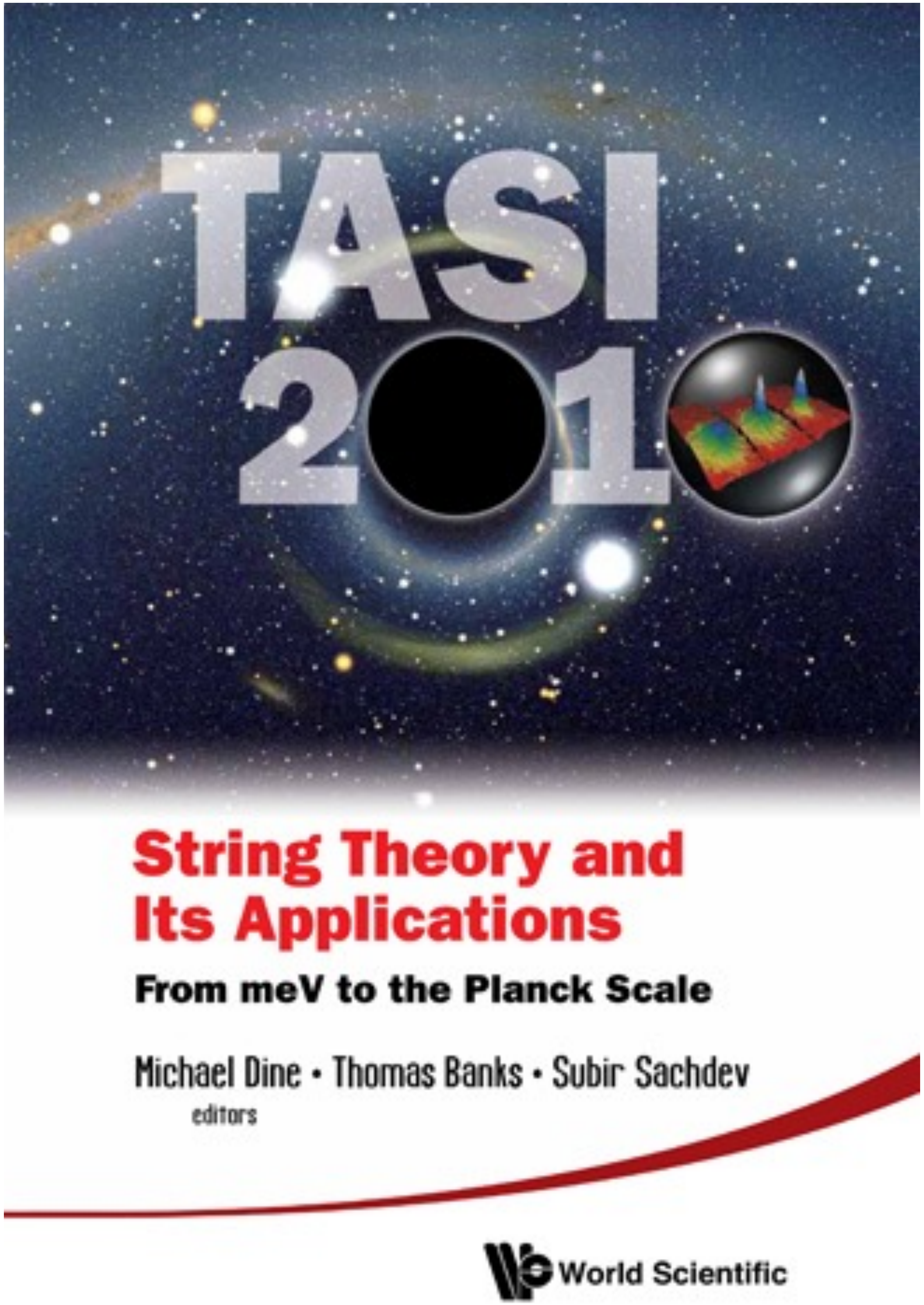
Hong Liu: to appear

McGreevy: 0909.0518

Nishioka, Ryu, Takayanagi: 0905.0932

Sachdev: 1012.0299

Reviews



TASI
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Outline

1. Quantum criticality and conformal field theories

2. Compressible quantum matter

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The AdS_4 - Schwarzschild black brane

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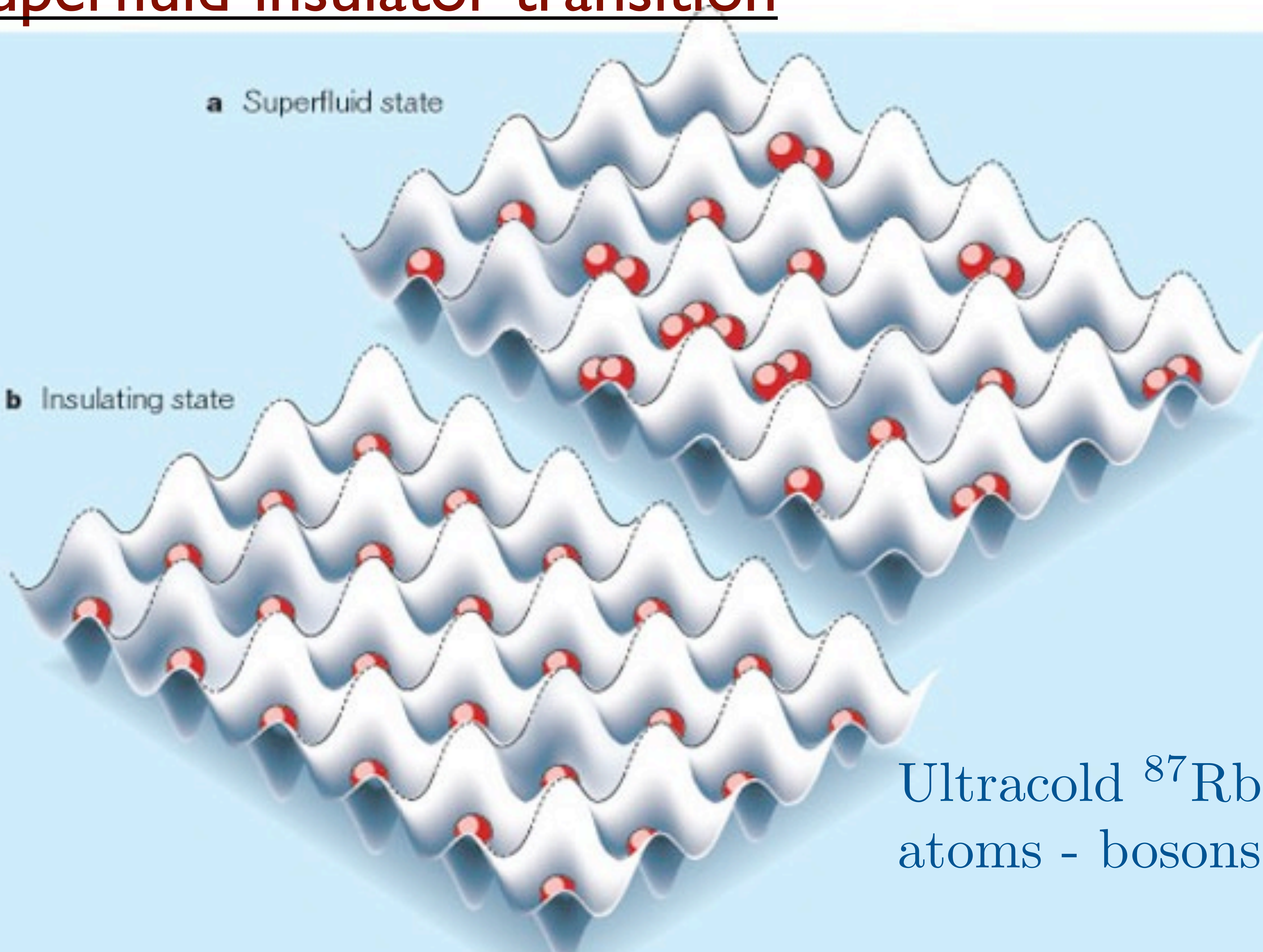
Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

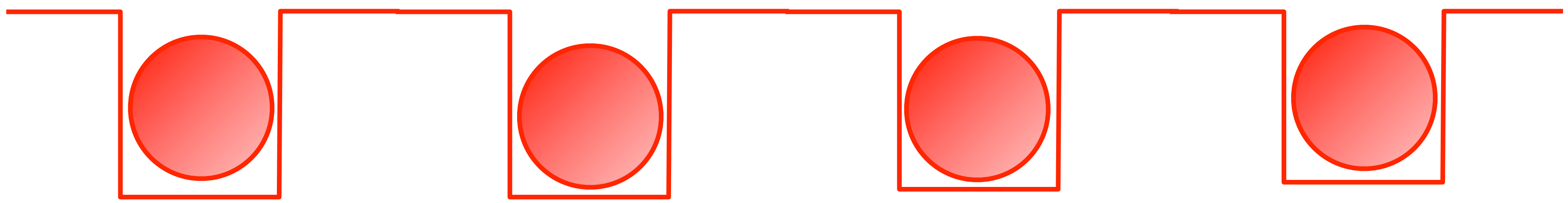
Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j (n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

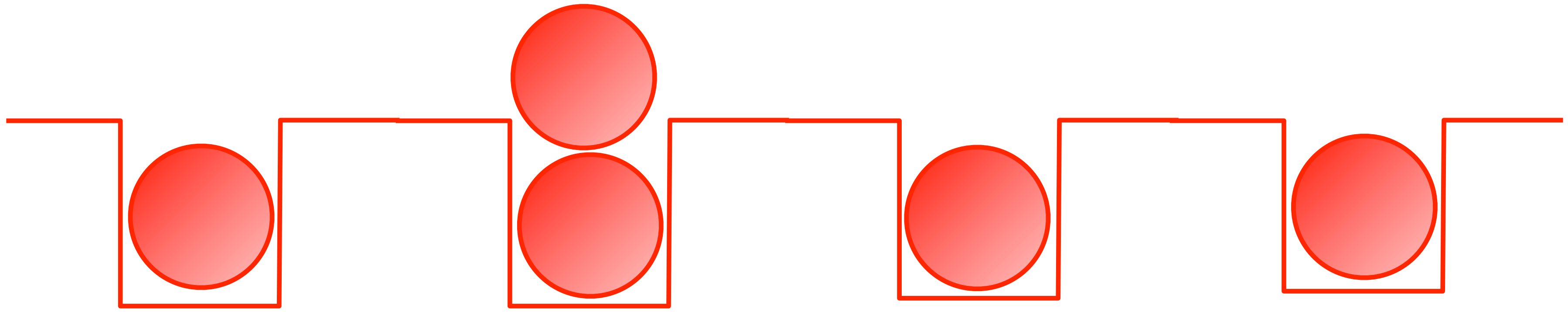
$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).



Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:

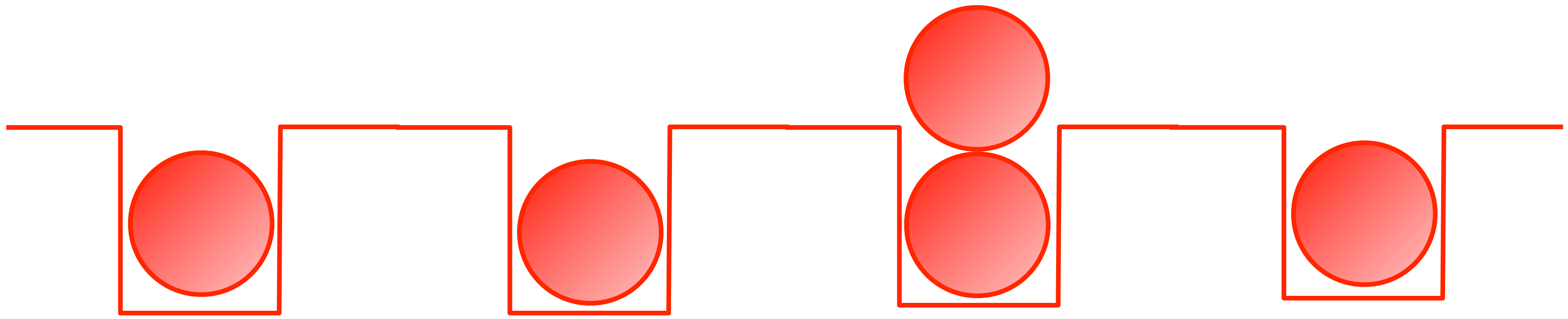


Particles $\sim \psi^\dagger$

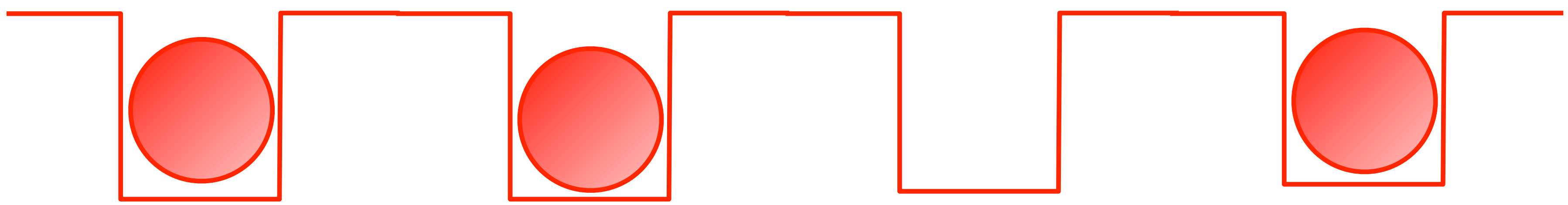
Excitations of the insulator:

Holes ~ ψ

Excitations of the insulator:



Particles ~ $\psi^\dagger$

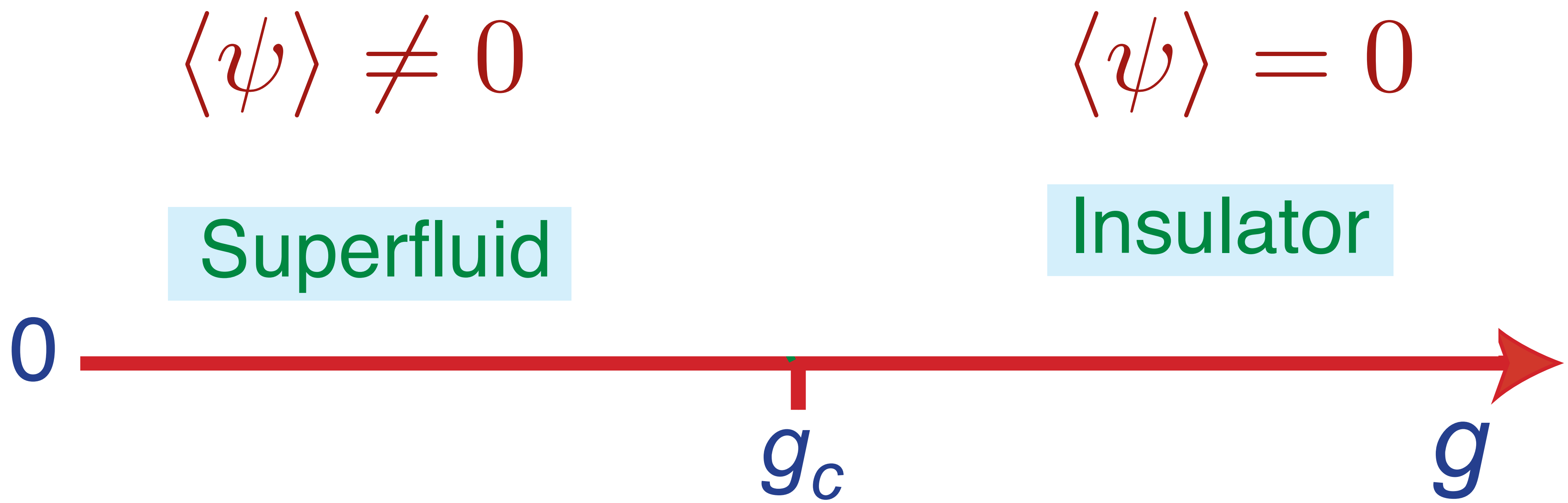


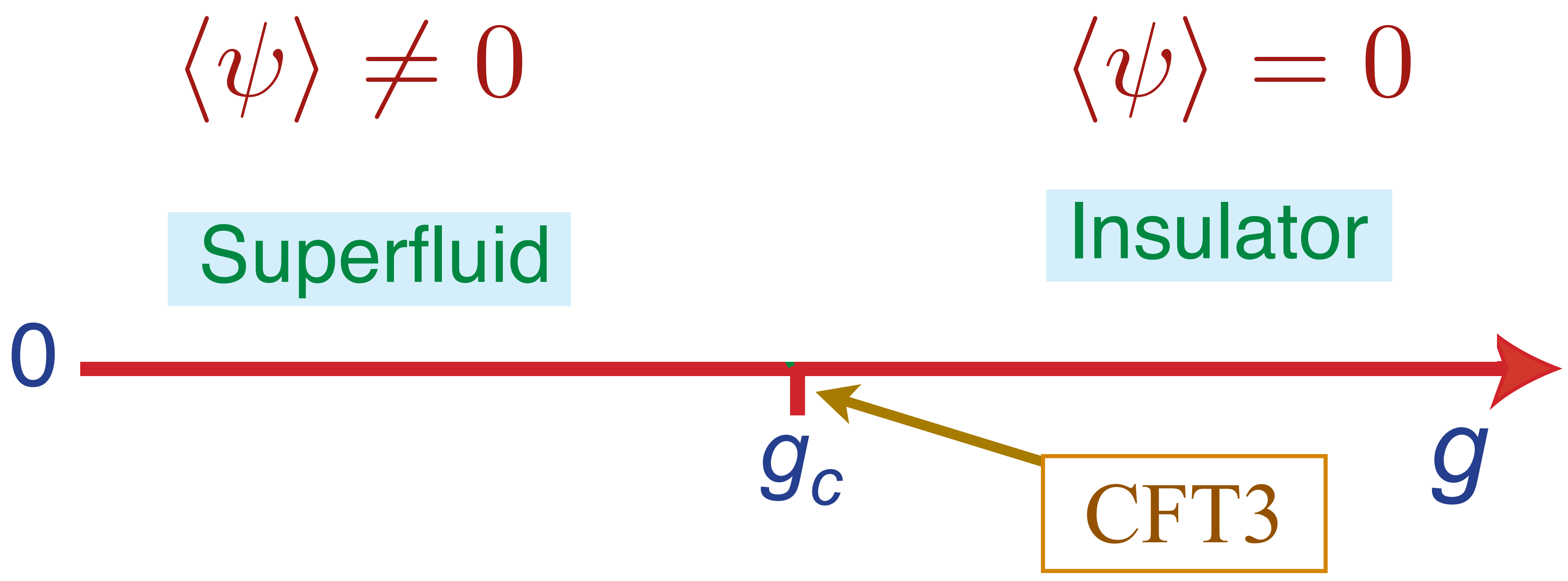
Holes ~ ψ

Density of particles = density of holes ⇒ "relativistic" field theory for ψ:

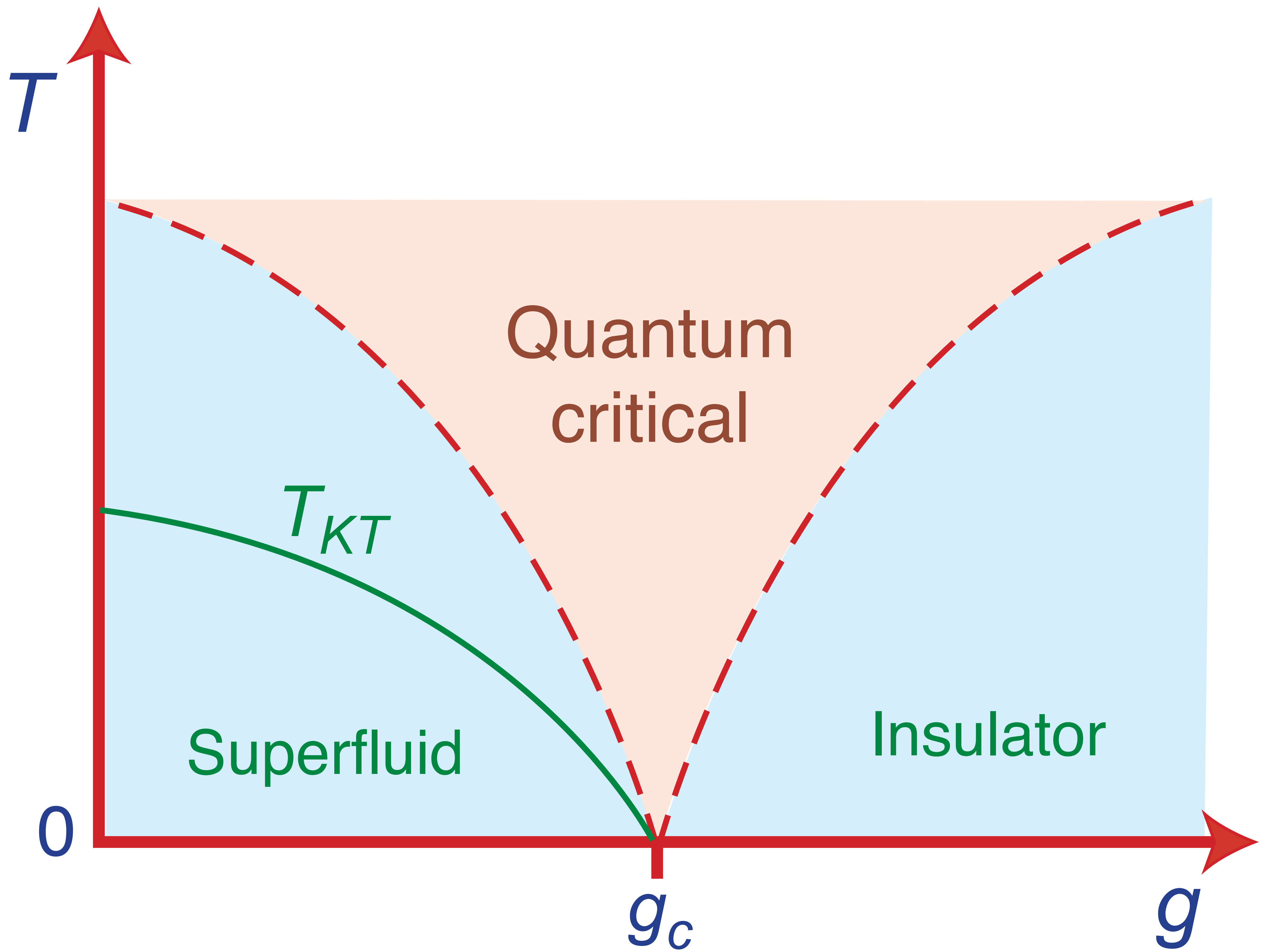
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).


