1. Quantum matter with quasiparticles:
random matrix model

2. Quantum matter without quasiparticles:
the complex SYK model

3. Fluctuations, and the Schwarzian

4. Models of strange metals

5. Einstein-Maxwell theory of charged
black holes in AdS space

The complex SYK model

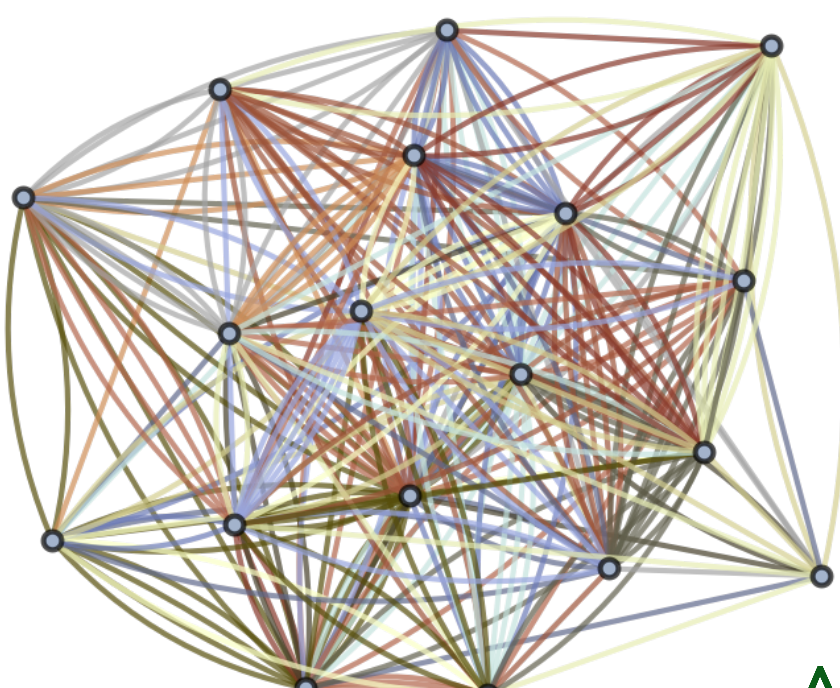
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

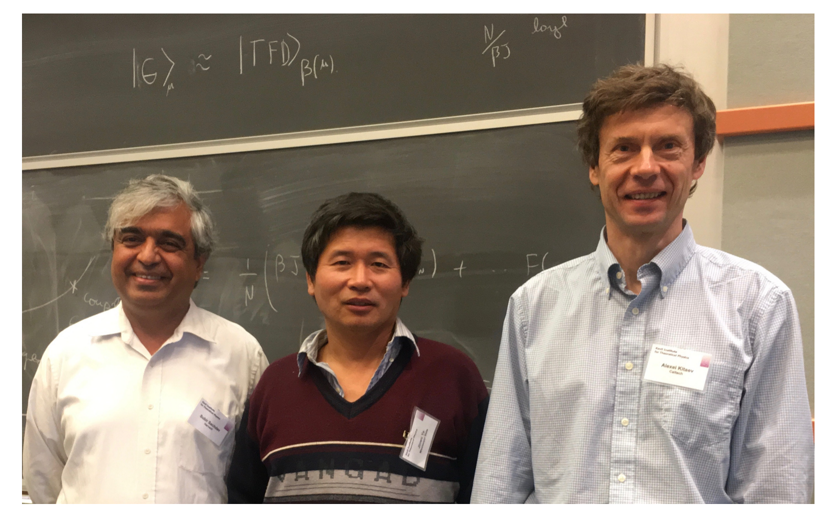
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + \epsilon \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.





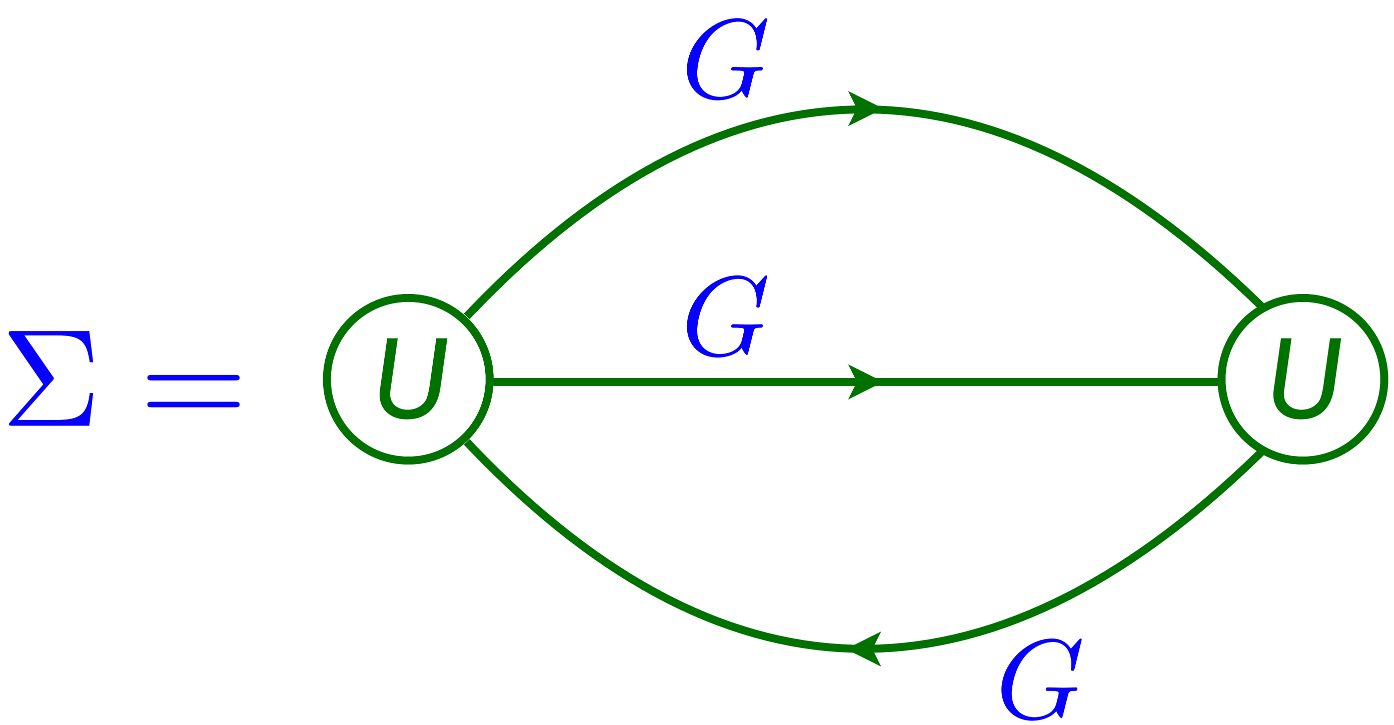
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega - \epsilon - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$



S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The complex SYK model

The large N limit is given by the sum of "melon" Feynman graphs

For long times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = \frac{A}{\sqrt{\tau}}$$

$$\left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = e^{-2\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter $\mathcal{E} = \mathbb{C}\left(\epsilon/U\right)$ determines the particle-hole asymmetry, and has a universal "Luttinger" relation to $\mathcal{Q}$.

In a Fermi liquid,

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = \left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = \widetilde{A}/\tau$$

The complex SYK model

Solution of these equations, and of the free energy, yields universal results for the SYK model with q fermion terms. These results are *quantitatively* unchanged by adding additional higher q fermion terms:

- At long times, and at $T = 0$, $G(\tau) \sim |\tau|^{-2\Delta}$ with $\Delta = 1/q$ ($\Rightarrow$ indication there are no quasiparticles)

- At general charge $\mathcal{Q}$, there is a spectral symmetry determined by a parameter $\mathcal{E}$:

$$G(\tau) \sim \left\{ \begin{array}{cc} -\tau^{-2\Delta} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-2\Delta} & \tau < 0 \end{array} \right. , \quad T = 0$$