$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g



$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g
CFT3



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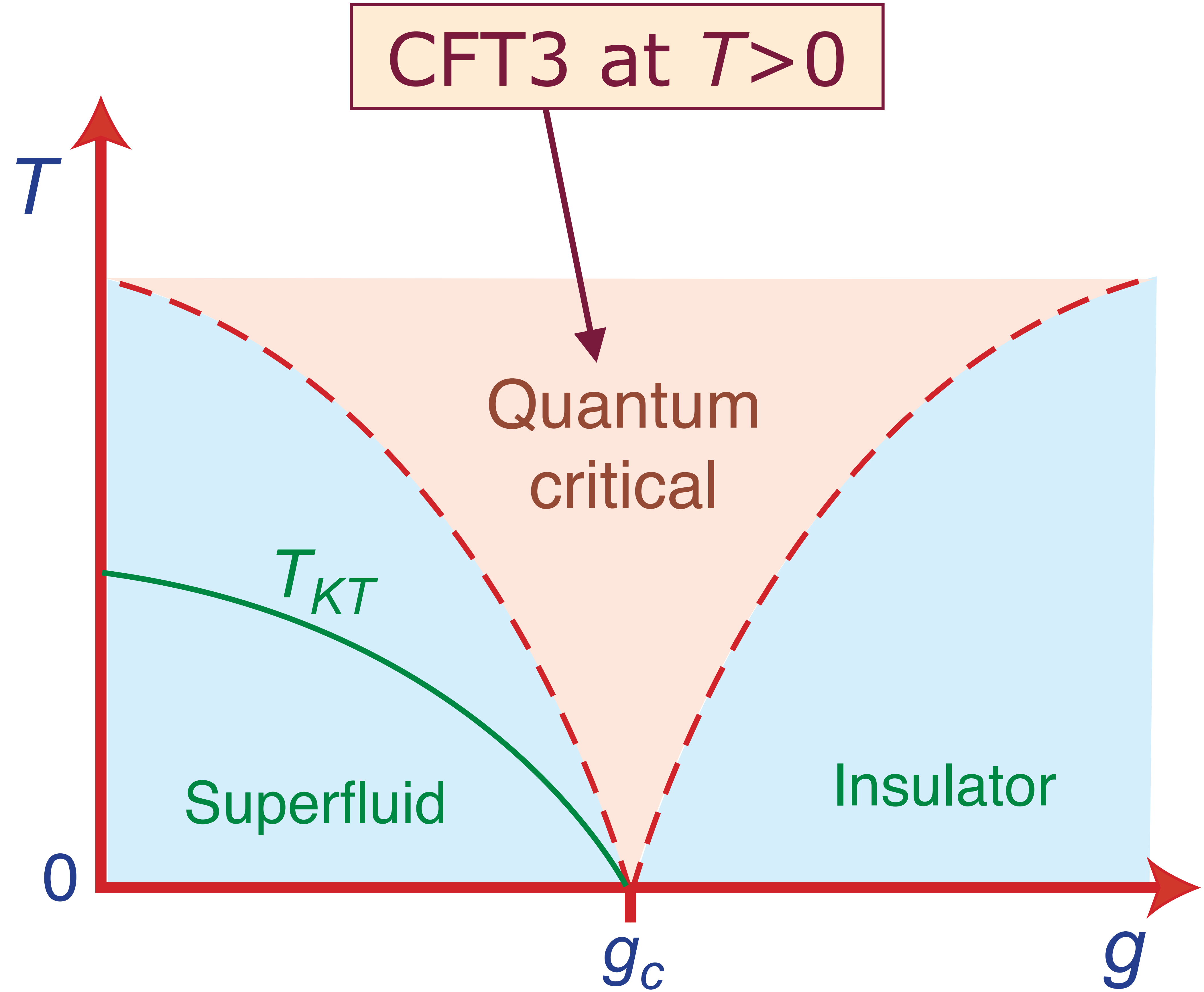

Thursday, June 30, 2011

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C}\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1) \,]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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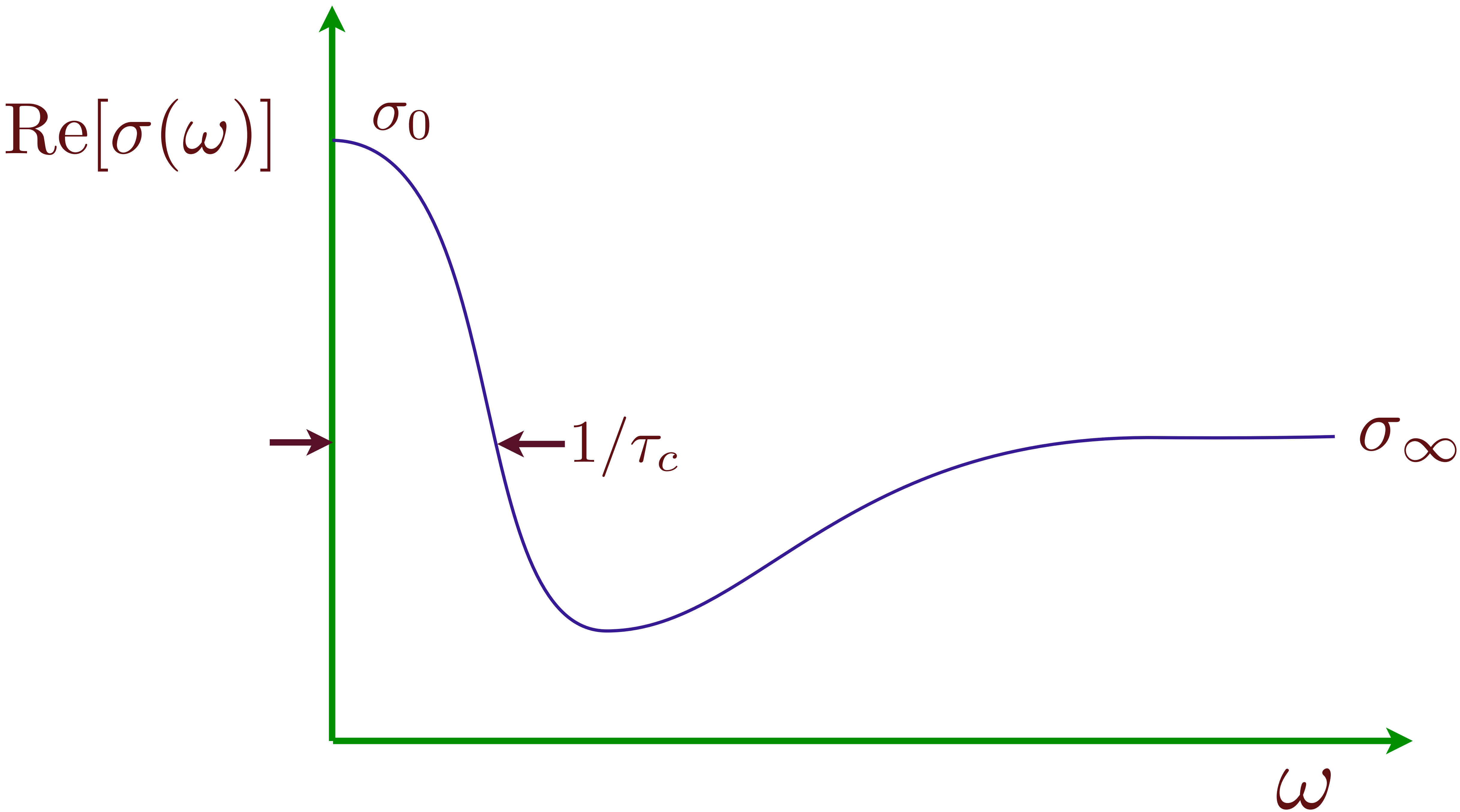
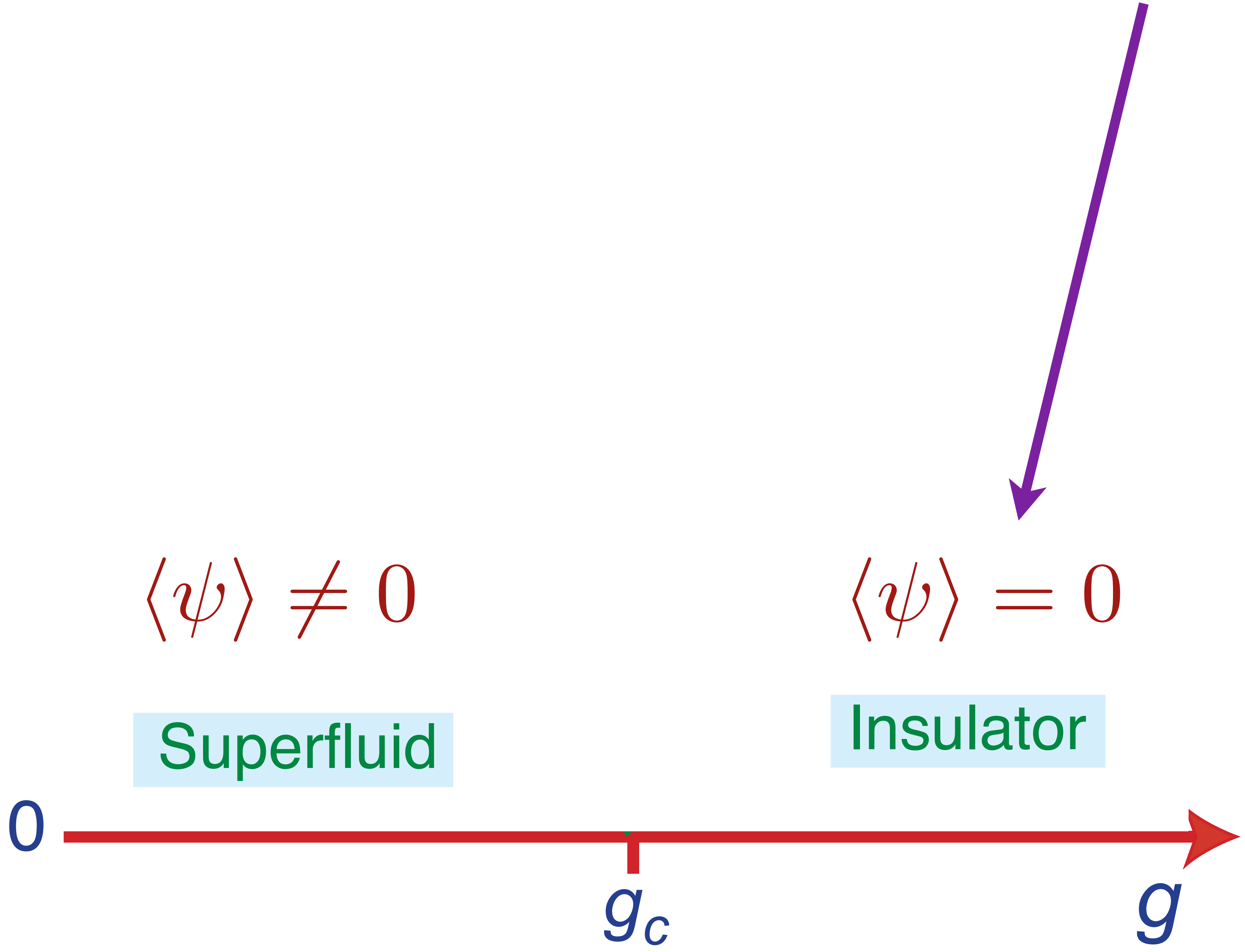
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

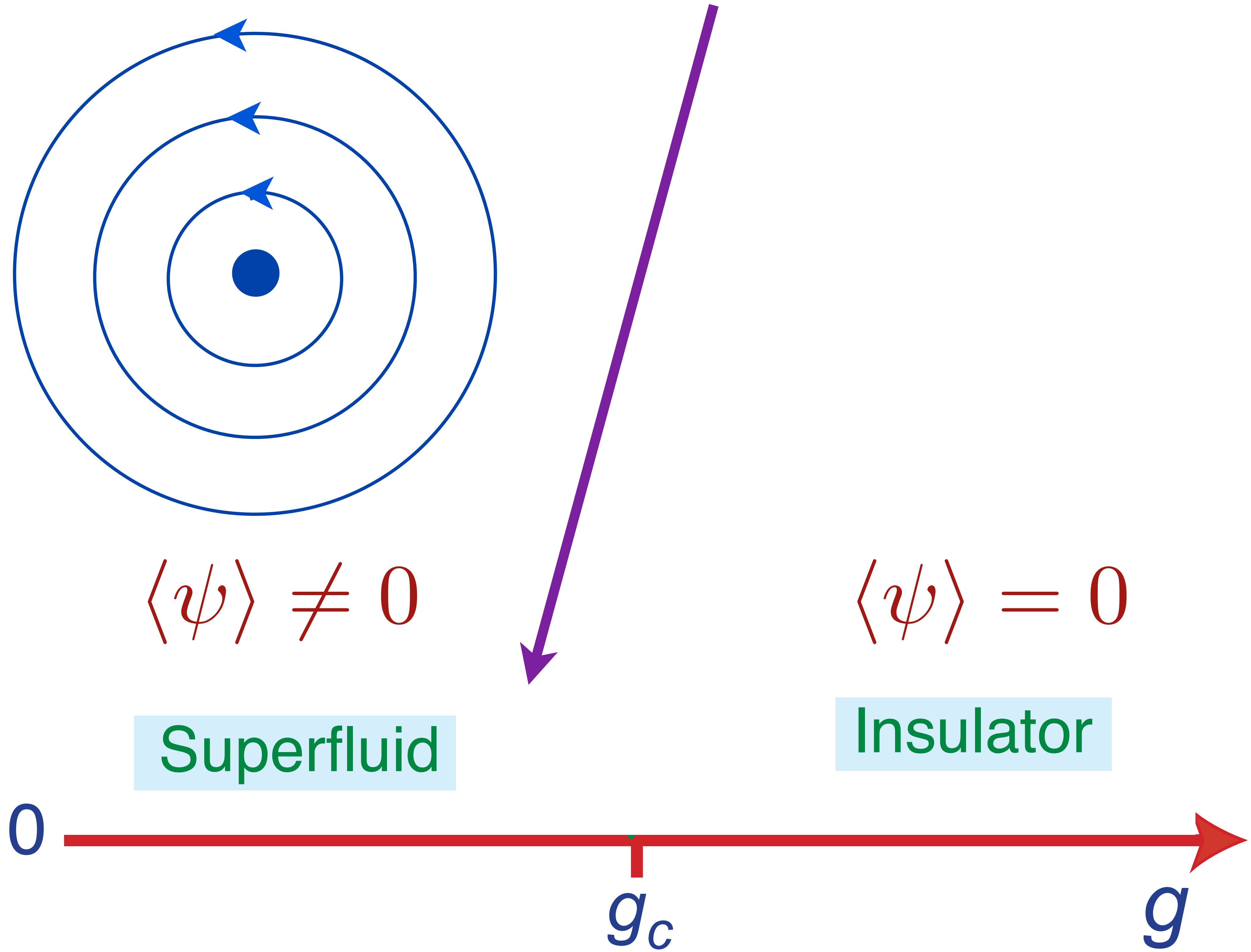
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



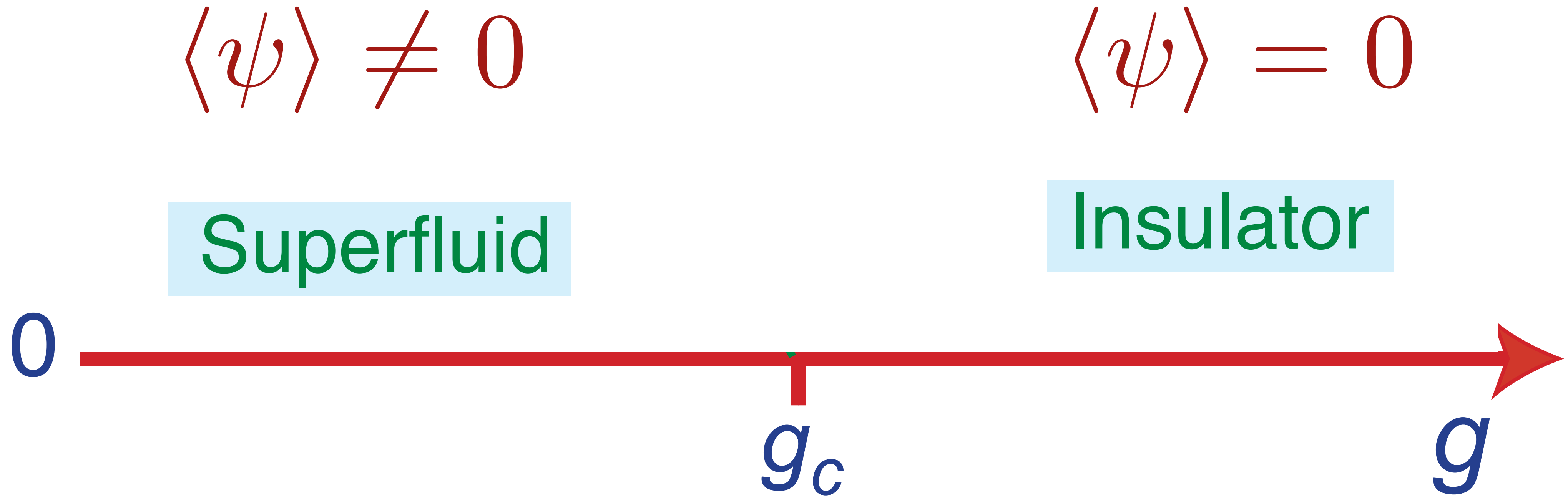
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



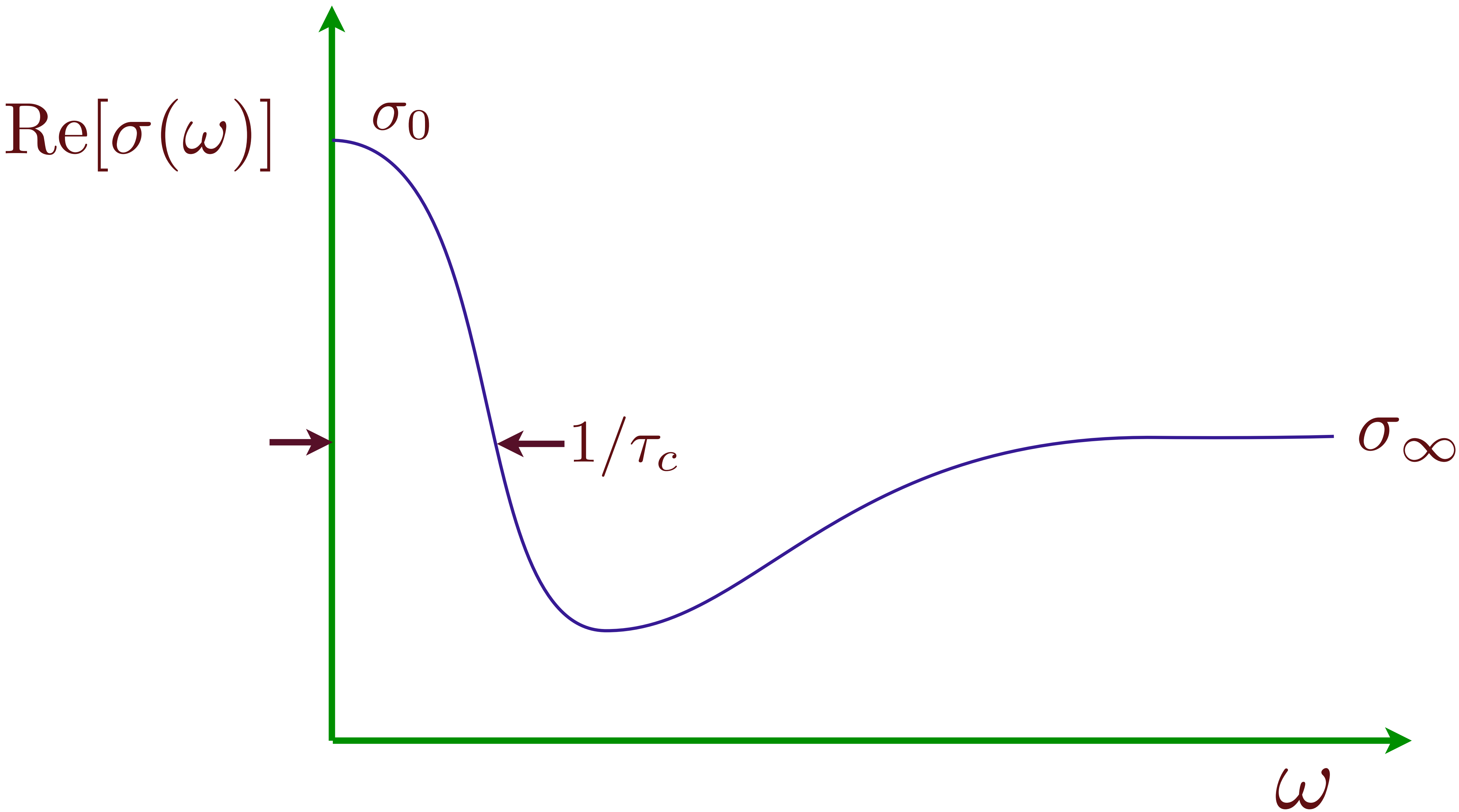
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

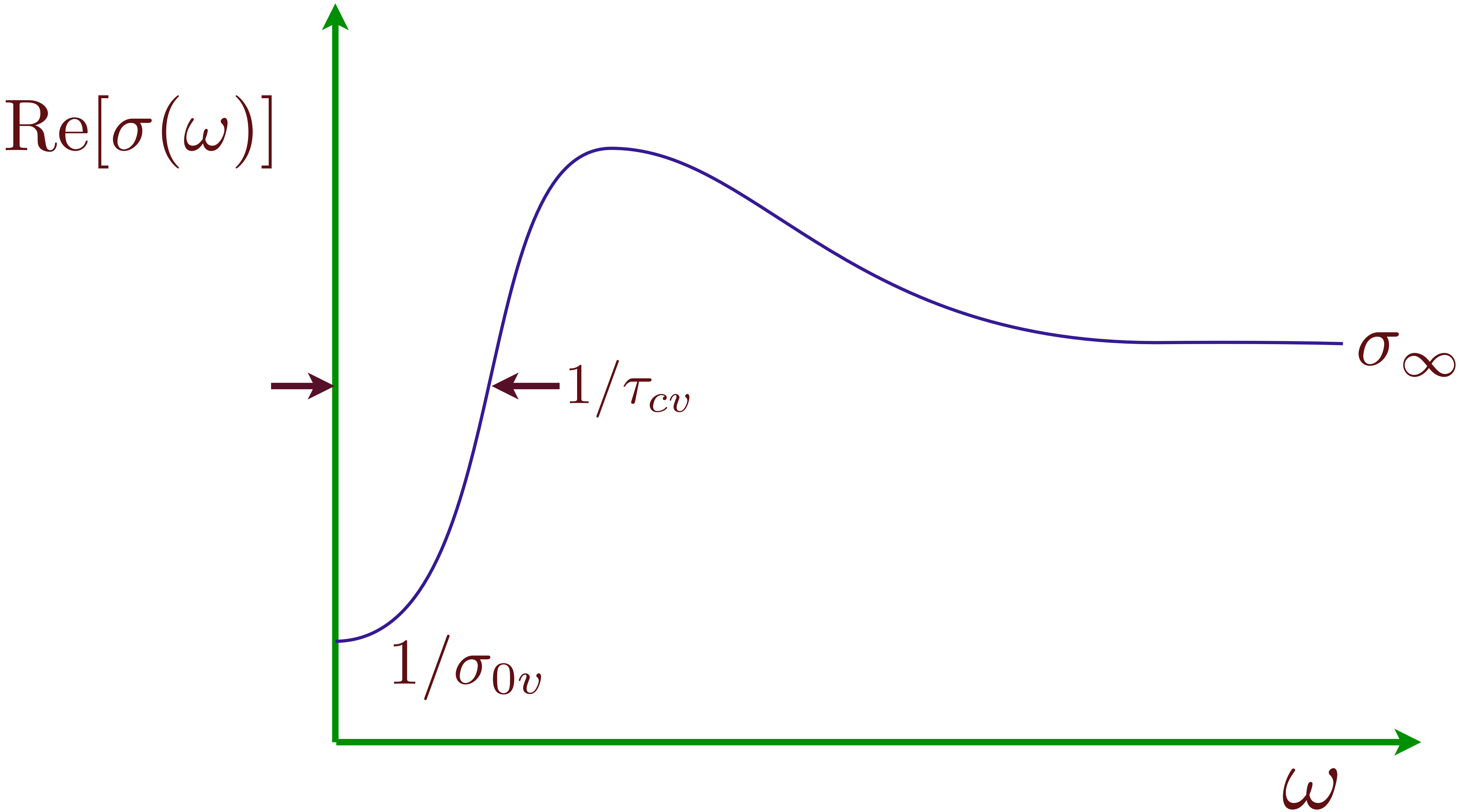
$$\text{Conductivity} = \text{Resistivity of vortices}$$