- There is a universal 'Luttinger relation' between $-\infty < \mathcal{E} < \infty$ and the total charge $0 < \mathcal{Q} < 1$

$$\begin{aligned} e^{2\pi\mathcal{E}} &= \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)} \\ \mathcal{Q} &= \frac{1}{2} - \frac{\theta}{\pi} + \left(\Delta - \frac{1}{2}\right) \frac{\sin(2\theta)}{\sin(2\pi\Delta)} \end{aligned}$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

The complex SYK model

Solution of these equations, and of the free energy, yields universal results for the SYK model with q fermion terms. These results are *quantitatively* unchanged by adding additional higher q fermion terms:

- At $T > 0$, we obtain a solution with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi \mathcal{E} T \tau}}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \,,$$

where the 'particle-hole asymmetry' is determined by $\mathcal{E}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

The equations for the Green's function can also be solved at non-zero T. At $\epsilon = \mathcal{E} = 0$ we "guess" the solution

$$G(\tau) = B\,\mathrm{sgn}(\tau)\left|\frac{\pi T}{\sin(\pi T \tau)}\right|^{\rho}$$

Then the self-energy is

$$\Sigma(\tau) = U^2 B^3 \mathrm{sgn}(\tau)\left|\frac{\pi T}{\sin(\pi T \tau)}\right|^{3\rho}$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

The complex SYK model

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Taking Fourier transforms, we have as a function of the Matsubara frequency ω_n

$$\begin{aligned} G(i\omega_n) &= [iB\Pi(\rho)]\,\frac{T^{\rho-1}\,\Gamma\left(\frac{\rho}{2}+\frac{\omega_n}{2\pi T}\right)}{\Gamma\left(1-\frac{\rho}{2}+\frac{\omega_n}{2\pi T}\right)} \\ \Sigma_{\rm sing}(i\omega_n) &= \left[iU^2B^3\Pi(3\rho)\right]\,\frac{T^{3\rho-1}\,\Gamma\left(\frac{3\rho}{2}+\frac{\omega_n}{2\pi T}\right)}{\Gamma\left(1-\frac{3\rho}{2}+\frac{\omega_n}{2\pi T}\right)}\,, \end{aligned}$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

The complex SYK model

$$G(i\omega_n) = [iB\Pi(\rho)] \frac{T^{\rho-1}\,\Gamma\left(\frac{\rho}{2} + \frac{\omega_n}{2\pi T}\right)}{\Gamma\left(1 - \frac{\rho}{2} + \frac{\omega_n}{2\pi T}\right)}$$

$$\Sigma_{\rm sing}(i\omega_n) = \left[iU^2 B^3 \Pi(3\rho)\right] \frac{T^{3\rho-1}\,\Gamma\left(\frac{3\rho}{2} + \frac{\omega_n}{2\pi T}\right)}{\Gamma\left(1 - \frac{3\rho}{2} + \frac{\omega_n}{2\pi T}\right)},$$

where we have dropped a less-singular term in Σ, and

$$\Pi(s) \equiv \pi^{s-1} 2^s \cos\left(\frac{\pi s}{2}\right) \Gamma(1-s).$$

Now the singular part of Dyson's equation is

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

$$G(i\omega_n)\Sigma_{\rm sing}(i\omega_n) = -1$$

Remarkably, the Γ functions appear with just the right arguments, so that there is a solution of the Dyson equation at $\rho = 1/2$!

So the Green's functions display thermal 'damping' at a scale set by T alone, which is independent of U.

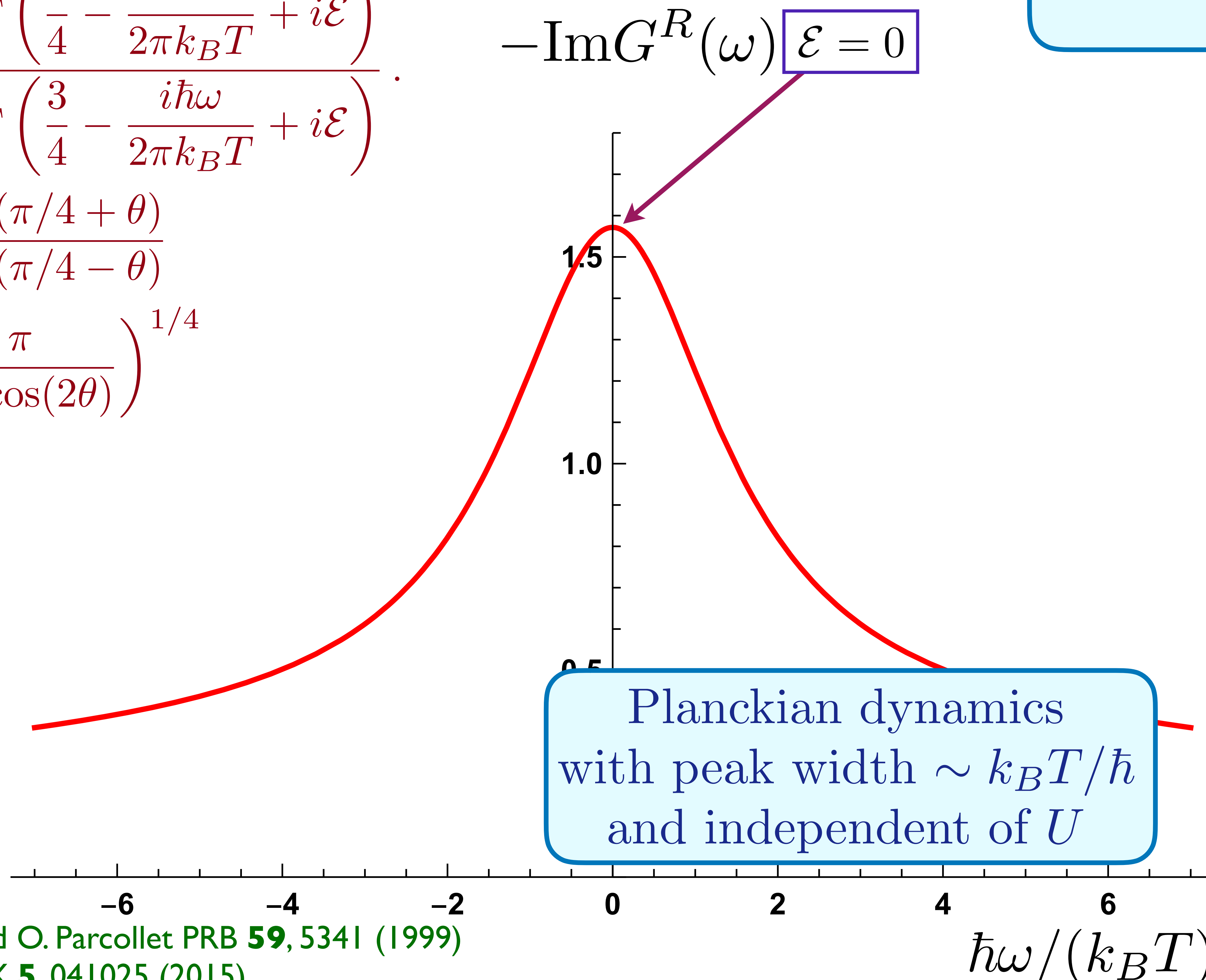
The complex SYK model

$$G^R_{\rm SYK}(\epsilon, \hbar\omega/(k_B T)) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$$\mathcal{E} = \mathbb{C}\frac{\epsilon}{U}$$