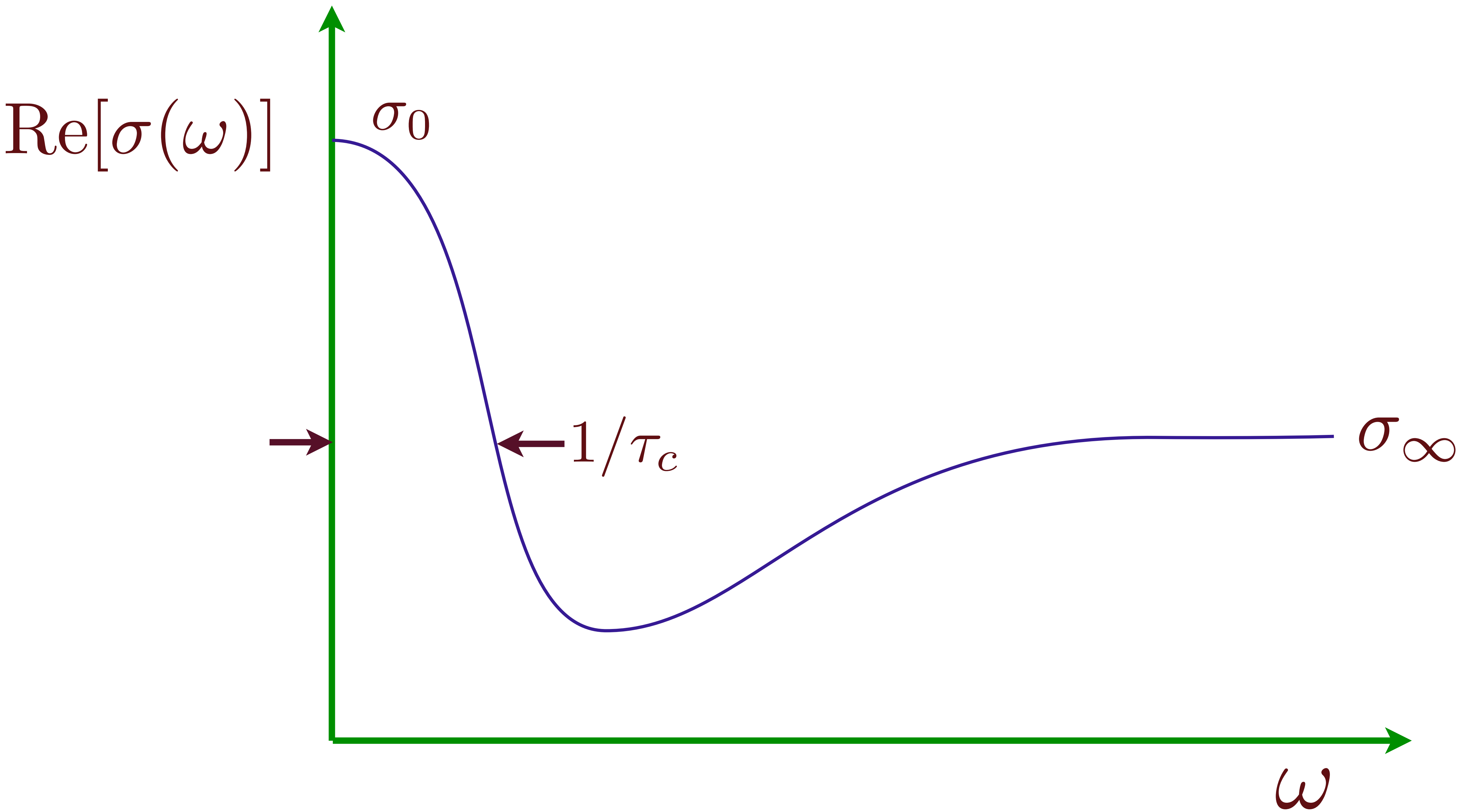
Boltzmann theory of bosons



Boltzmann theory of vortices

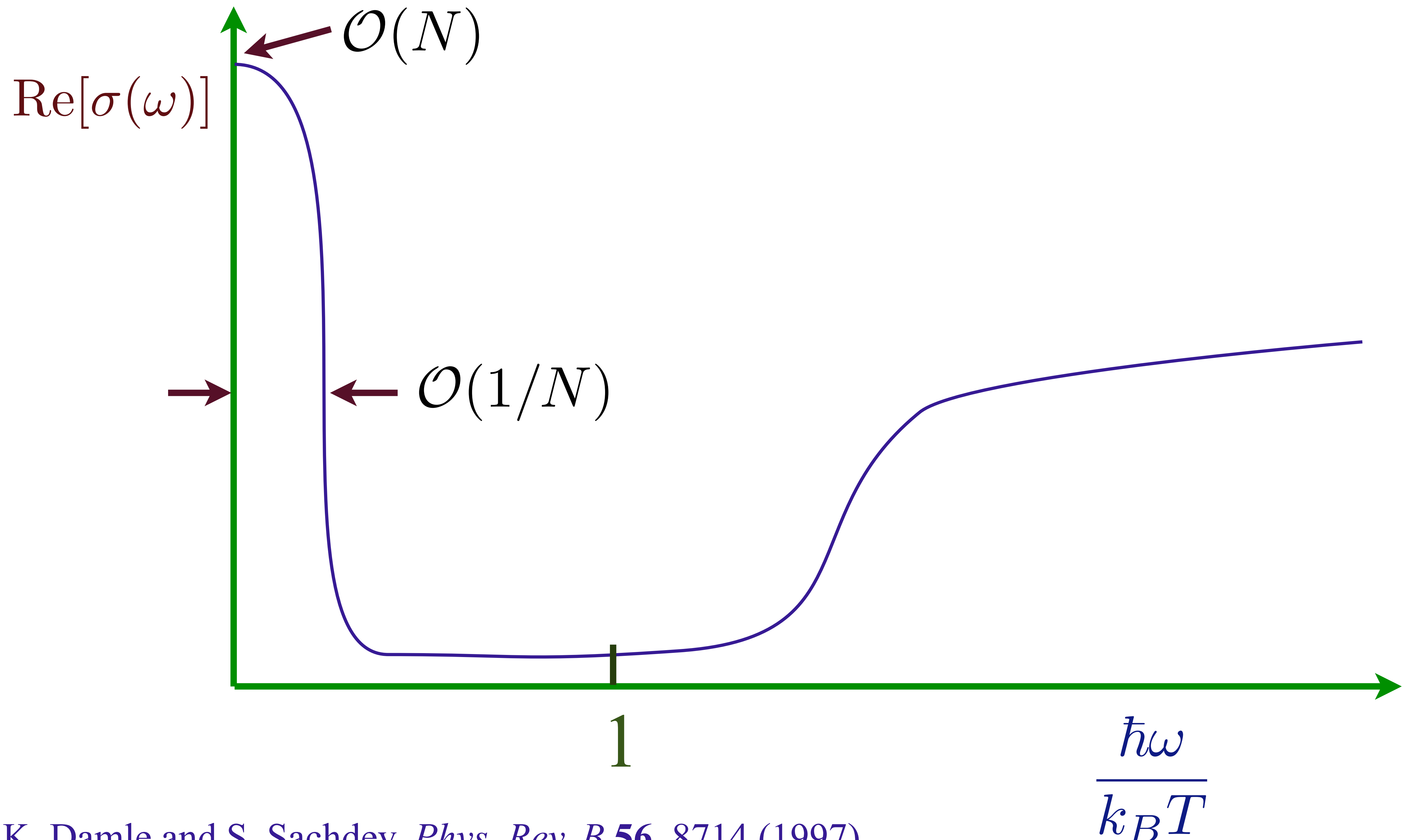


Boltzmann theory of bosons



Vector large *N* expansion for CFT3

$$\sigma = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_BT}\right); \quad \Sigma \rightarrow \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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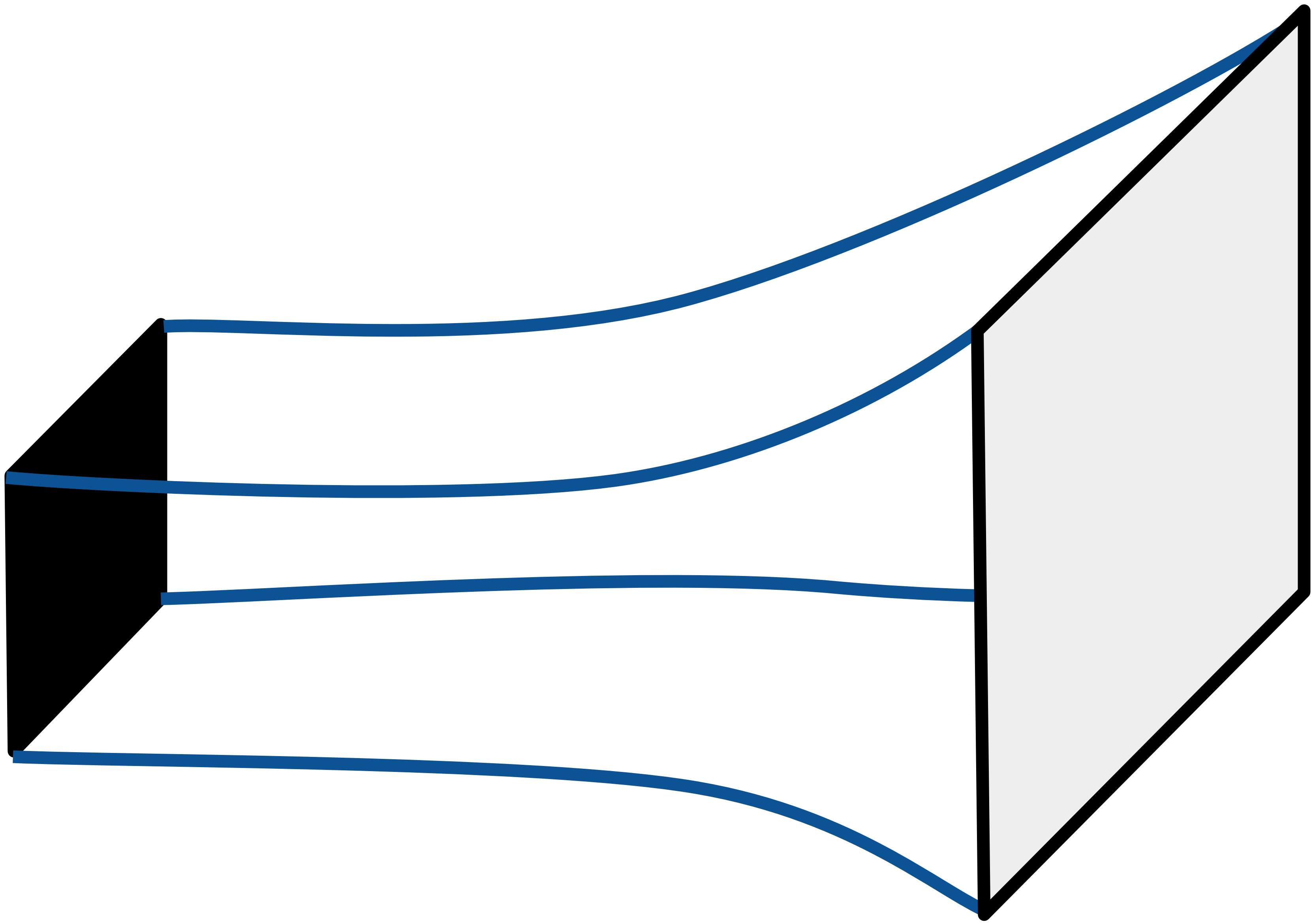
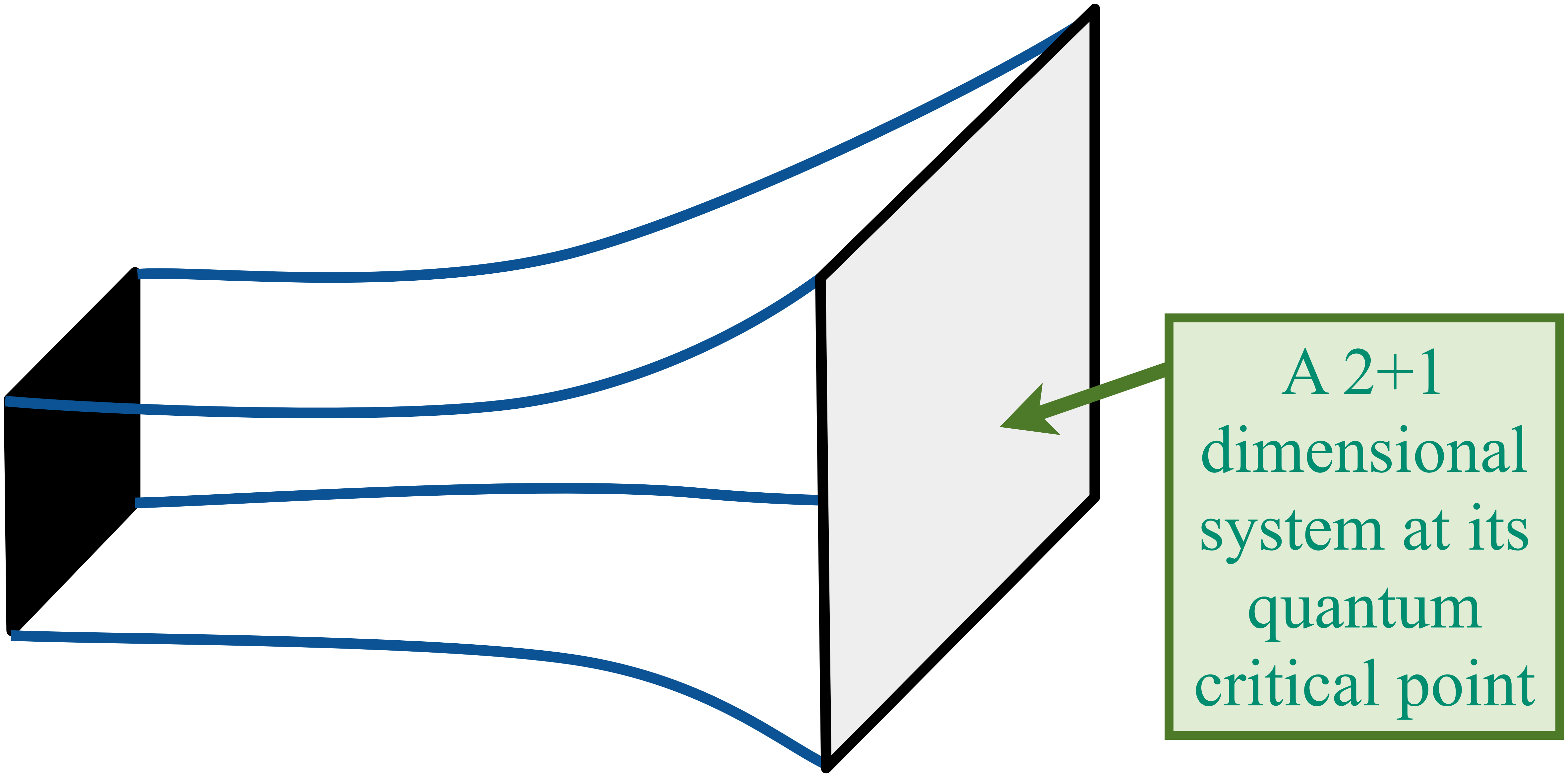
AdS_4-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence

AdS$_4$-Schwarzschild black-brane



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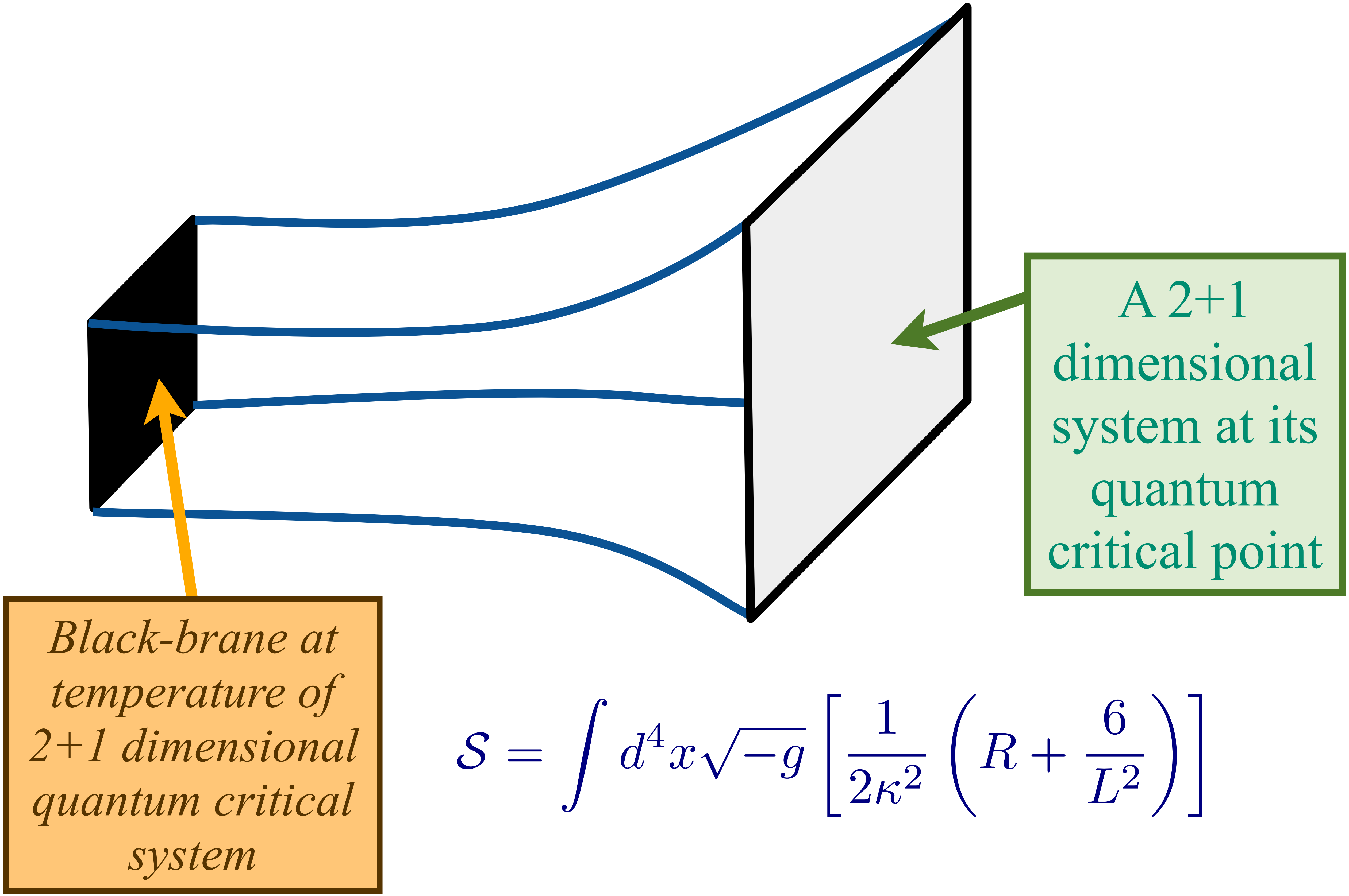

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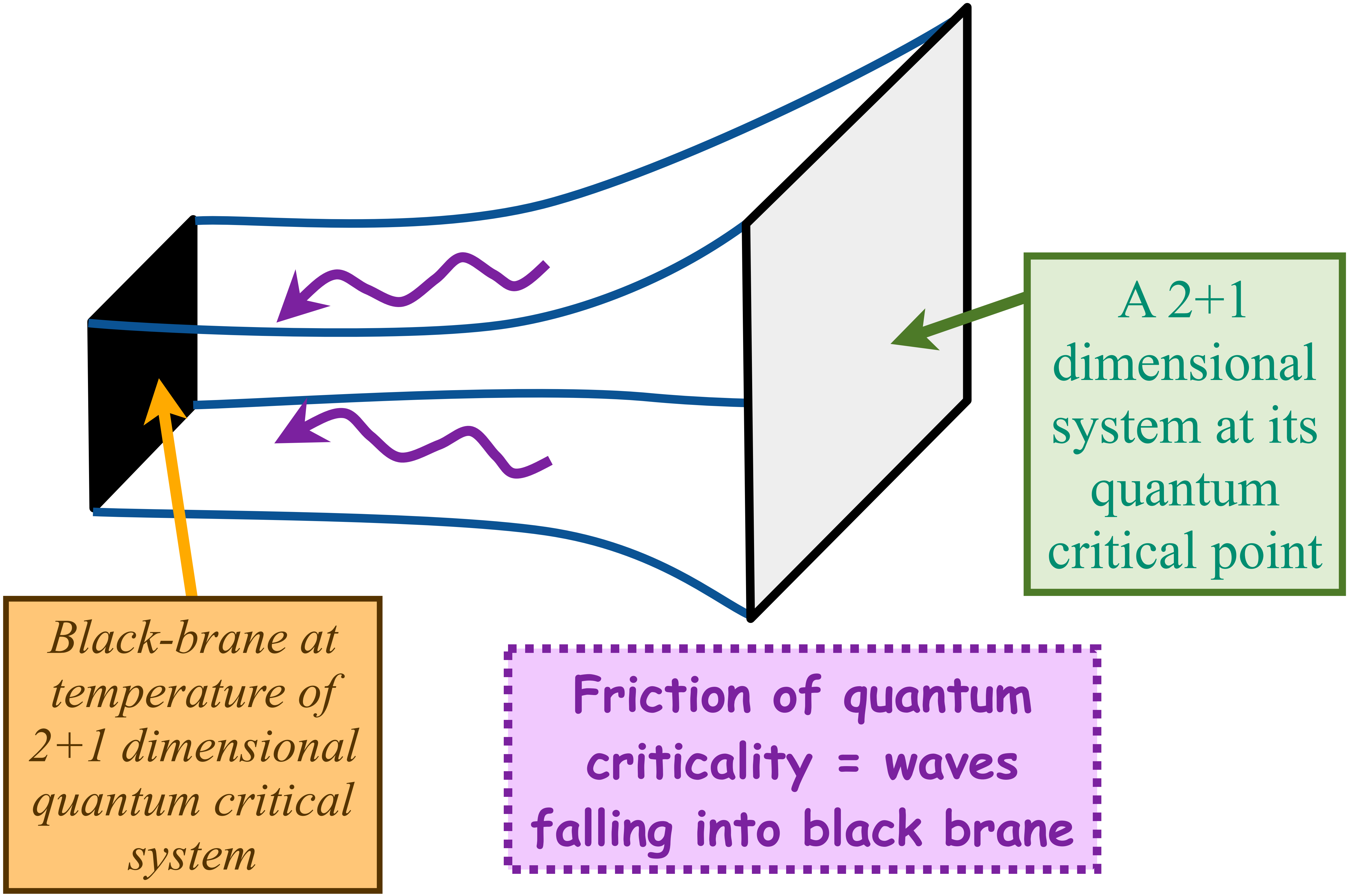

AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

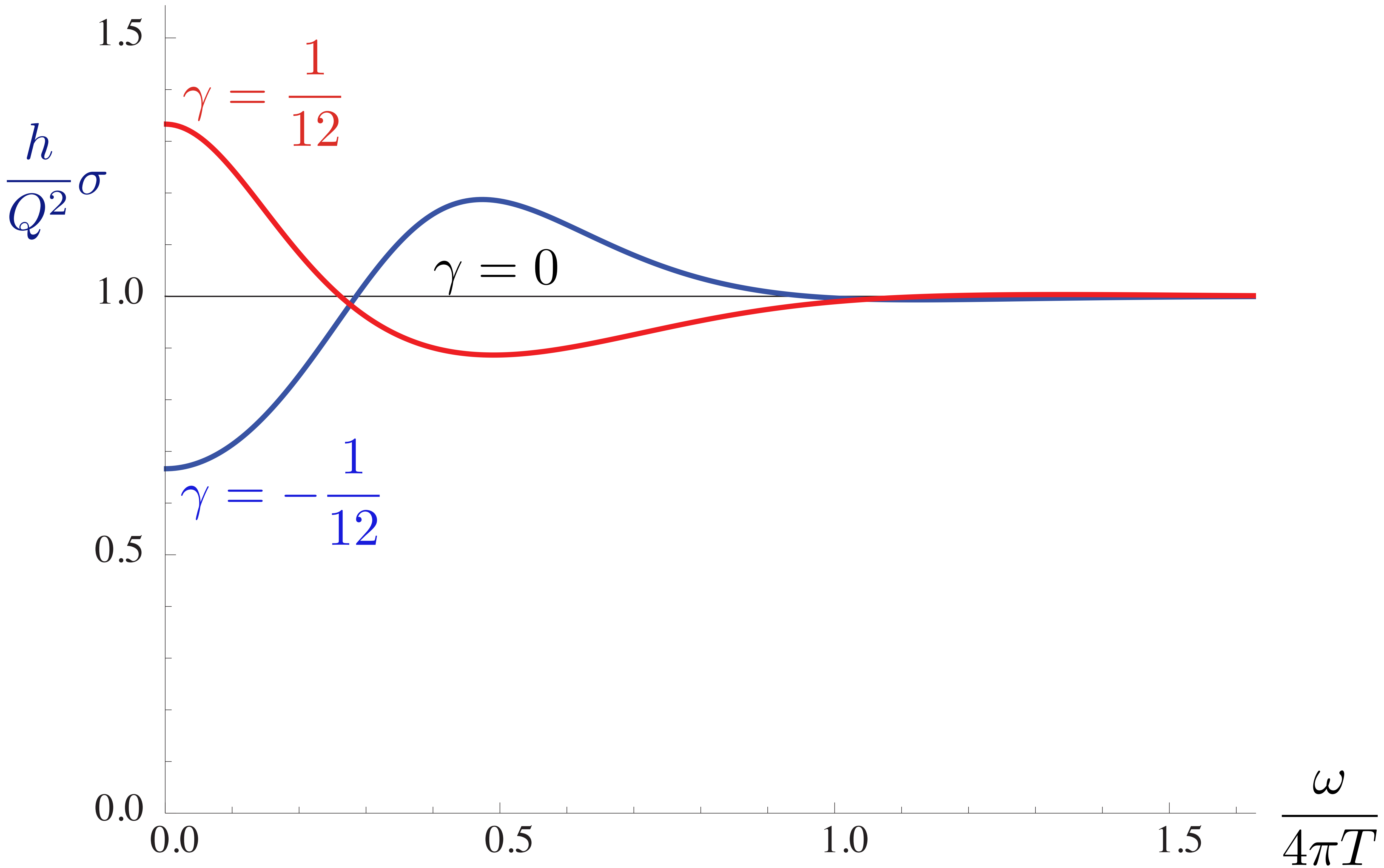
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab}F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"