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

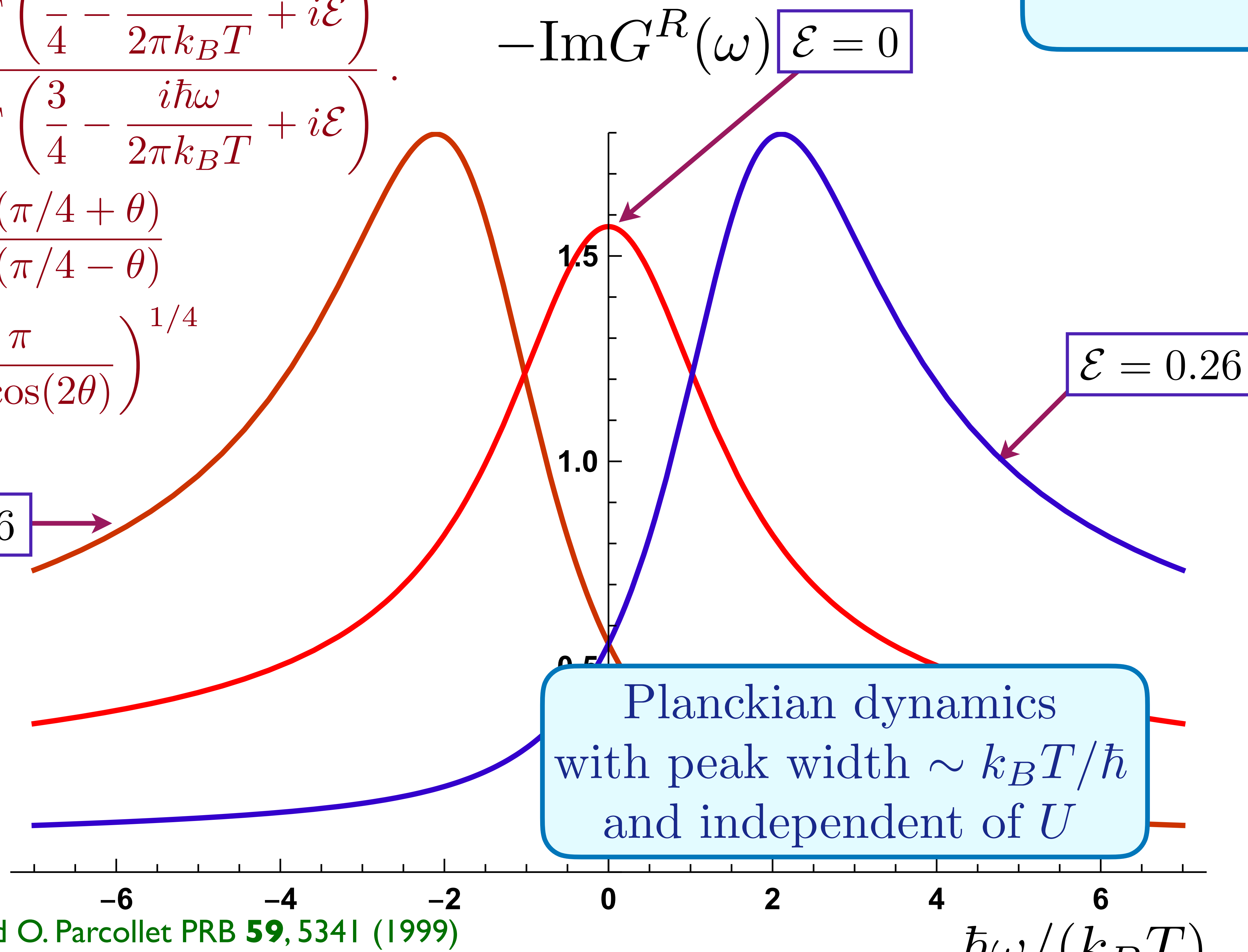
$$G^R_{\rm SYK}(\epsilon, \hbar\omega/(k_B T)) =$$

$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\dfrac{1}{4} - \dfrac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{\Gamma\left(\dfrac{3}{4} - \dfrac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$

$$\mathcal{E} = \mathbb{C}\,\frac{\epsilon}{U}$$



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

We now examine the behavior of the chemical potential, μ, as $T \to 0$ at fixed $\mathcal{Q}$. For this we relate the long-time 'conformal' Greens function, (valid for $\tau \gg 1/U$) to its short-time behavior. In particular at $|\omega_n| \gg U$ we have

$$G(i\omega_n) = \frac{1}{i\omega_n} - \frac{\mu}{(i\omega_n)^2} + \ldots$$

which implies for the spectral density of the Green's function, $\rho(\Omega)$

$$\mu = -\int_{-\infty}^{\infty} \frac{d\Omega}{\pi}\, \Omega \rho(\Omega),$$

which makes it evident that μ depends only upon the particle-hole asymmetric part of the spectral density. Next, we can relate the Ω integrals to the derivative of the imaginary time correlator

$$\mu = -\partial_\tau G(\tau = 0^+) - \partial_\tau G(\tau = (1/T)^-).$$

The complex SYK model

We pull out an explicitly particle-hole asymmetric part of $G(\tau)$ by defining

$$G(\tau) \equiv e^{-2\pi\mathcal{E}T\tau} G_c(\tau) \quad , \quad 0 < \sigma < \frac{1}{T}.$$

where G_c will be given by a particle-hole symmetric conformal form at low T and low ω. Then we obtain

$$\begin{aligned} \mu &= 2\pi\mathcal{E}T \left[G(\tau = 0^+) + G(\tau = (1/T)^-) \right] \\ &\qquad + \text{terms dependent on } G_c \\ &= -2\pi\mathcal{E}T + \text{terms dependent on } G_c \end{aligned}$$

It can be shown that all the terms dependent upon G_c have a T dependence that is weaker than linear in T provided $\mathcal{Q}$ is held fixed. Hence we have

$$\mu = \mu_0 - 2\pi\mathcal{E}T + \text{terms vanishing as } T^p \text{ with } p > 1$$

with μ_0 a non-universal constant. From this relation we obtain

The complex SYK model

with μ_0 a non-universal constant. From this relation we obtain

$$\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = -2\pi\mathcal{E} \quad , \quad T \to 0,$$

Using a Maxwell relation we then have

$$\frac{1}{N}\left(\frac{\partial S}{\partial \mathcal{Q}}\right)_{T} = 2\pi\mathcal{E} \neq 0 \;\; \text{as } T \to 0.$$

The complex SYK model

Solution of these equations and corresponding evaluation of the free energy yields the following universal results (*i.e.* all results are *quantitatively* unchanged by adding additional higher q fermion terms):

- There is a non-vanishing entropy in the zero temperature limit

$$S(T \to 0) = N s_0 + \ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The complex SYK model

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- There is a non-vanishing entropy in the zero temperature limit

$$S(T \to 0) = N s_0 + \dots$$

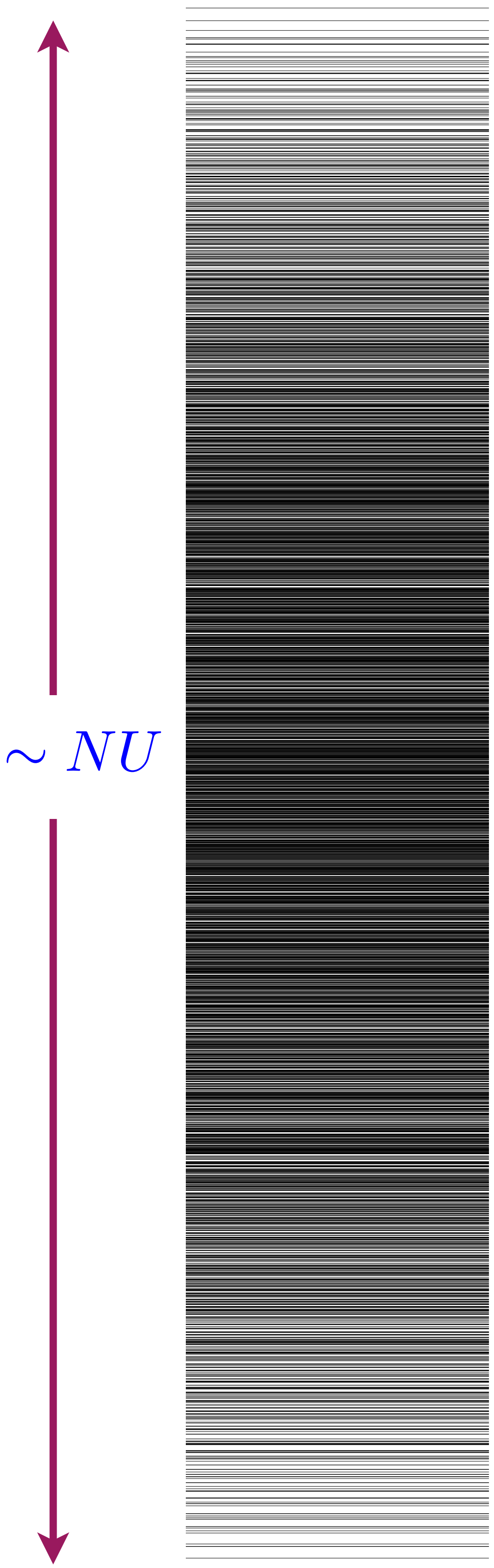
- The saddle point equations imply the relation