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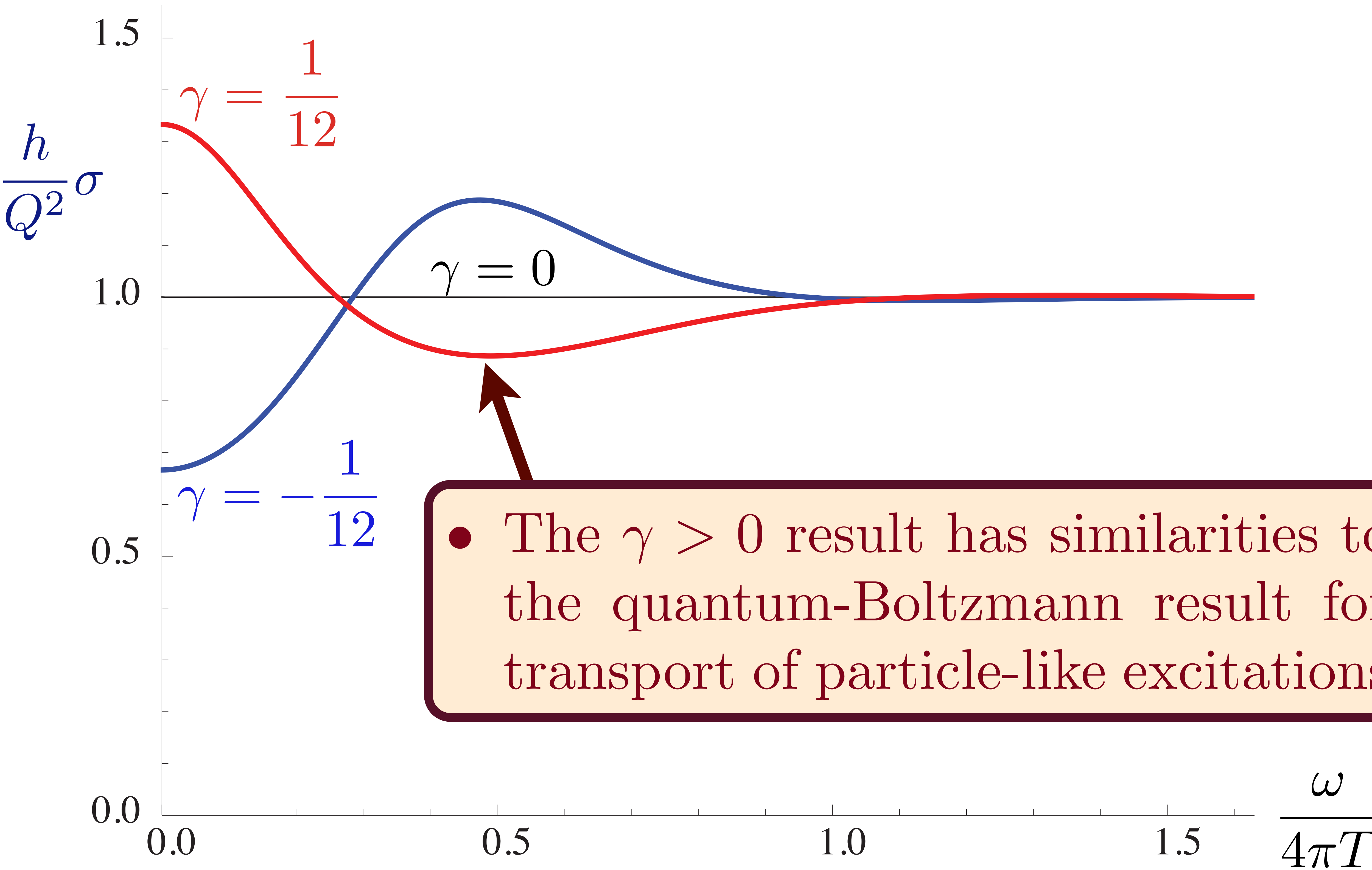

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

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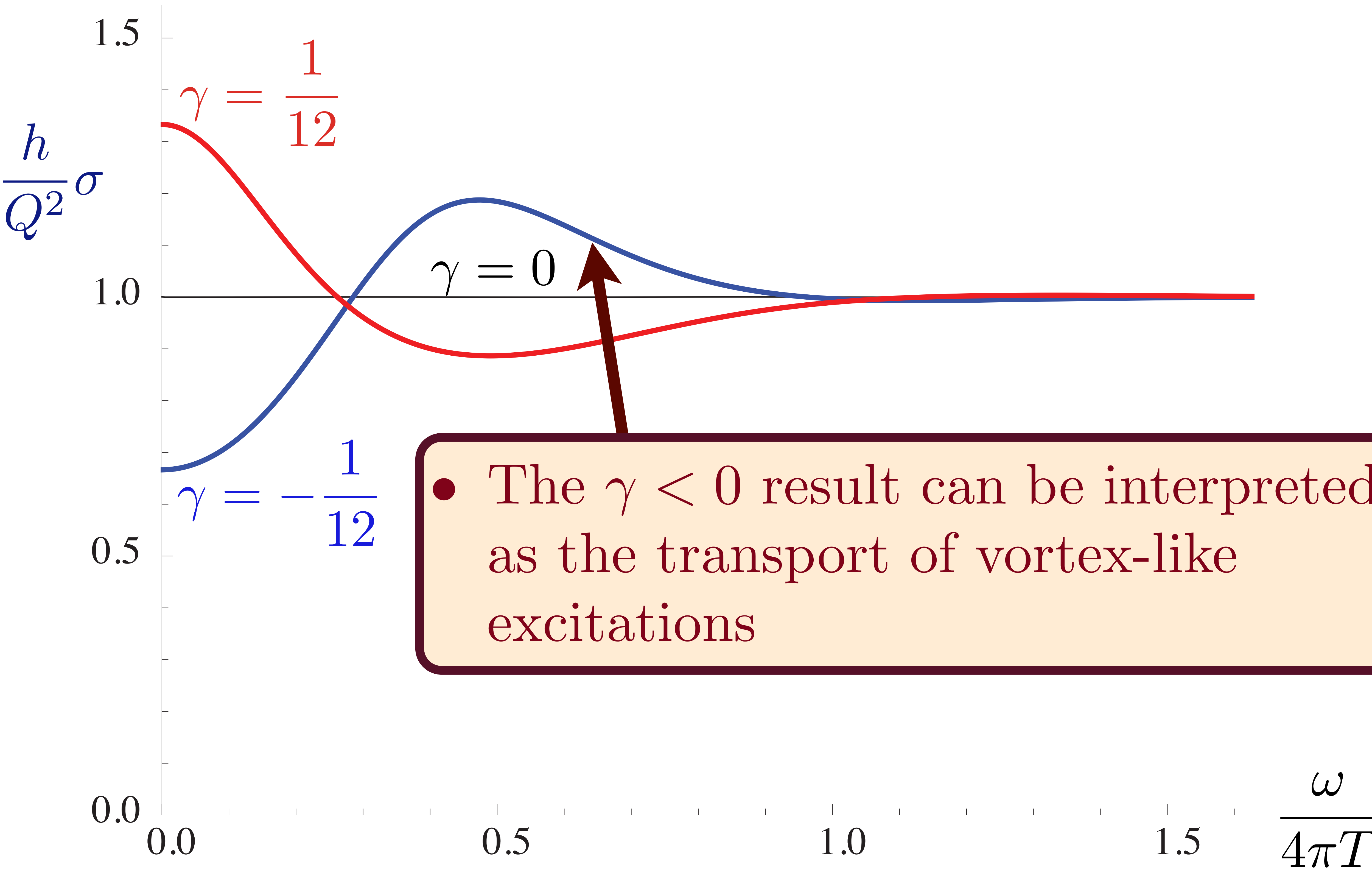

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

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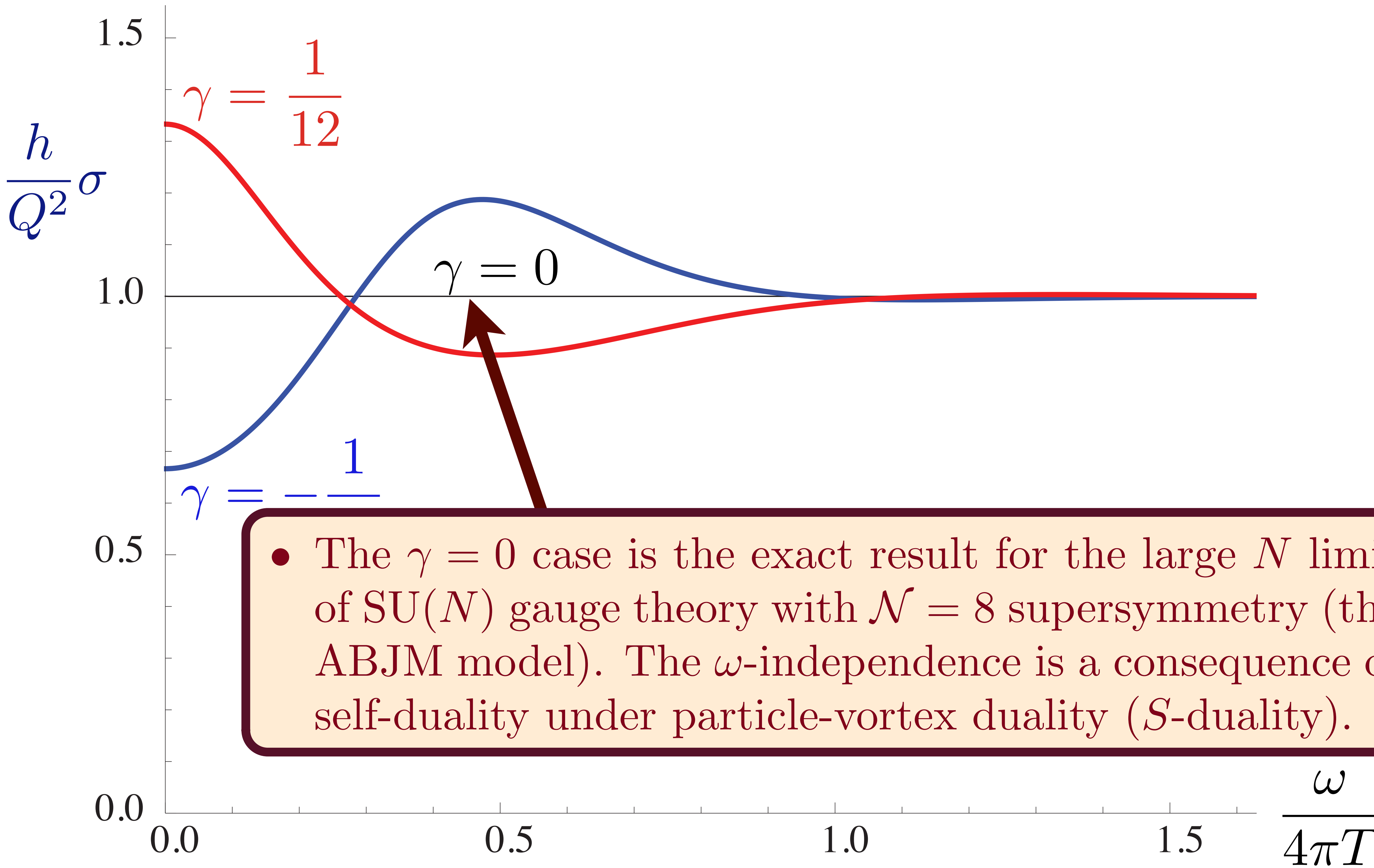

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

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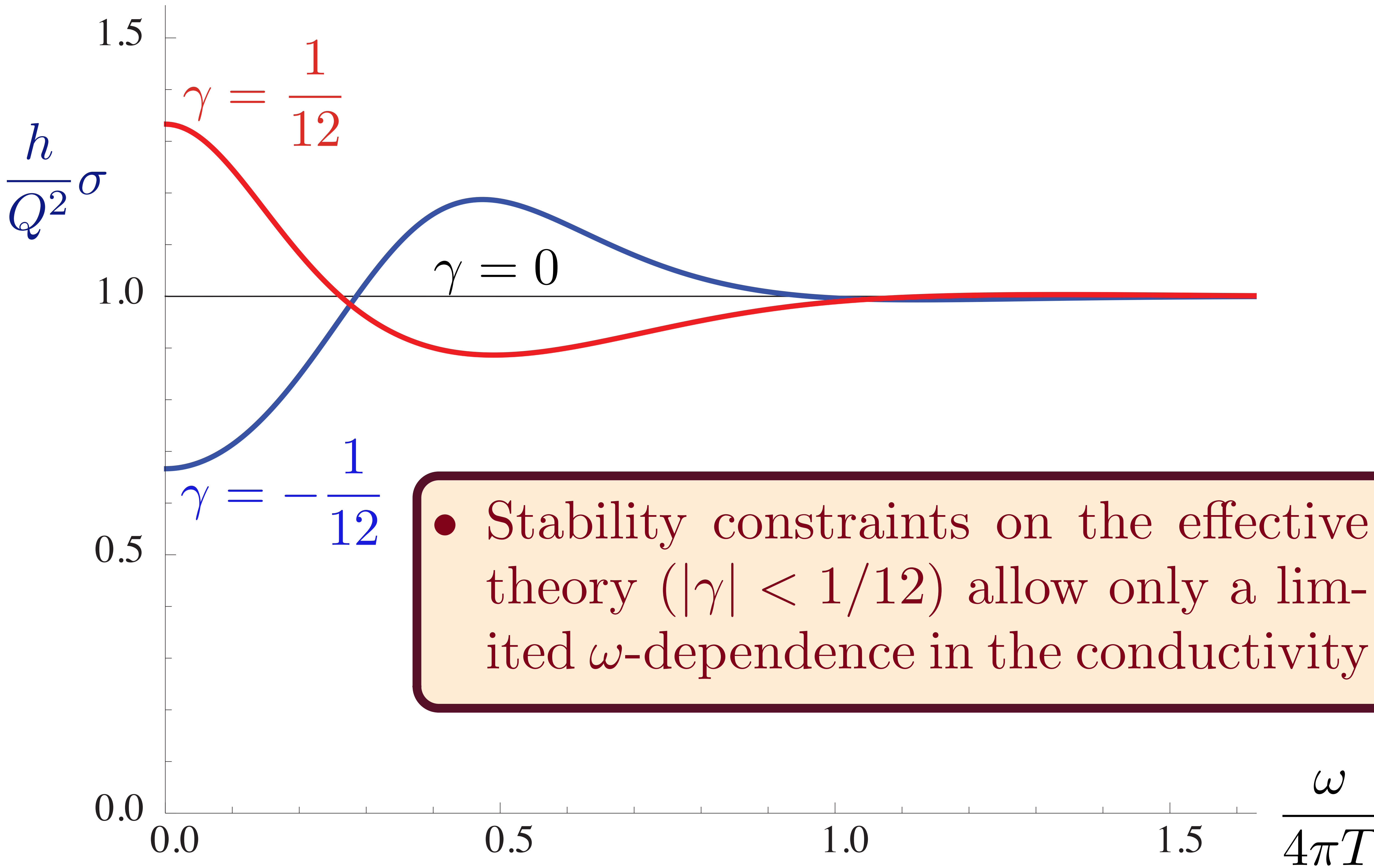
AdS$_4$ theory of strongly interacting "perfect fluids"



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Other studies

Chern-Simons terms and quantum Hall effect:

Alanen, Keski-Vakkuri, Kraus, Suur-Uski;
Bayntun, Burgess, Dolan, Lee

Shear Viscosity

Mueller, Fritz, Schmalian

Non-linear transport:

Karch, Sondhi

Topological insulators:

Ryu, Takayanagi
Hoyos, Jensen, Karch

Non-equilibrium transport:

....

Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value

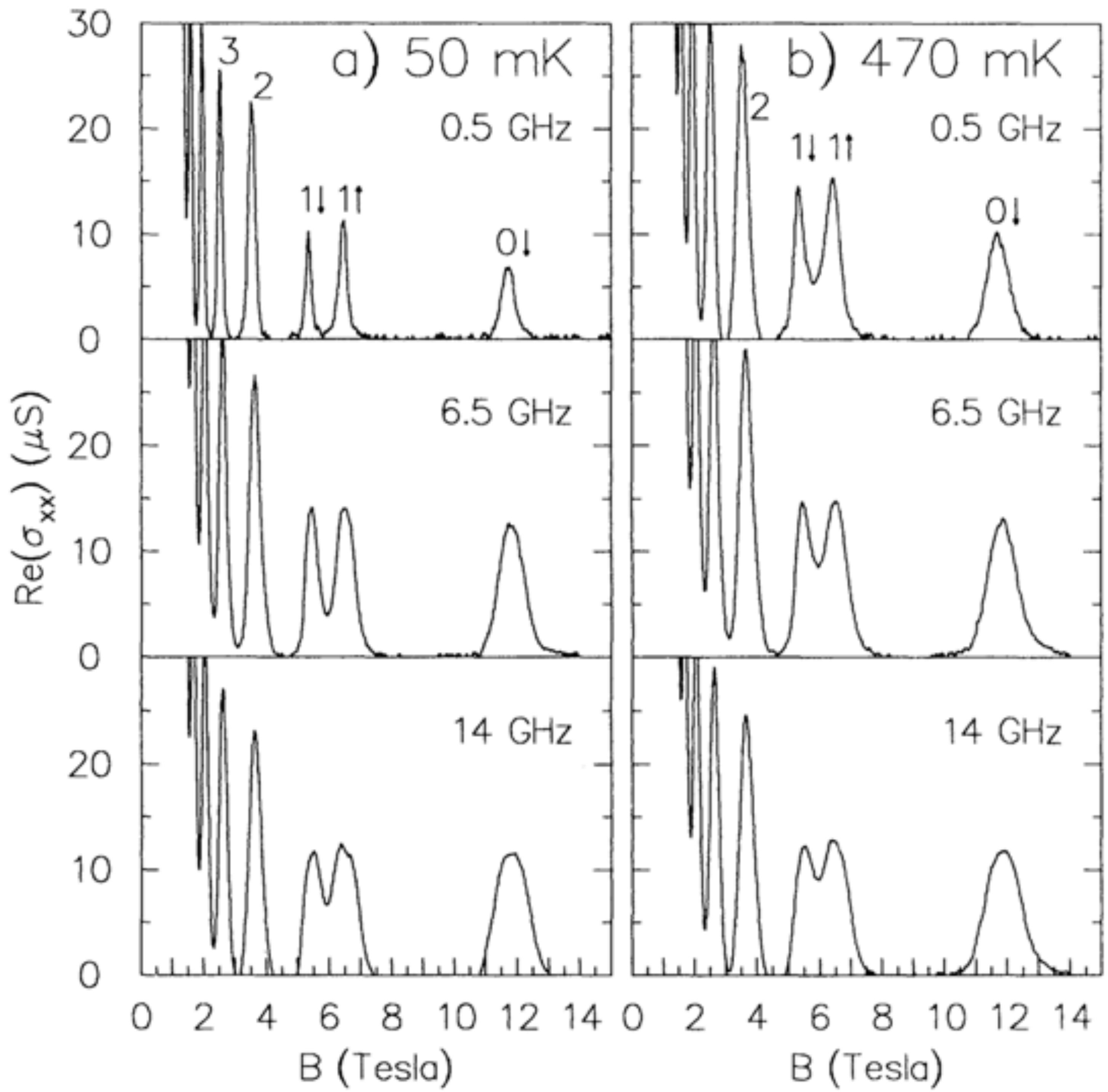


FIG. 3. Re(σ_{xx}) vs B at three frequencies and two temperatures. Peaks are marked with Landau level index N and spin.

L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui, *Physical Review Letters* **71**, 2638 (1993).

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Compressible quantum matter

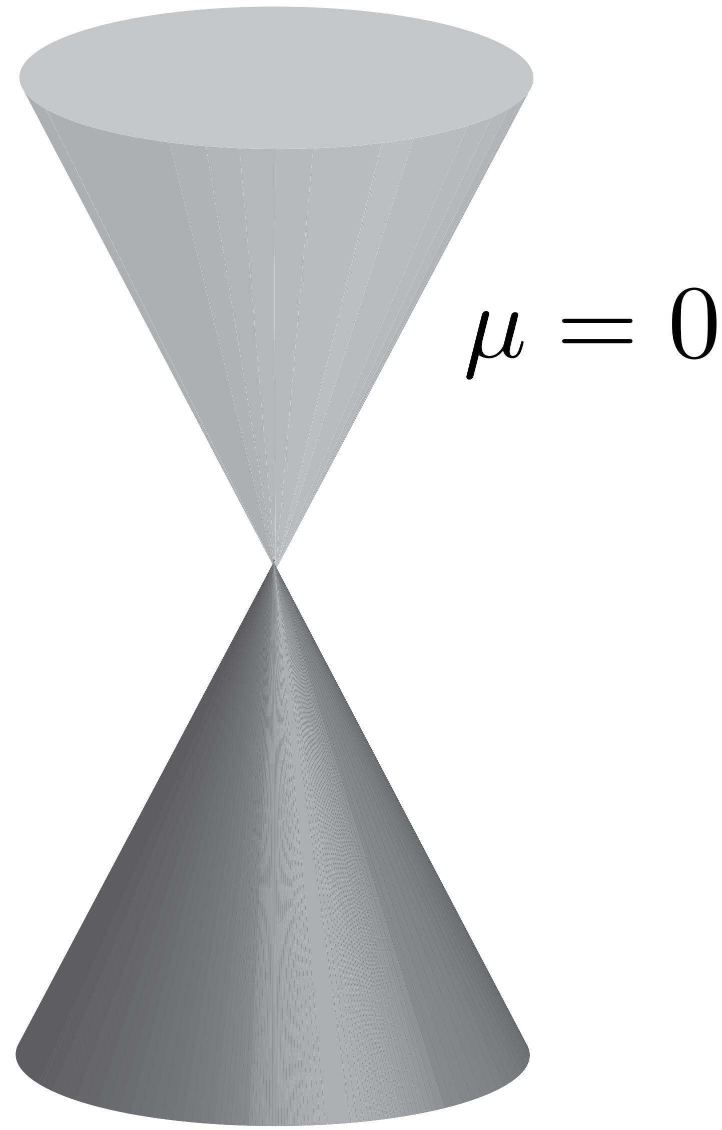
- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

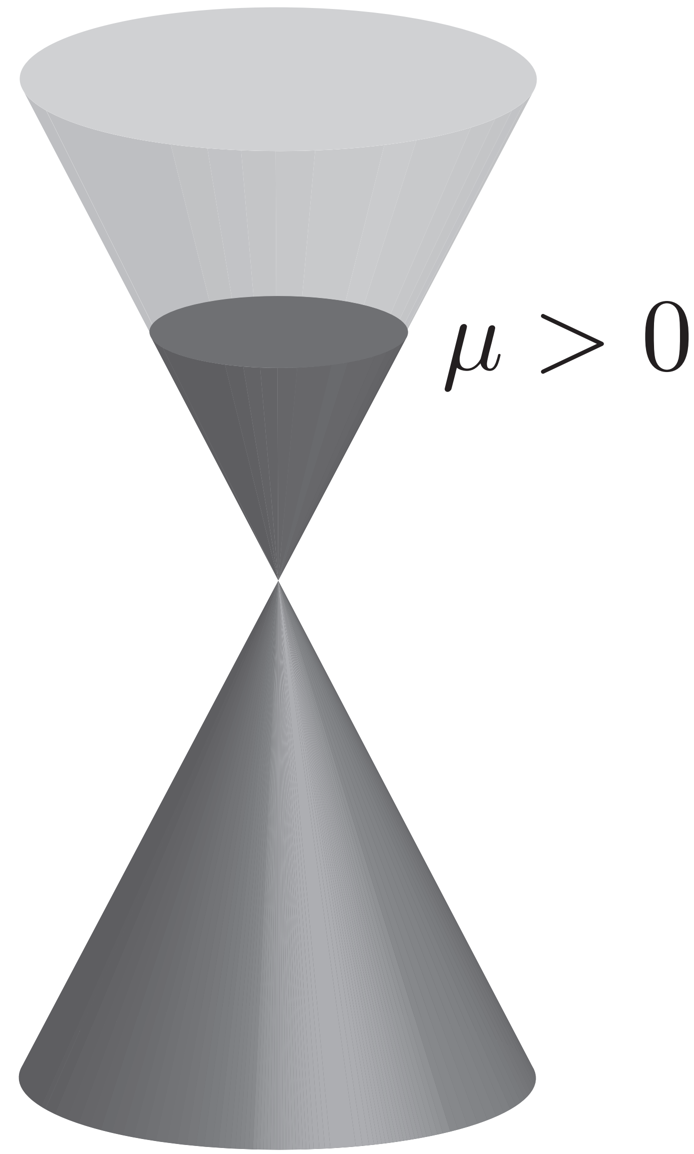
- Describe *zero temperature* phases where $\langle \mathcal{Q} \rangle$ varies smoothly as a function of any external parameter μ (the "chemical potential"). For simplicity, we assume μ couples linearly to $\mathcal{Q}$.