$$\boxed{\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}}$$

Integrating this relation from $s_0 = 0$, $\mathcal{Q} = 0$, allows us to compute s_0 as a function of $\mathcal{Q}$.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The complex SYK model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}\,.$$

At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

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The SYK model

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The SYK model

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll U$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations. The singular part of the self-energy and the Green's function obey

$$\int_0^\beta d\tau_2\, \Sigma_{\rm sing}(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma_{\rm sing}(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

The complex SYK model

$$\int_0^\beta d\tau_2 \, \Sigma_(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

These equations are invariant under

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.
By using $f(\sigma) = \tan(\pi T \sigma)/(\pi T)$ and
$g(\sigma) = e^{-2\pi \mathcal{E} T \sigma}$, we can now obtain
the $T > 0$ solution from the $T = 0$ solution.

A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

The SYK model

Let us write the large N saddle point solutions of S as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

The saddle point will be invariant under a reperamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\widetilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

A. Kitaev

Fluctuations

- The saddle-point

$$G(\tau_1 - \tau_2) = -A \frac{e^{-2\pi\mathcal{E}T(\tau_1-\tau_2)}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T(\tau_1 - \tau_2))} \right)^{2\Delta}$$

is invariant only under PSL(2, R) transformations which map the thermal circle onto itself, and an associated gauge transformation

$$\frac{\tan(\pi T f(\tau))}{\pi T} = \frac{a \dfrac{\tan(\pi T \tau)}{\pi T} + b}{c \dfrac{\tan(\pi T \tau)}{\pi T} + d} \quad , \quad ad - bc = 1,$$

$$-i\phi(\tau) = -i\phi_0 + 2\pi\mathcal{E}T(\tau - f(\tau))$$

A. Kitaev, 2015

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

Infinite-range (SYK) model without quasiparticles

After introducing replicas $a = 1 \dots n$, and integrating out the disorder, the partition function can be written as

$$Z = \int \mathcal{D}c_{ia}(\tau) \exp \left[- \sum_{ia} \int_0^\beta d\tau \, c_{ia}^\dagger \left(\frac{\partial}{\partial \tau} - \mu \right) c_{ia} \right.$$
$$\left. - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c_{ia}^\dagger(\tau) c_{ib}(\tau') \right|^4 \right] .$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp \left[-N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left(G(\tau_2, \tau_1) \right. \right.$$
$$\left. \left. + \frac{1}{N} \sum_i c_i(\tau_2) c_i^\dagger(\tau_1) \right) \right] .$$

Infinite-range (SYK) model without quasiparticles

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln \det \left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right]$$

$$+ \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

The SYK model

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1)\mathcal{D}\Sigma(\tau_1, \tau_2)e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4}G_s(f(\tau_1) - f(\tau_2))e^{i\phi(\tau_1)-i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)e^{-E_0/T+Ns_0-NS_{\rm eff}[f,\phi]},$$

where $E_0 \propto N$ is the ground state energy.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Fluctuations

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\rm eff}[f,\phi] = \frac{NK}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau f)^2 - \frac{N\gamma}{4\pi^2}\int_0^{1/T} d\tau \left\{\tan(\pi T f(\tau)), \tau\right\},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 .$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\rm eff}\left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0\right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017);
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746