Turning on a chemical potential on a CFT



Massless Dirac fermions
(e.g. graphene)

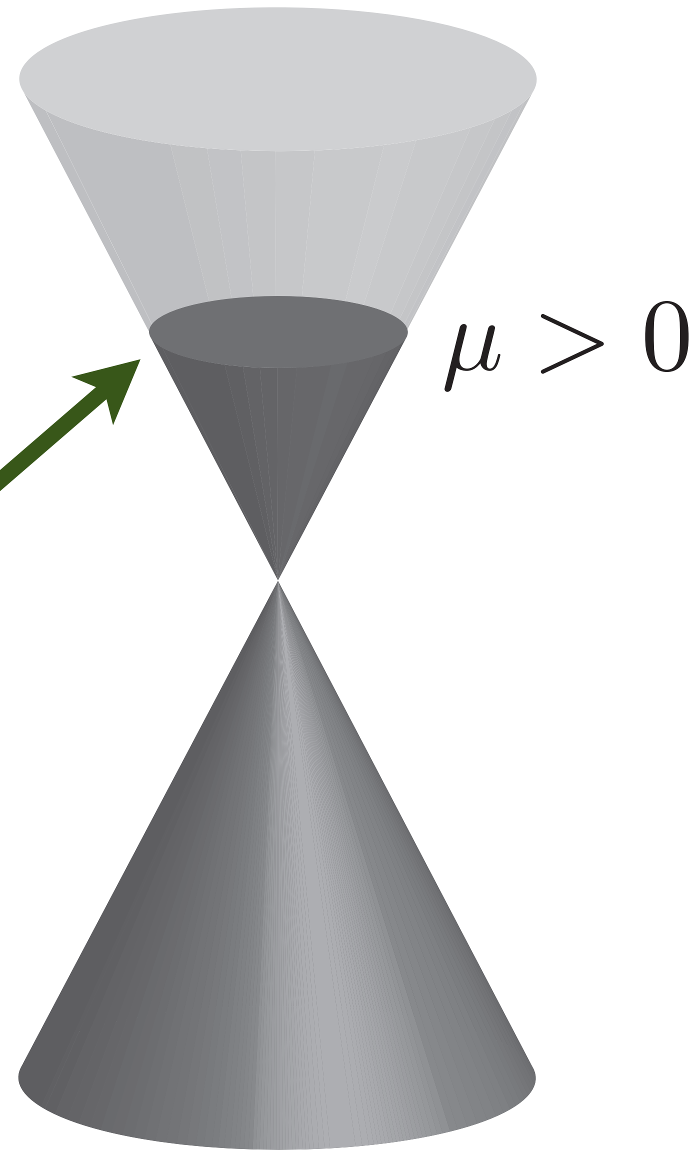
Turning on a chemical potential on a CFT



Massless Dirac fermions
(e.g. graphene)

Turning on a chemical potential on a CFT

Compressible phase is a **Fermi Liquid** with a **Fermi surface**



Massless Dirac fermions
(e.g. graphene)

The Fermi surface

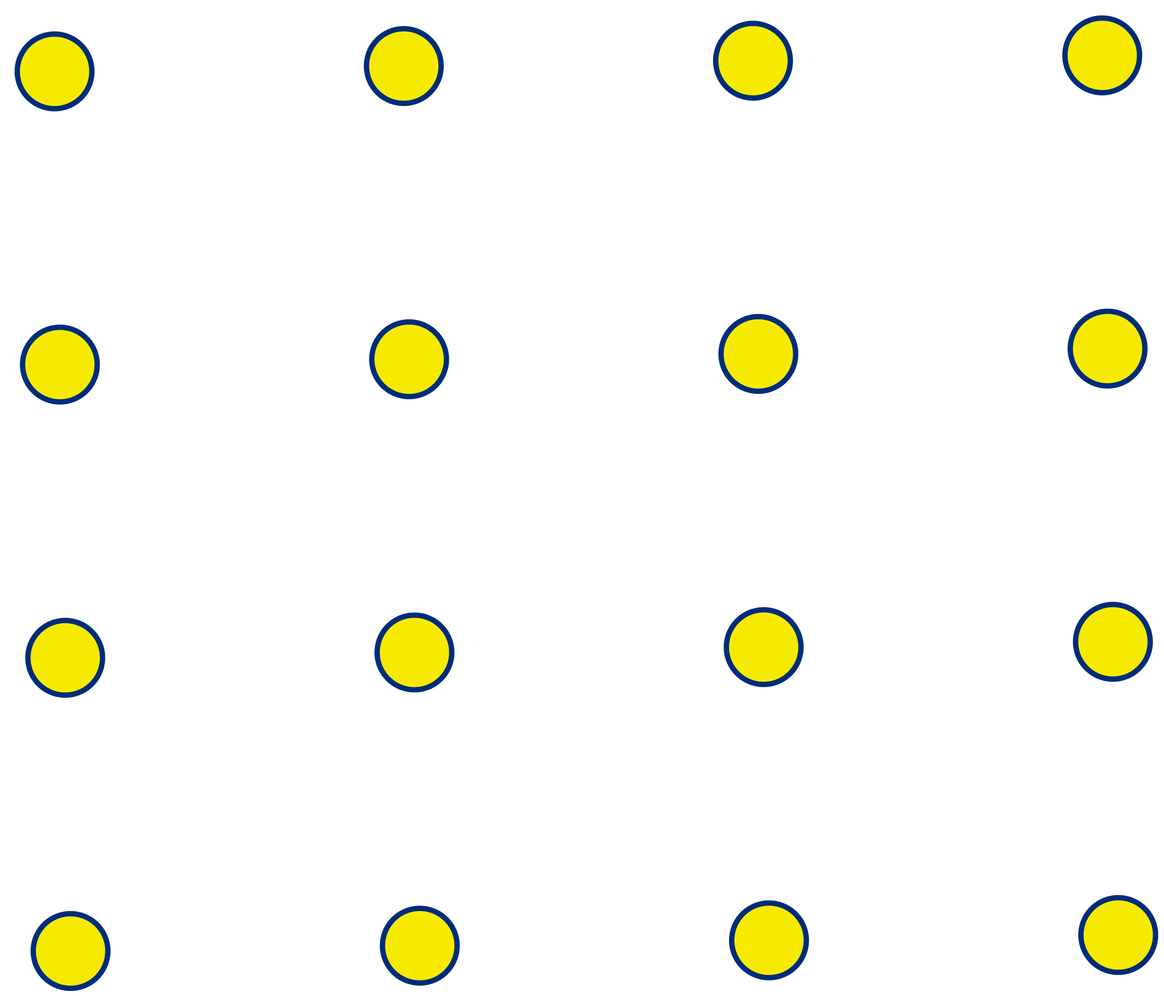
This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge $\mathcal{Q}$.

$$G^{-1}_{\text{fermion}}(k = k_F, \omega = 0) = 0.$$

Luttinger relation: The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$. This is a *key* constraint which allows extrapolation from weak to strong coupling.

Compressible quantum matter

Another compressible state is the **solid**
(or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

The only other familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

Compressible quantum matter

Conjecture: All compressible states which preserve translational and global $U(1)$ symmetries must have FERMI SURFACES, *but they are not necessarily Fermi liquids.*

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- Such states obey the Luttinger relation

$$\sum_\ell q_\ell \mathcal{A}_\ell = \langle \mathcal{Q} \rangle,$$

where the ℓ'th Fermi surface has fermionic quasiparticles with global U(1) charge q_ℓ and encloses area $\mathcal{A}_\ell$.

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- Non-Fermi liquids have quasiparticles coupled to deconfined gauge fields (or gapless bosonic modes at quantum critical points).

ABJM theory in D=2+1 dimensions

- $4N^2$ Weyl fermions carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $4N^2$ complex bosons carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $\mathcal{N}=6$ supersymmetry

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- $\mathcal{N} = 6$ supersymmetry

Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance

Theory similar to ABJM

- U(1) gauge invariance and U(1) global symmetry
- Fermions, f_+ and f_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- Bosons, b_+ and b_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- No supersymmetry

L. Huijse and S. Sachdev, arXiv:1104.5022

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- Bosons, b_+ and b_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- No supersymmetry
- Fermions, c, gauge-invariant bound states of fermions and bosons carrying global U(1) charge 2.

L. Huijse and S. Sachdev, arXiv:1104.5022

Theory similar to ABJM

$$\begin{aligned} \mathcal{L} &= f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma \\ &+ b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma \\ &+ \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right) \end{aligned}$$

The index $\sigma = \pm 1$

L. Huijse and S. Sachdev, arXiv:1104.5022

Theory similar to ABJM

$$
\begin{aligned}
\mathcal{L} &= f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma \\
&+ b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma \\
&\quad + \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right) \\
&+ c^\dagger \left[\partial_\tau - \frac{\nabla^2}{2m_c} + \epsilon_2 - 2\mu \right] c \\
&\quad - g_2 \left[c^\dagger \left(f_+ b_- + f_- b_+ \right) + \text{H.c.} \right]
\end{aligned}
$$

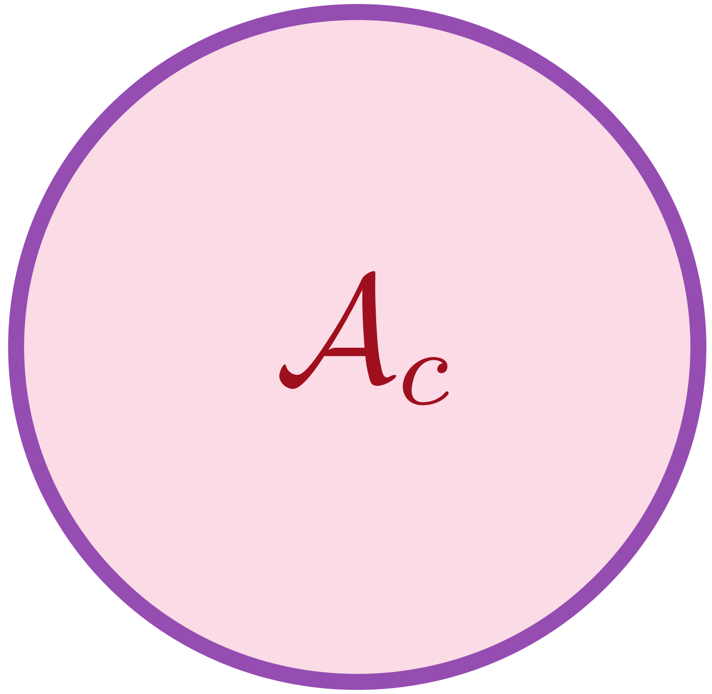
The index $\sigma = \pm 1$, and $\epsilon_{1,2}$ are tuning parameters of phase diagram

Conserved U(1) charge: $\mathcal{Q} = f_\sigma^\dagger f_\sigma + b_\sigma^\dagger b_\sigma + 2c^\dagger c$

L. Huijse and S. Sachdev, arXiv:1104.5022

Phases of ABJM-like theories

$$\langle b_{\pm} \rangle = 0$$



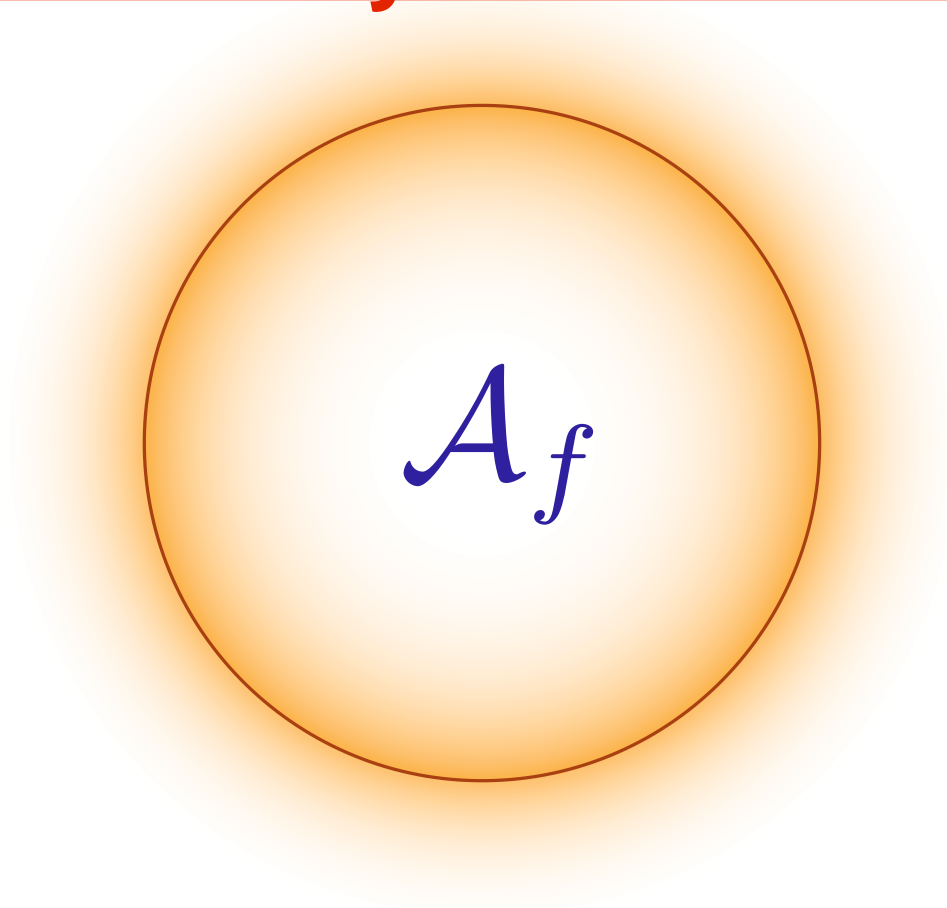
$$2\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

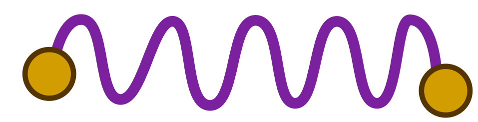
Fermi liquid (FL) of gauge-neutral particles

U(1) gauge theory is in *confining* phase

Phases of ABJM-like theories

$$\langle b_{\pm} \rangle = 0$$





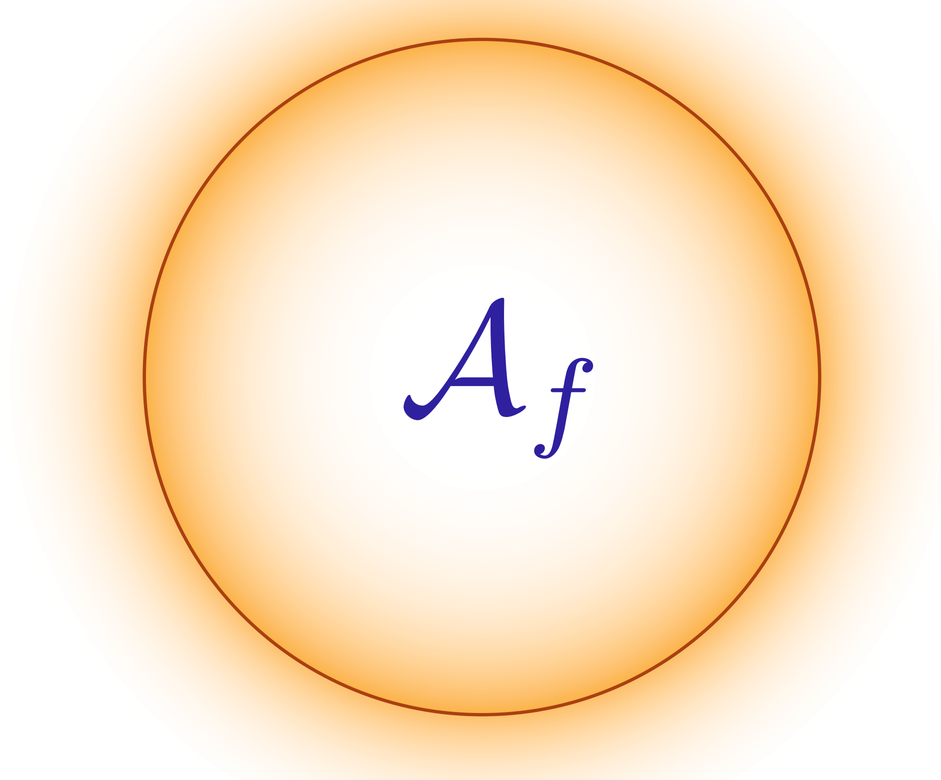
$$2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

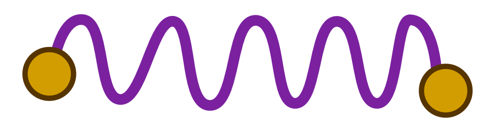
non-Fermi liquid (NFL)

U(1) gauge theory is in *deconfined* phase

Phases of ABJM-like theories

$\langle b_{\pm} \rangle = 0$





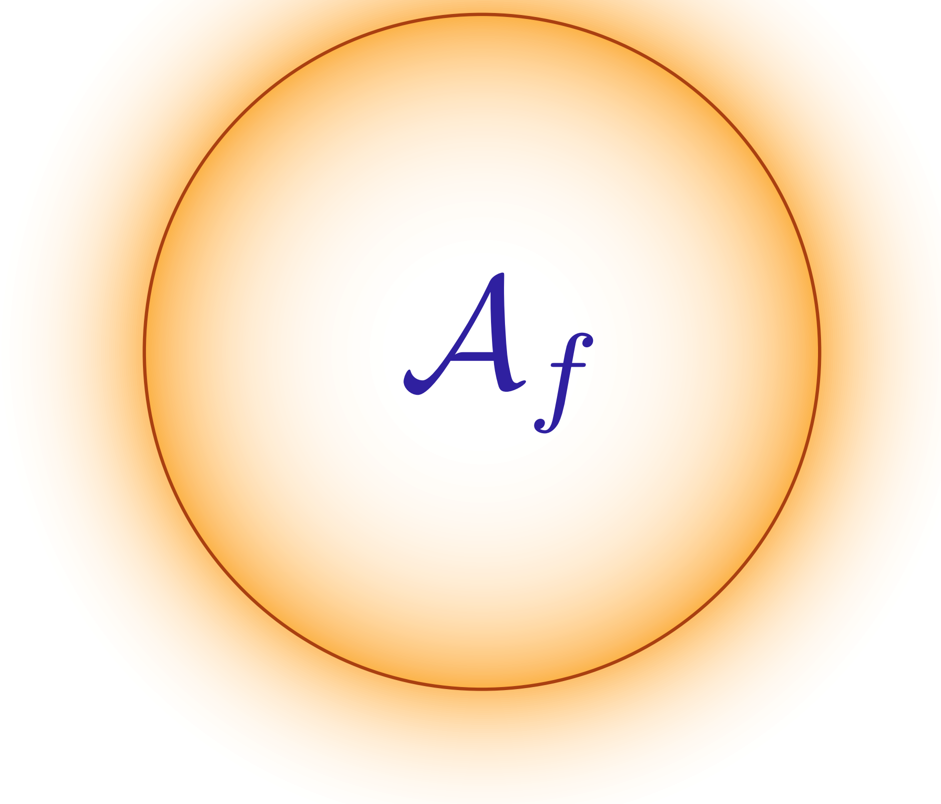
Fermi surface coupled to Abelian or non-Abelian gauge fields:

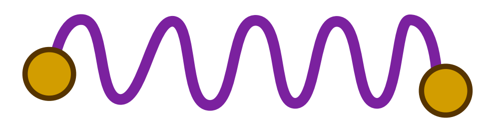
- Longitudinal gauge fluctuations are screened by the fermions.
- Transverse gauge fluctuations are unscreened, and Landau-damped. They are IR fluctuations with dynamic critical exponent $z > 1$.
- Theory is *strongly coupled in two spatial dimensions*.
- "Non-Fermi liquid" broadening of the fermion quasiparticle pole.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Phases of ABJM-like theories

$$\langle b_{\pm} \rangle = 0$$





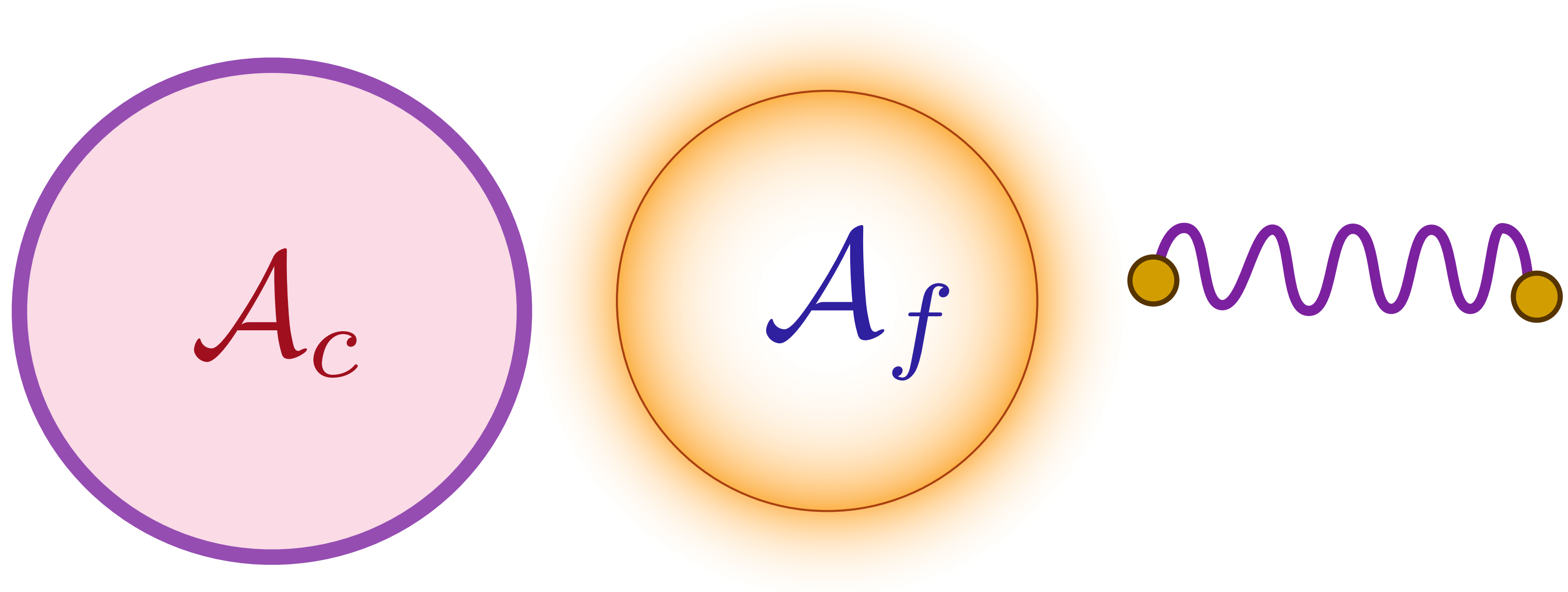
$$2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

non-Fermi liquid (NFL)

U(1) gauge theory is in *deconfined* phase

Phases of ABJM-like theories

$$\langle b_\pm \rangle = 0$$



$$2\mathcal{A}_c + 2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

Fractionalized Fermi liquid (FL*)

U(1) gauge theory is in *deconfined* phase

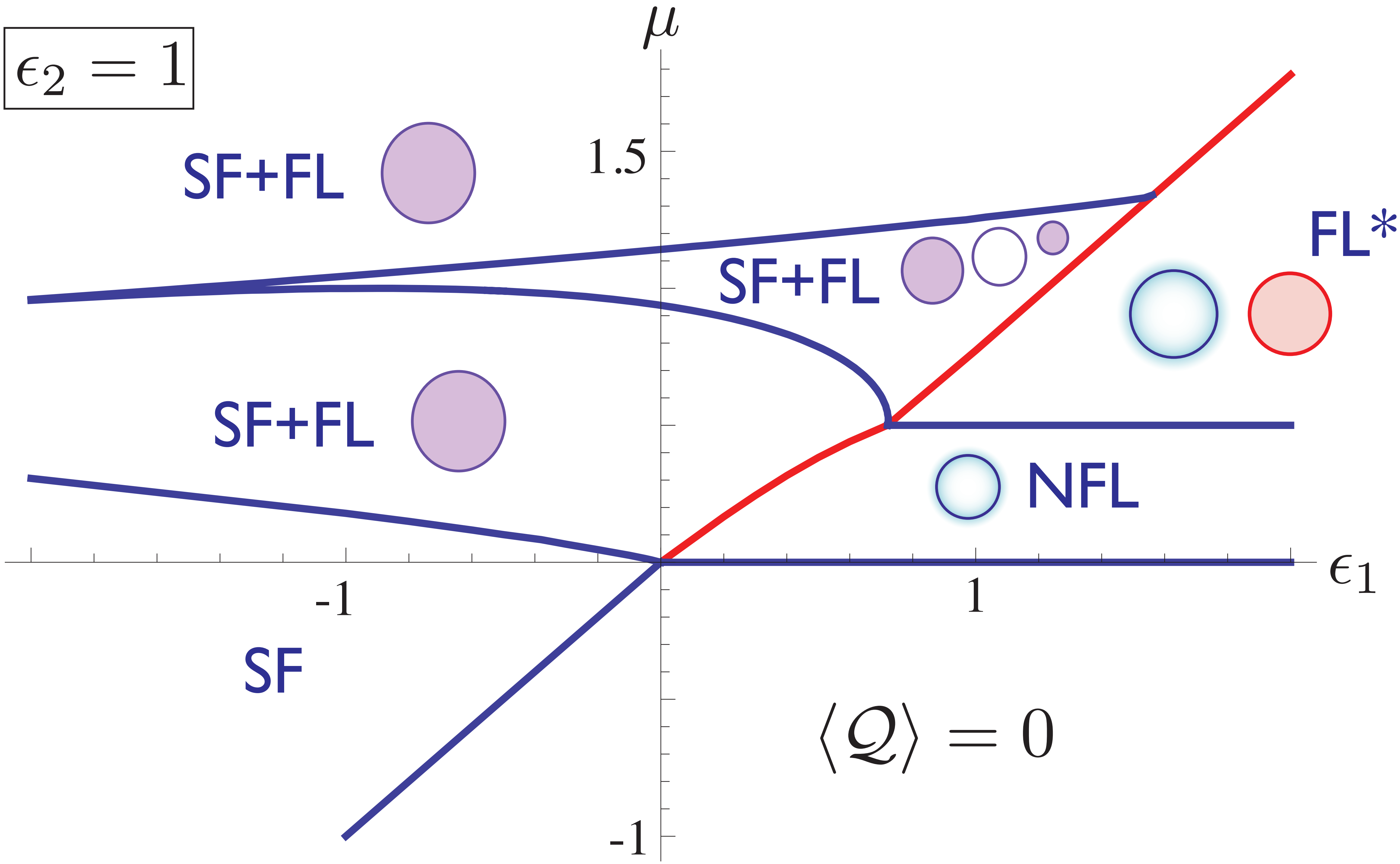
Phases of ABJM-like theories

$$\langle b_\pm \rangle \neq 0$$
$$\langle b_+ b_- \rangle \neq 0$$
$$\langle f_+ f_- \rangle \neq 0$$

No constraint on Fermi surface area, which can be zero

Superfluid (SF)

U(1) gauge theory is in *Higgs* phase, due to condensation of fermion pairs, and global U(1) is broken



L. Huijse and S. Sachdev, arXiv:1104.5022

Outline

1. Quantum criticality and conformal field theories

The AdS_4 - Schwarzschild black brane

2. Compressible quantum matter

A. Condensed matter overview

B. The AdS_4 - Reissner-Nordström black-brane and $AdS_2 \times R^2$

C. Beyond $AdS_2 \times R^2$

Outline

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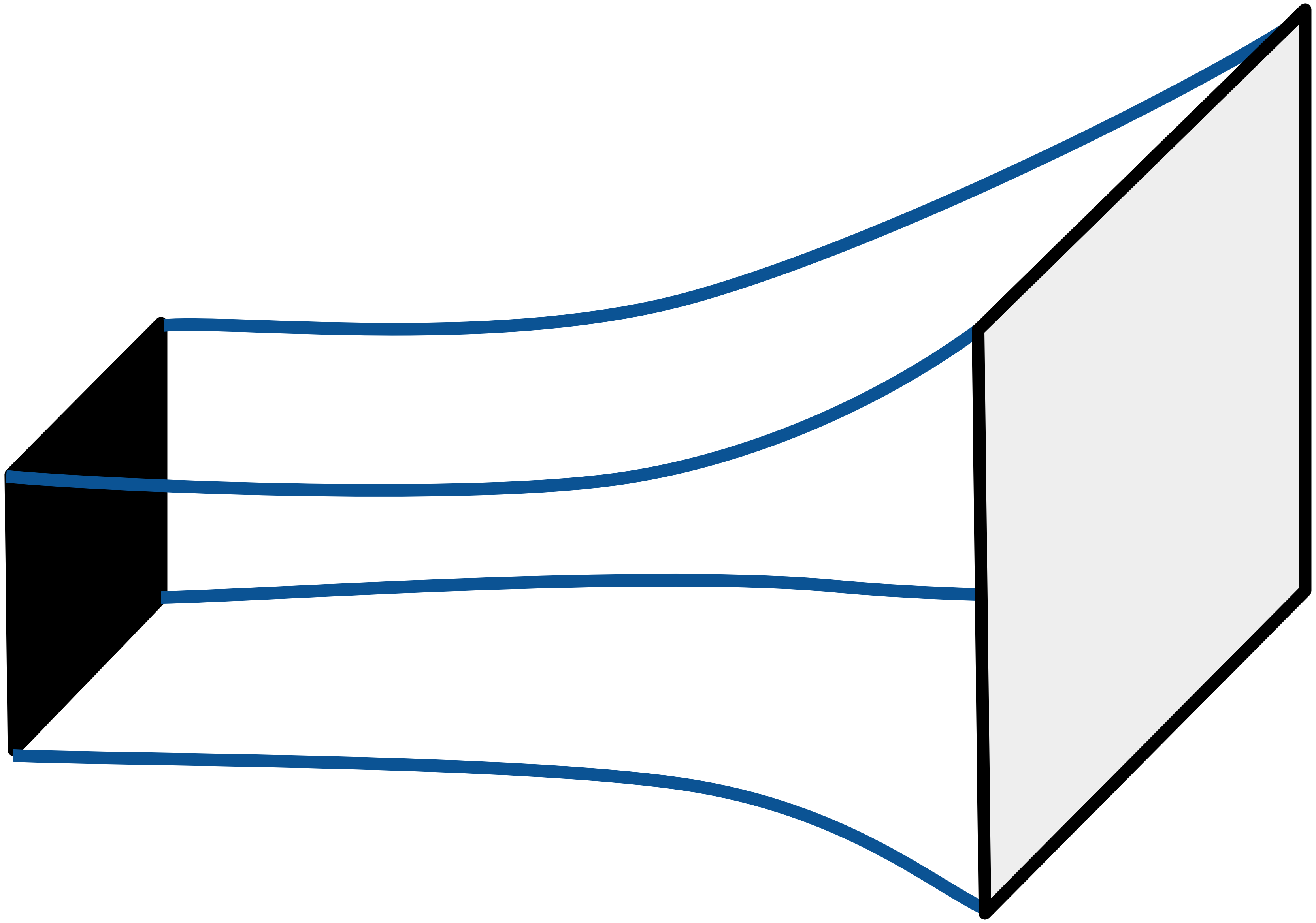
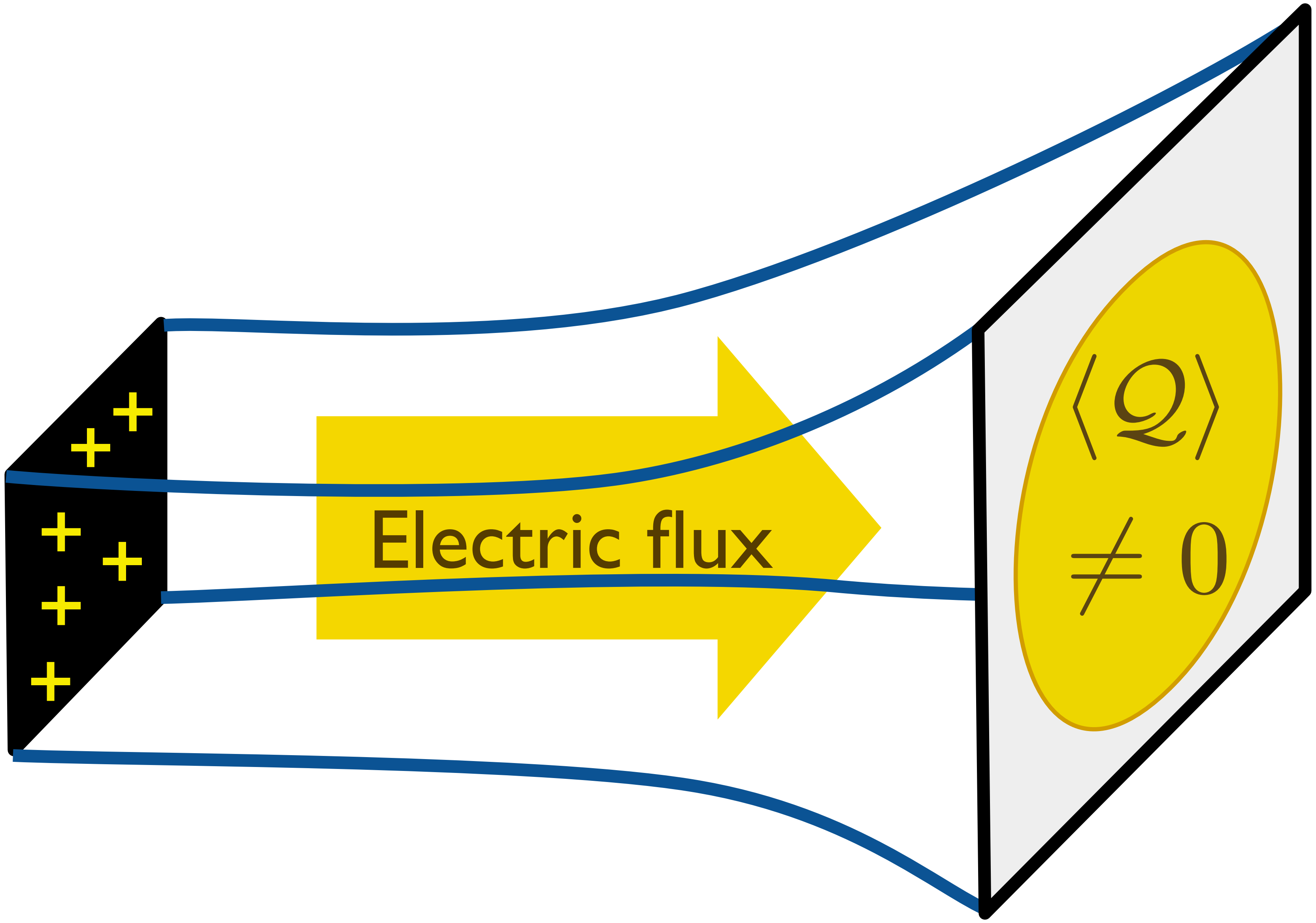
AdS$_4$-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

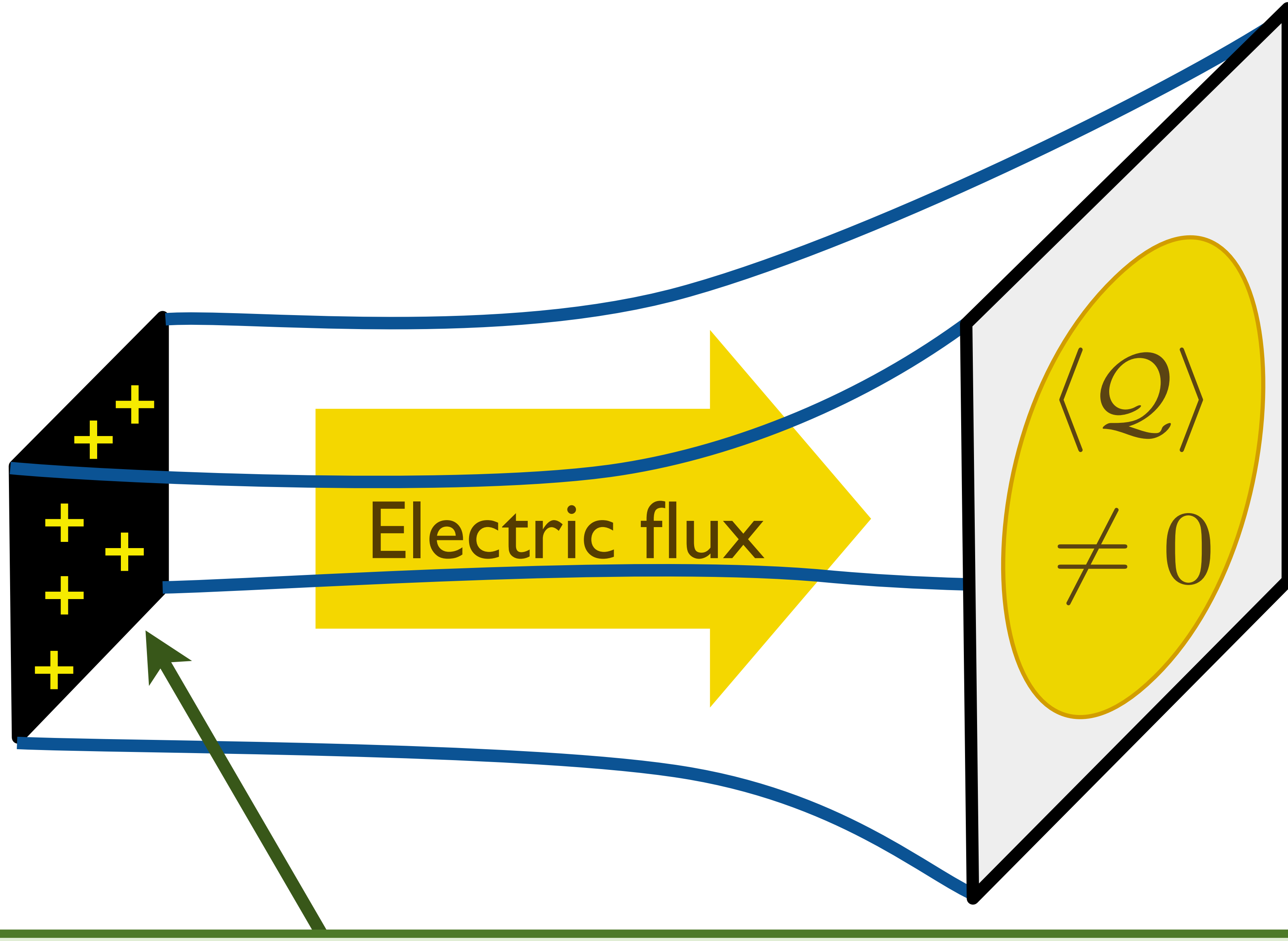
AdS$_4$-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

AdS$_4$-Reissner-Nordtröm black-brane



At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\mathrm{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

Properties of AdS_2 X R^2

This state appears stable in the presence of matter fields (with large enough bulk mass). The single-particle Green's function of the boundary theory has the IR (small ω) limit

$$G^{-1}(k,\omega) = A(k) + B(k)\omega^{\nu_k}$$

where $A(k)$, $B(k)$, and ν_k are smooth functions of k.

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

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T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
Lee; Denef, Hartnoll, Sachdev; Cubrovic, Zaanen, Schalm; Faulkner, Polchinski

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Properties of $AdS_2 \times R^2$

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For… …ve hav… …is non-Fermi liquid like.

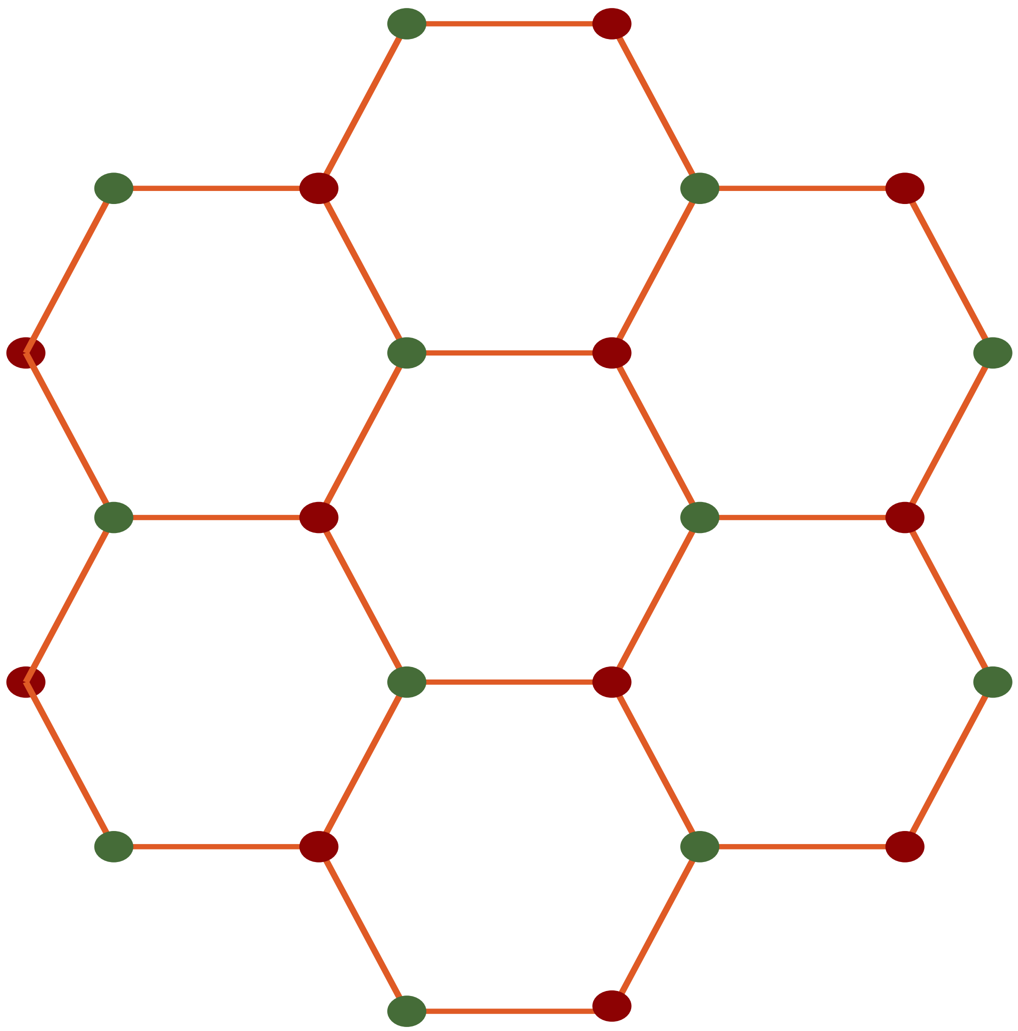
In both cases, there is a large deficit (~ order N^2) in the Luttinger count. This suggests there are ``hidden Fermi surfaces" of gauge-charged particles.

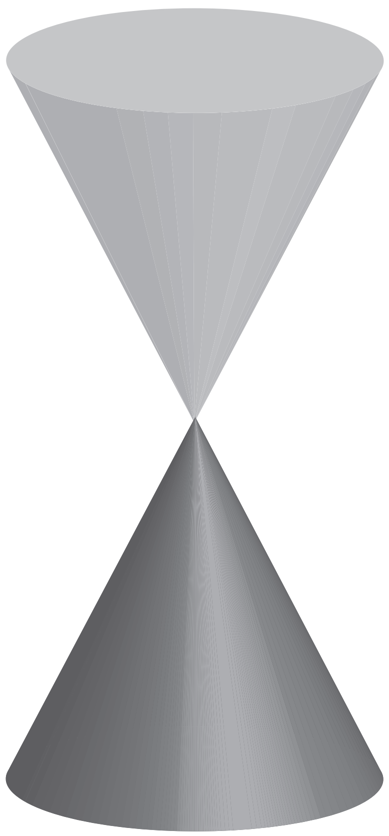
S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Lee; Denef, Hartnoll, Sachdev; Cubrovic, Zaanen, Schalm; Faulkner, Polchinski

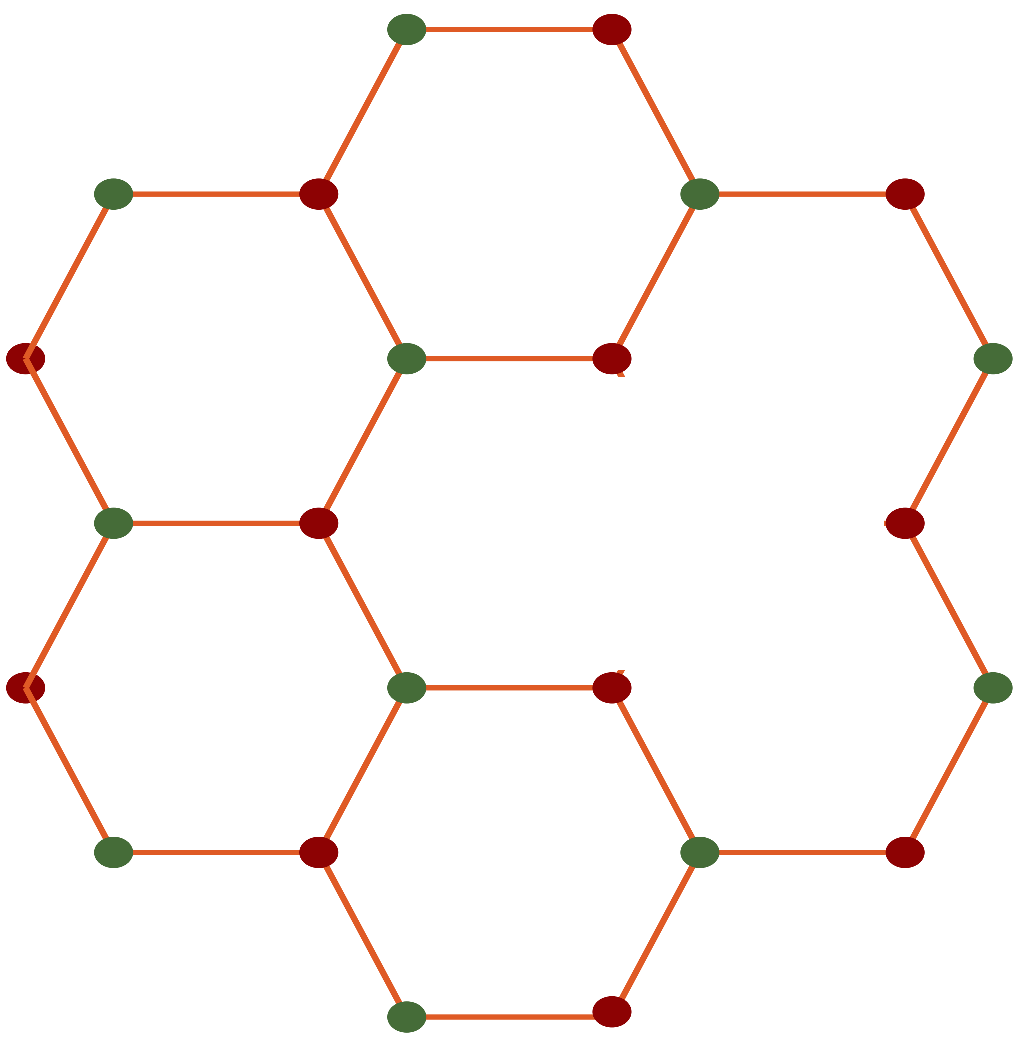
Interpretation of AdS_2

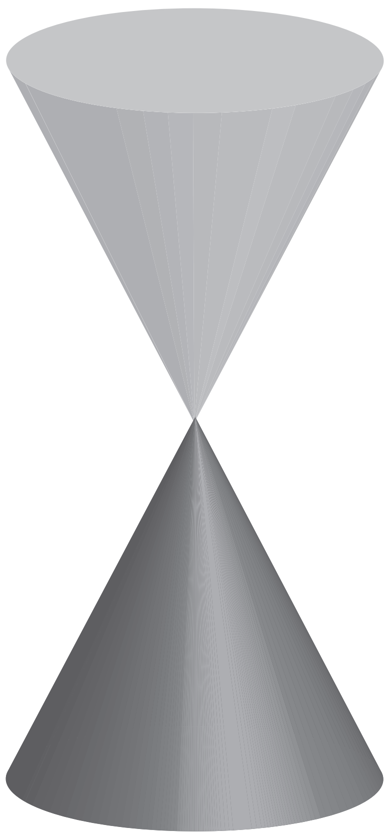




CFT on graphene

Interpretation of AdS_2

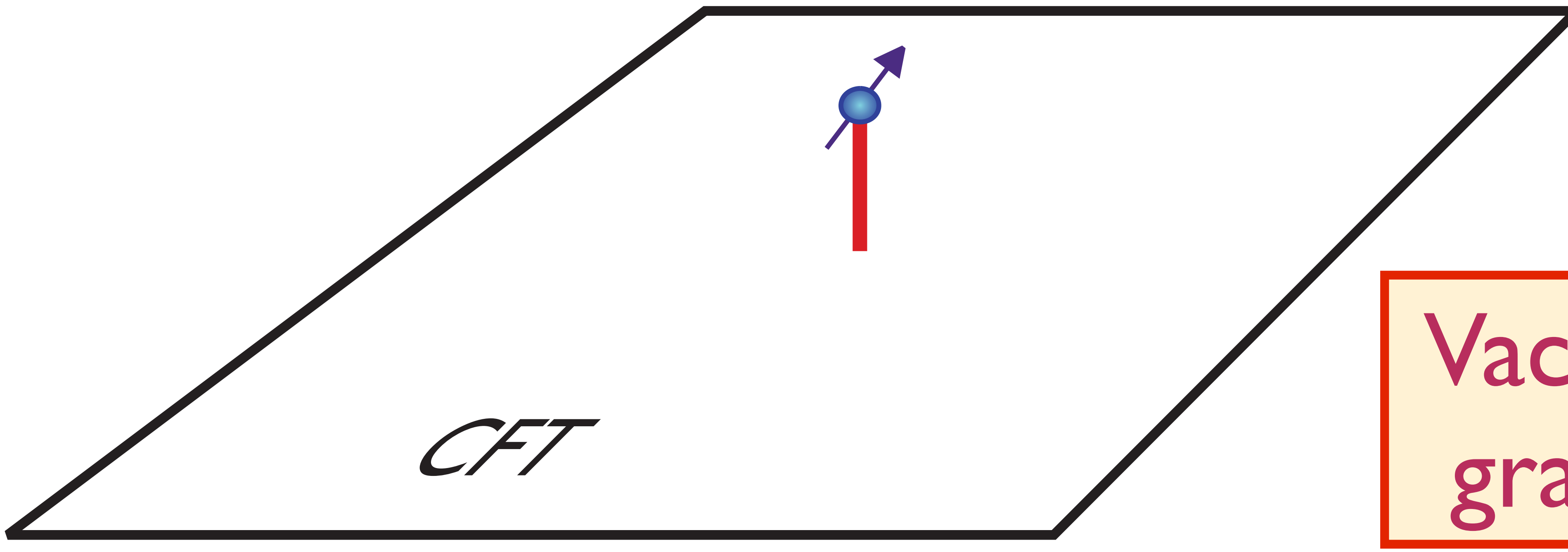




Add "matter" one-at-a-time: honeycomb lattice with a vacancy.

There is a zero energy quasi-bound state with $|\psi(r)| \sim 1/r$.
We represent this by a localized fermion field $\chi_\alpha(\tau)$.

Interpretation of AdS_2



Vacancy in graphene

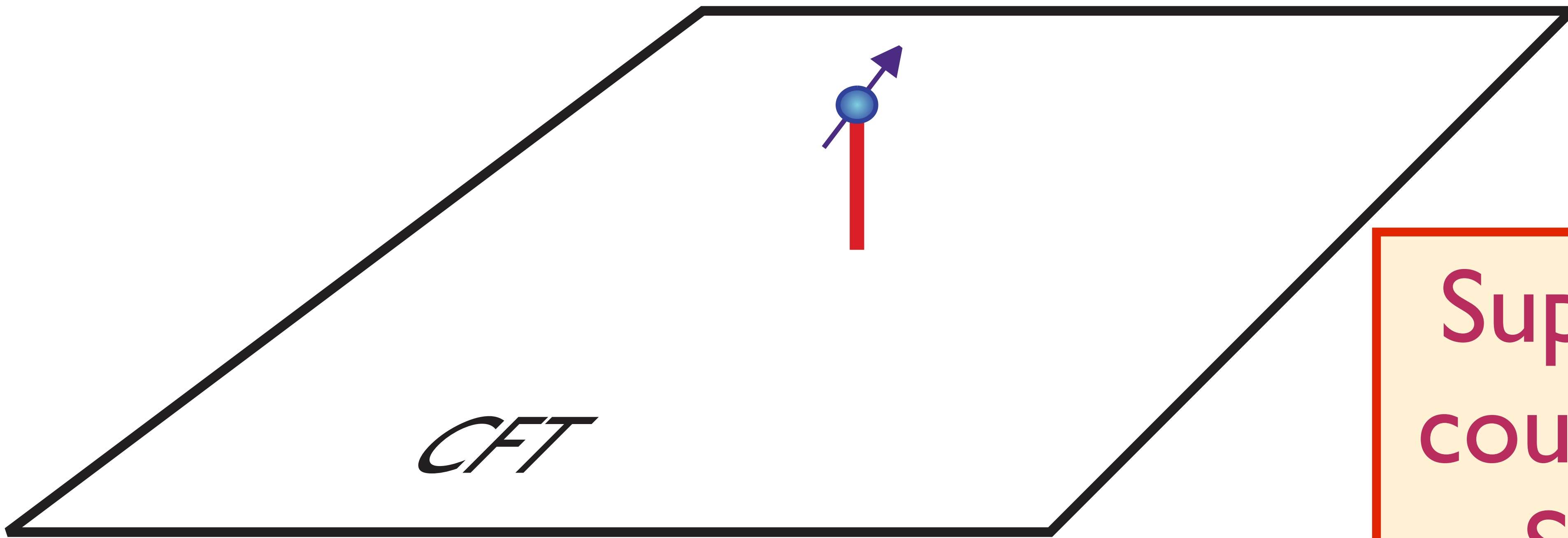
$$\mathcal{S} = \int d^3x\, \mathcal{L}_{\rm CFT} - \int d\tau \mathcal{L}_{\rm imp}$$

$$\mathcal{L}_{\rm imp} = \chi_\alpha^\dagger \frac{\partial \chi_\alpha}{\partial \tau} - \kappa\, \chi_\alpha^\dagger \sigma^a_{\alpha\beta} \chi_\beta\, \varphi^a(\mathbf{r} = 0, \tau)$$

AdS_2: "Boundary" conformal field theory obtained when κ flows to a fixed point $\kappa \to \kappa^*$.

S. Sachdev, C. Buragohain, and M. Vojta, *Science* **286**, 2479 (1999)

Interpretation of AdS_2



Superspin coupled to SYM4

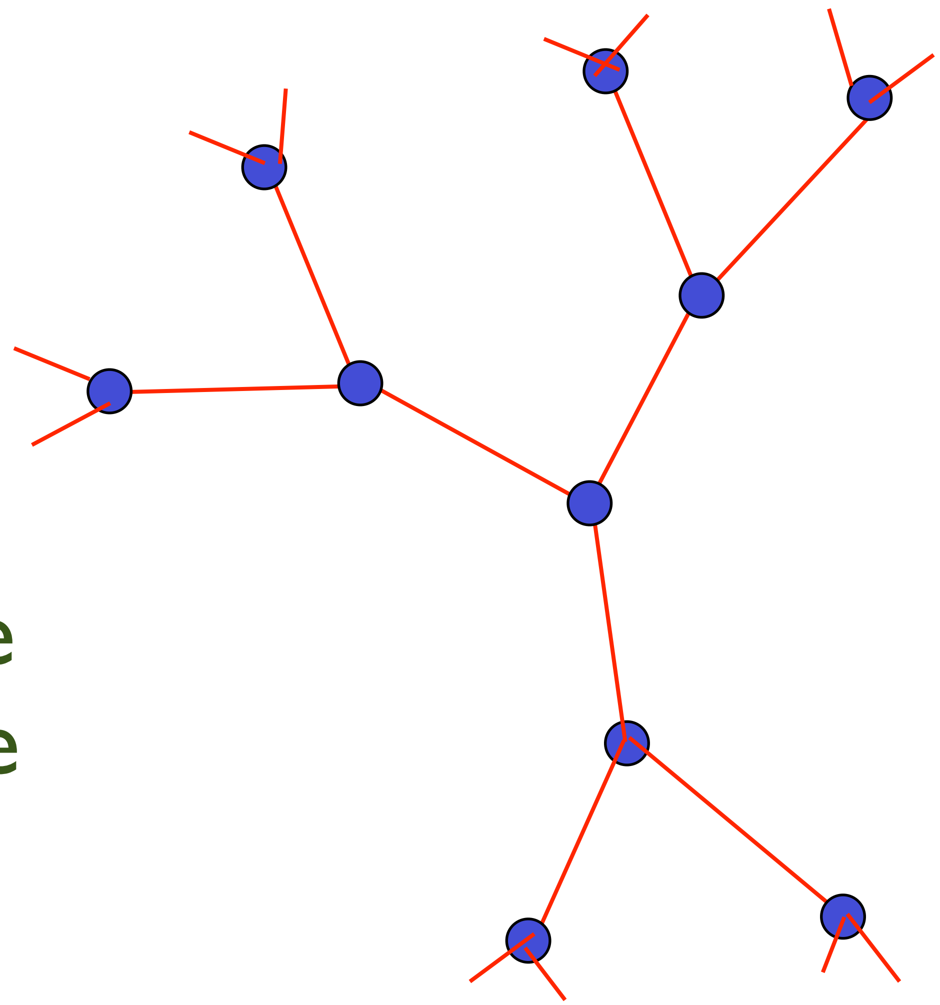
$$\mathcal{S} = \int d^4x\, \mathcal{L}_{\rm SYM} + \int d\tau\, \mathcal{L}_{\rm imp}$$

$$\mathcal{L}_{\rm imp} = \chi_b^\dagger \frac{\partial \chi^b}{\partial \tau} + i\chi_b^\dagger \left[(A_\tau(0,\tau))^b_c + v^I\, (\phi_I(0,\tau))^b_c \right] \chi^c$$

Solution in large N limit shows low energy theory of impurity is described by AdS_2

S. Kachru, A. Karch, and S. Yaida, Phys. Rev. D **81**, 026007 (2010)

Interpretation of $AdS_2 \times R^2$

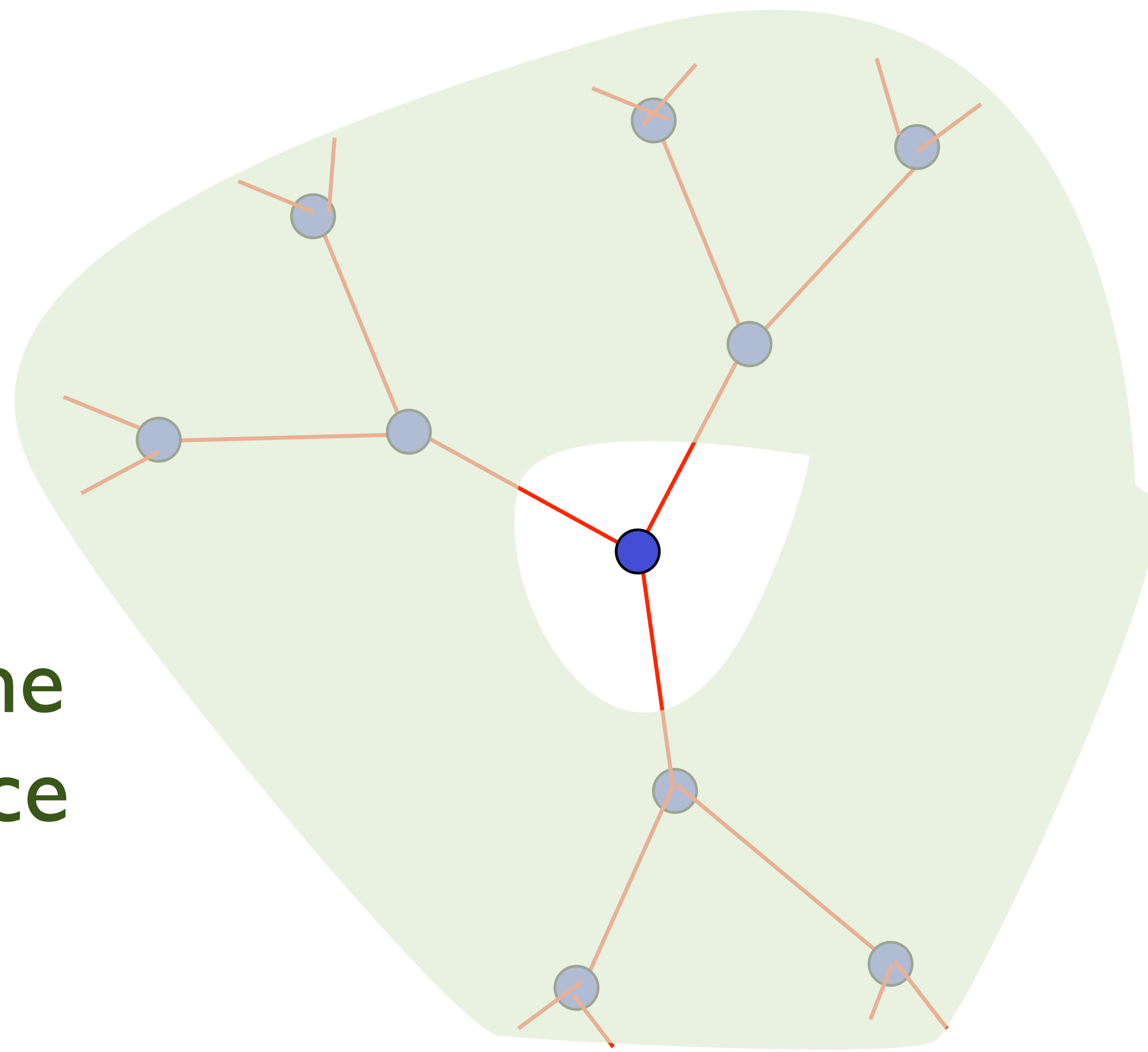


Bethe lattice

Solve electronic models in the limit of large number of nearest-neighbors

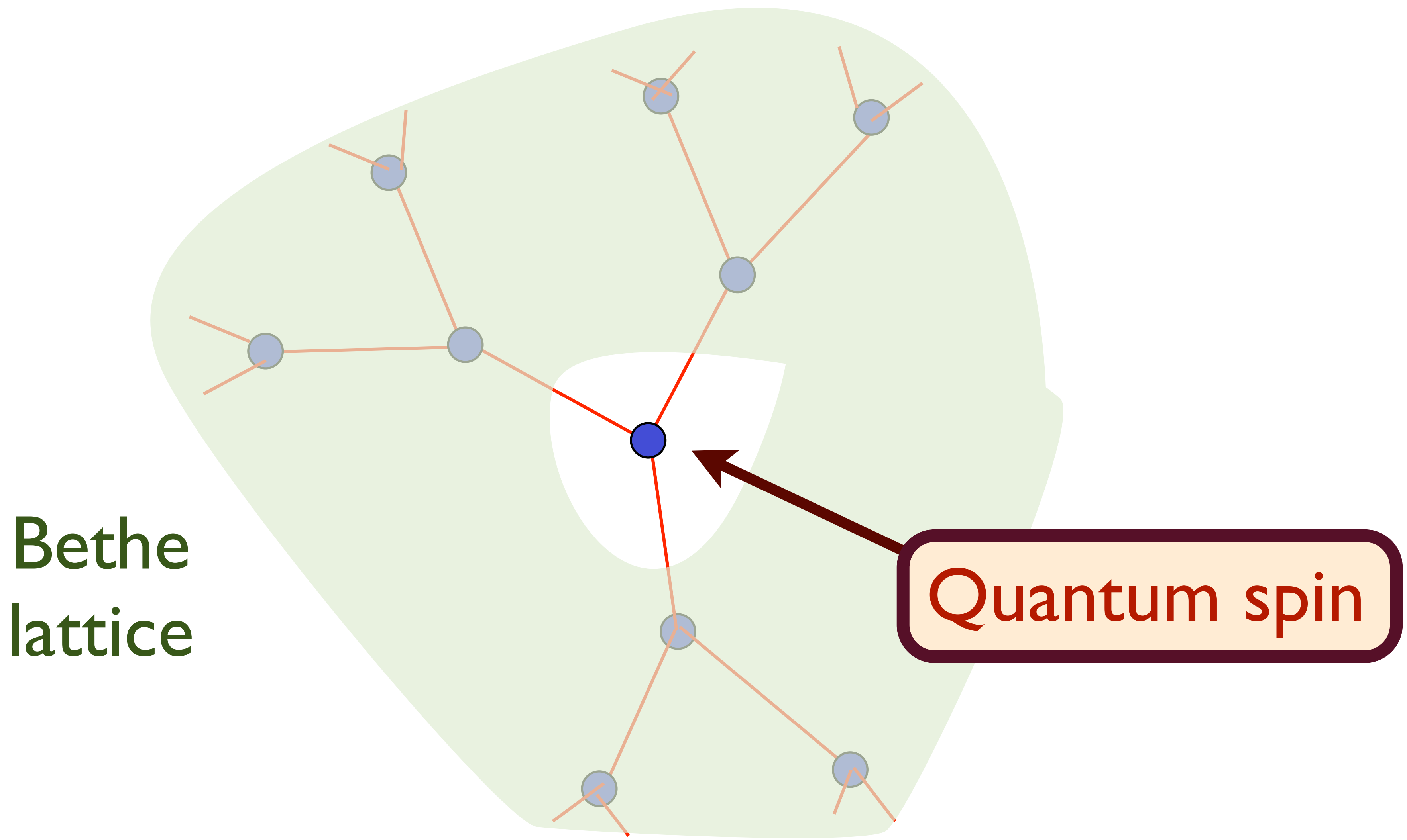
Interpretation of $AdS_2 \times R^2$

Bethe lattice



Theory is expressed as a "quantum spin" coupled to an "environment": solution is often a boundary CFT in 0+1 dimension

Interpretation of $AdS_2 \times R^2$



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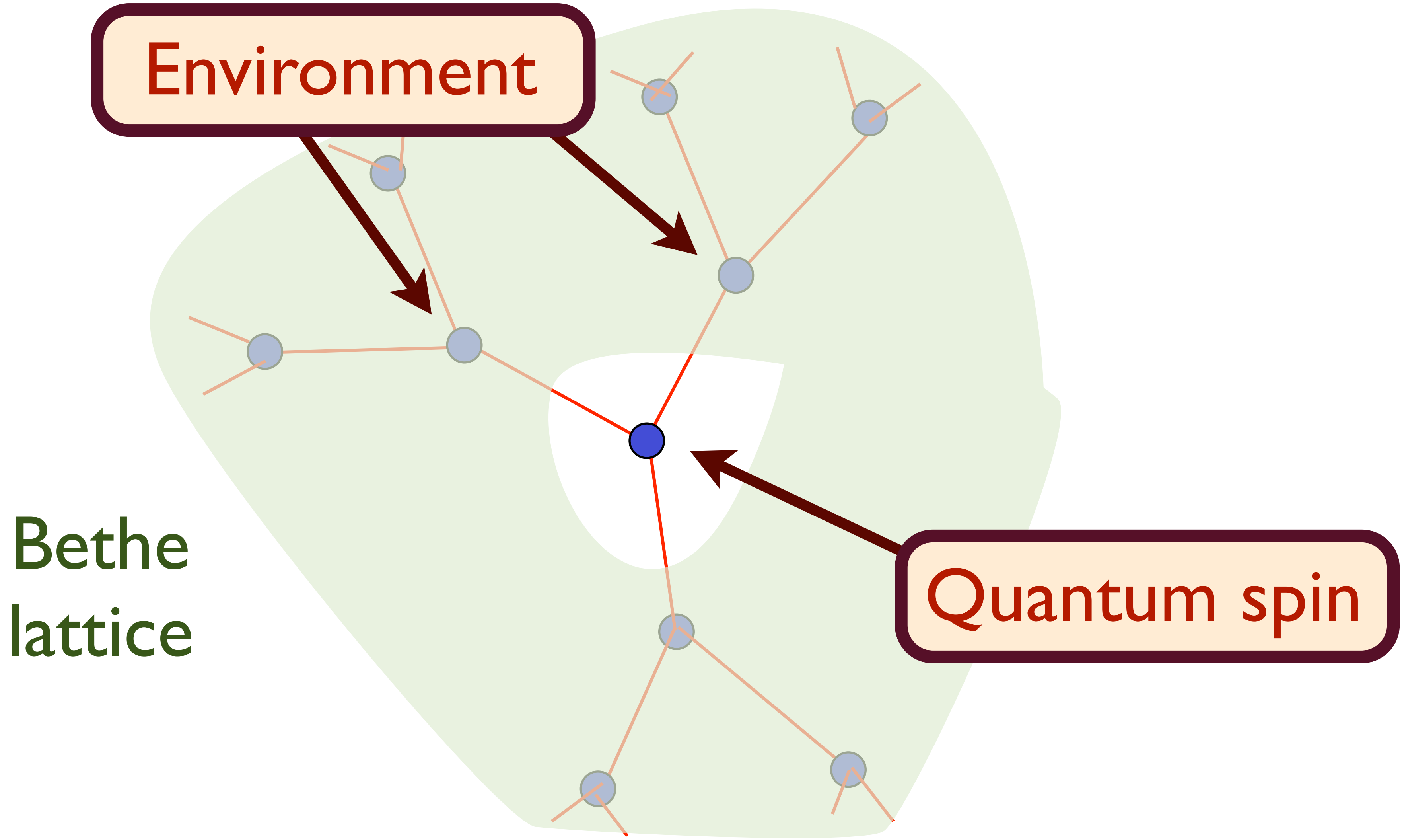

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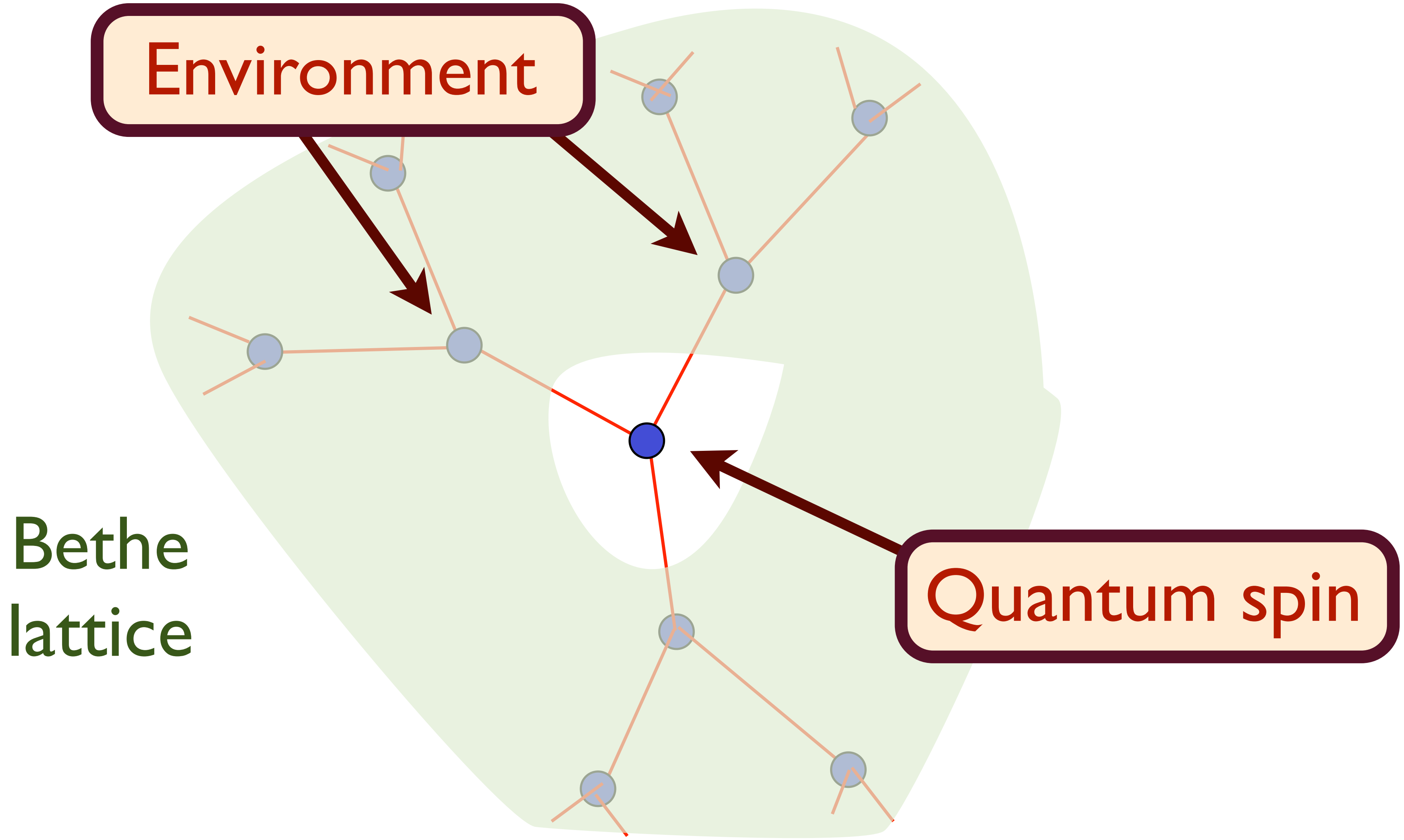

Exponents are determined by self-consistency condition between “spin” and “environment”.

Artifacts of AdS_2 X R^2

- The large-neighbor-limit solution matches with those of the $\mathrm{AdS}_2 \times R^2$ holographic solutions:
  - A non-zero ground state entropy.
  - Single fermion self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
  - A marginal Fermi liquid spectrum for fermionic quasi-particles (for the holographic solution, this requires tuning a free parameter).
  - The low energy sector has conformally invariant correlations.

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

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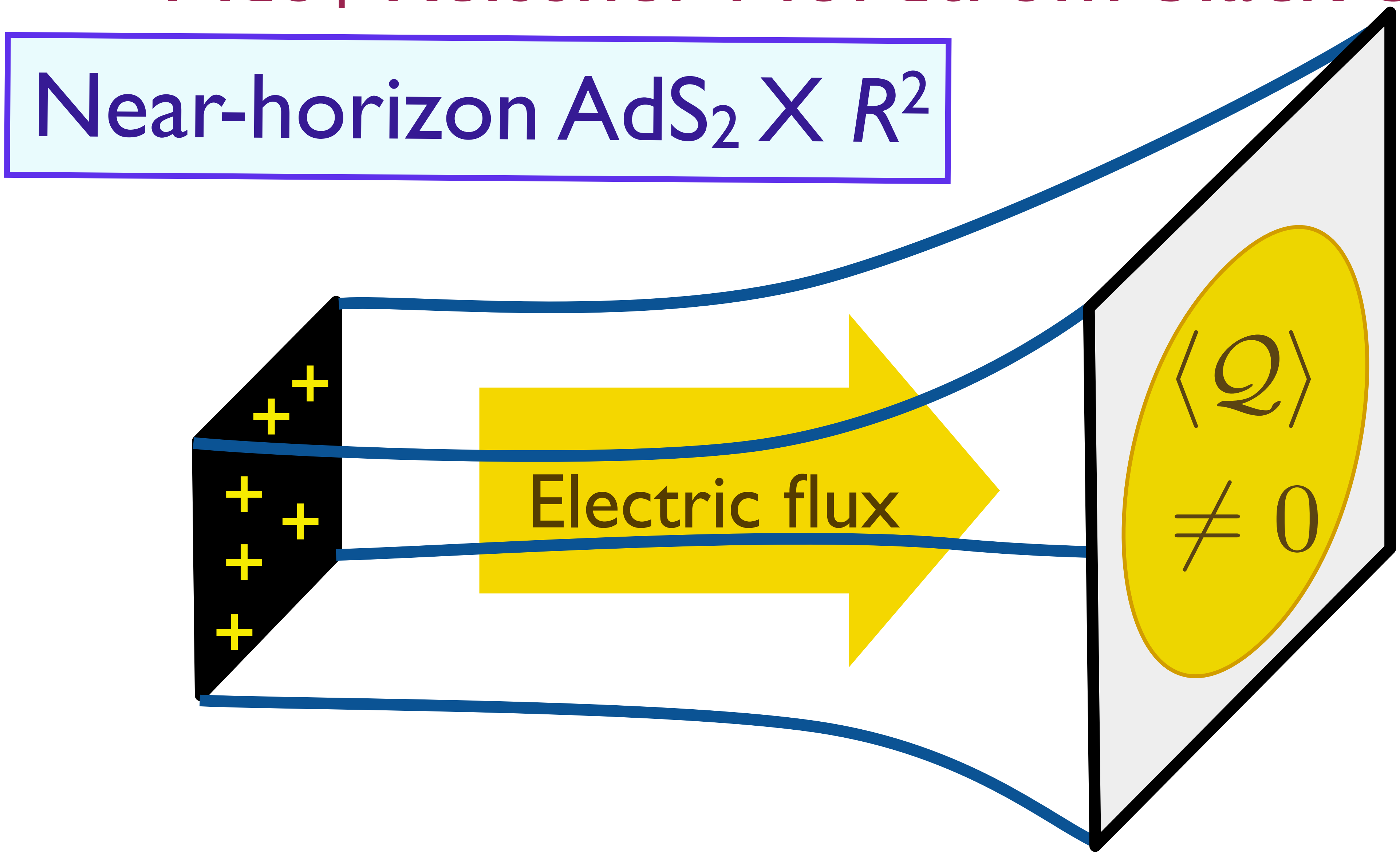
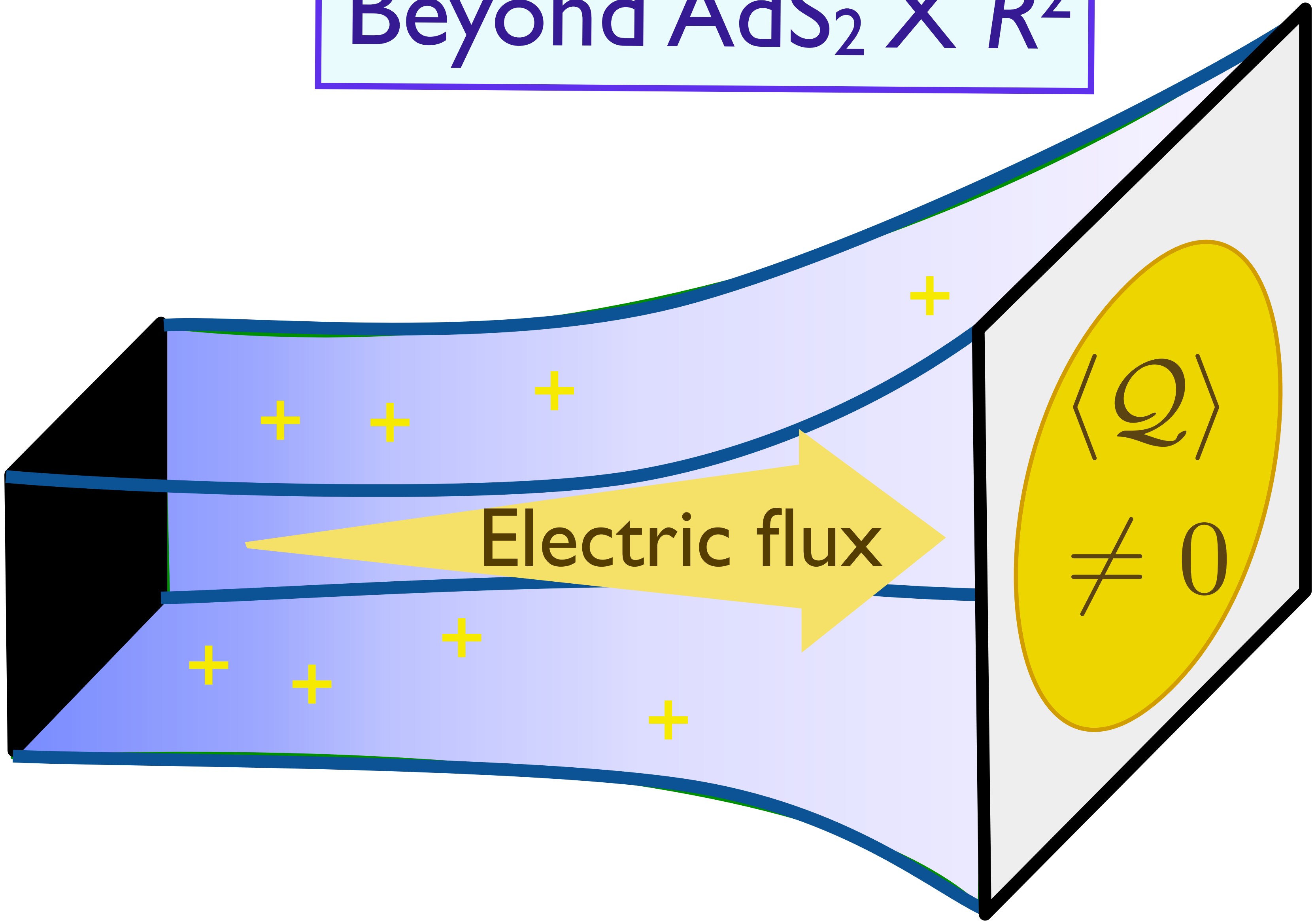
AdS$_4$-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

Beyond $AdS_2 \times R^2$



$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{ab}F^{ab} + \mathcal{L}_{\text{matter}}\right]$$

Sufficiently light matter undergoes Schwinger pair-creation, back-reacts on the metric, the horizon may disappear, and the charge density is delocalized in the bulk spacetime

Beyond $AdS_2 \times R^2$

- The metric often has a "Lifshitz" form in the IR:

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dr^2 + dx^2 + dy^2}{r^2}$$

with dynamic scaling exponent z. This possibly indicates Landau-damped transverse gauge modes. The $\mathrm{AdS}_2 \times R^2$ case corresponds to $z \to \infty$.

Kachru, Liu, Mulligan; Horowitz, Roberts; Gubser, Nellore; Hartnoll, Polchinski, Silverstein, Tong; Hartnoll, Tavanfar; Charmousis, Gouteraux, Kim, Kiritsis, Meyer; Goldtein, Iizuka, Kachru, Prakash, Trivedi, Westphal; Herzog, Klebanov, Pufu, Tesileanu

Beyond AdS_2 X R^2

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- For bosons, back-reaction on metric appears when bosons condense, leading to a holographic description of superfluids. The Lifshitz metric is mysterious, indicating the presence of additional low energy modes not found in traditional superfluids.

Gubser; Hartnoll, Herzog, Horowitz; Nishioka, Ryu, Takayanagi;
Gauntlett, Sonner, Wiseman; Gubser, Pufu, Rocha; Denef, Hartnoll;
Gusber, Herzog, Pufu, Tesileanu;
Faulkner, Horowitz, McGreevy, Roberts, Vegh;
Erdmenger, Grass, Kerner, Ngo; Ammon, Erdmenger, Kaminski, O'Bannon

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- For fermions, multiple Fermi surfaces are obtained, whose total enclosed area is <u>consistent</u> with the Luttinger count. This appears to be a Fermi liquid, but the Lifshitz metric is still mysterious.

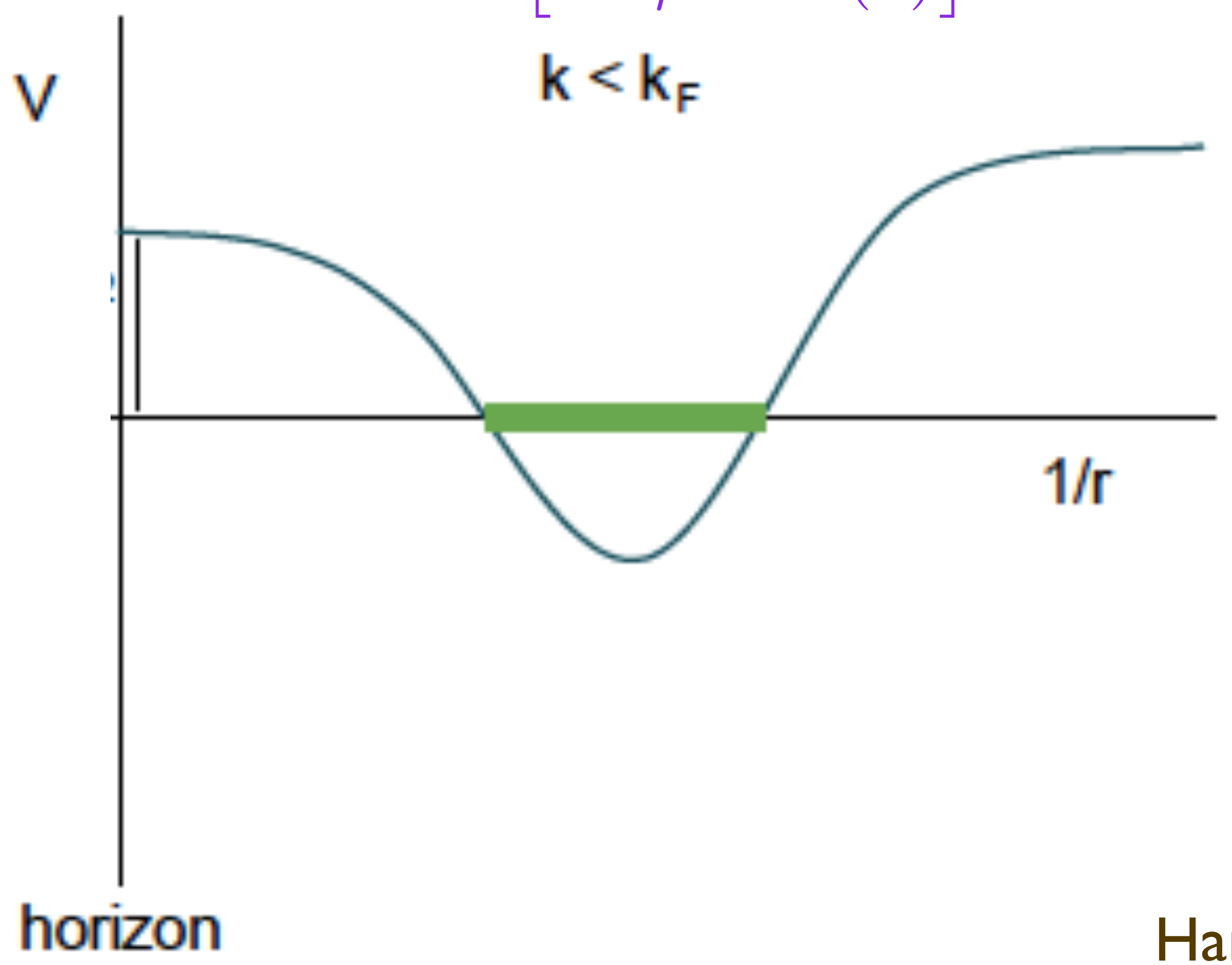
Arsiwalla, de Boer, Papadodimas, Verlinde; Hartnoll, Hofman, Vegh; Iqbal, Liu, Mezei; Cubrovic, Schalm, Sun, Zaanen

Beyond AdS_2 X R^2

- Account for the matter in a Thomas-Fermi approximation: the local chemical potential determines the local density and pressure, using the equation of state of a free Fermi gas: so determine the density, electric field, and metric as a function of r, the "extra" dimension.

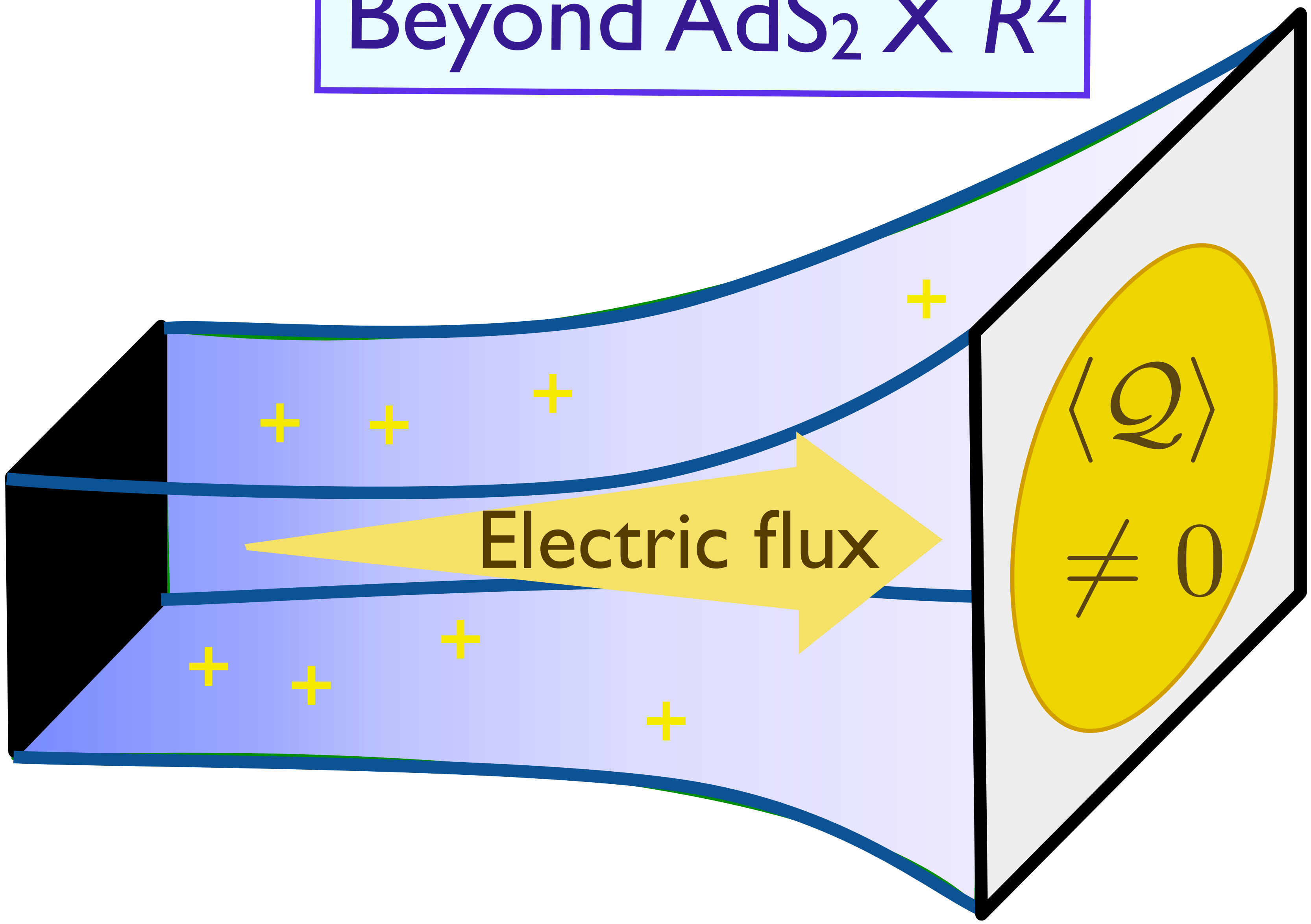
- Then compute the fermion Green's function in the background. The bulk equation for the fermion field leads to poles in Green's function at many $k = k_F^{(n)}$.

$$\left[-\partial_r^2 + V(r)\right] \Psi = 0$$



Hartnoll, Hofman, Vegh; Iqbal, Liu, Mezei;

Beyond $AdS_2 \times R^2$



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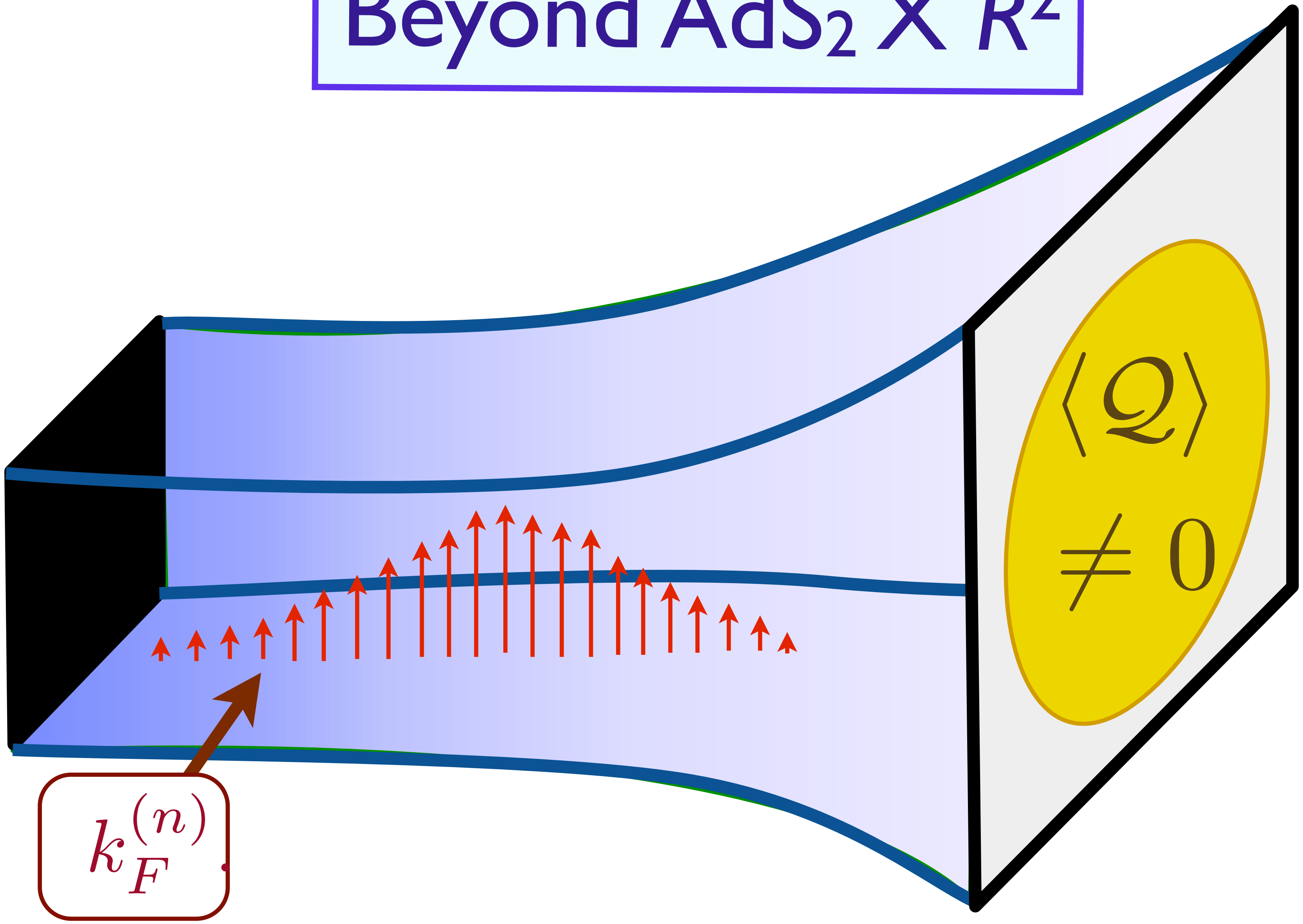
Beyond $AdS_2 \times R^2$



Luttinger relation is obeyed

$$\sum_n \pi (k_F^{(n)})^2 = \langle \mathcal{Q} \rangle$$

However, there are $\sim N^2$ Fermi surfaces, and low energy properties are dominated by $k_F^{(n)} \approx 0$.

Hartnoll, Hofman, Vegh; Iqbal, Liu, Mezei;

Conclusions

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Conclusions

Compressible quantum matter

- The Reissner-Nordström solution provides the simplest holographic theory of a compressible state

- The RN solutions has many problems: finite ground-state entropy density, violation of Luttinger relation.

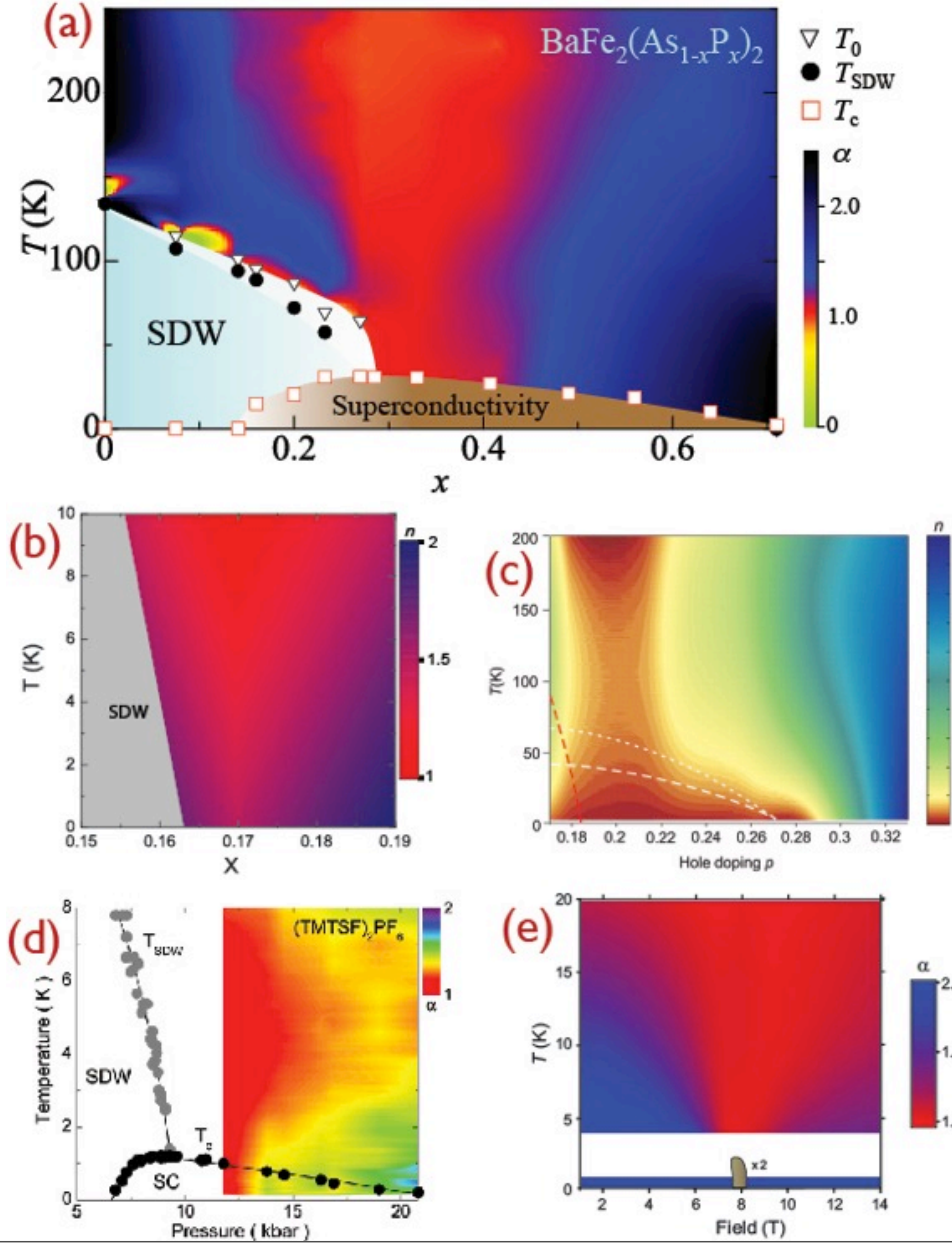
- Condensation of a scalar leads to the holographic theory of a superfluid. The IR metric has a Lifshitz form, indicating the presence of neutral gapless excitations not found in a superfluid.

Conclusions

Compressible quantum matter

- Fermion back-reaction leads to a Fermi liquid with many Fermi surfaces which do obey the Luttinger relation. However, the IR Lifshitz metric, and the very small Fermi wavevectors appear to be unwanted artifacts.

- Needed: a complete holographic theory of non-Fermi liquids and "fractionalized" Fermi liquids, obeying the Luttinger relations, to describe experiments on "strange metals".



Plots of the resistivity exponent

$$\frac{d\ln(\rho)}{d\ln T}$$

(a) Pnictide
(b) e-doped cuprate
(c) h-doped cuprate
(d) organic superconductor
(e) $Sr_2Ru_3O_7$

Umklapp scattering likely crucial