Outline

1. Coupled dimer antiferromagnets

   Landau-Ginzburg quantum criticality

2. Spin liquids and valence bond solids

   (a) Schwinger-boson mean-field theory - square lattice
   (b) Gauge theories of perturbative fluctuations
   (c) Non-perturbative effects: Berry phases
   (d) Schwinger-boson mean-field theory - triangular lattice
   (e) Visons and the Kitaev model

3. Cuprate superconductivity

   (a) Review of experiments, old and new
   (b) Fermi surfaces and the spin density wave theory
   (c) Fermi pockets and the underdoped cuprates

Outline

1. Coupled dimer antiferromagnets

   Landau-Ginzburg quantum criticality

2. Spin liquids and valence bond solids

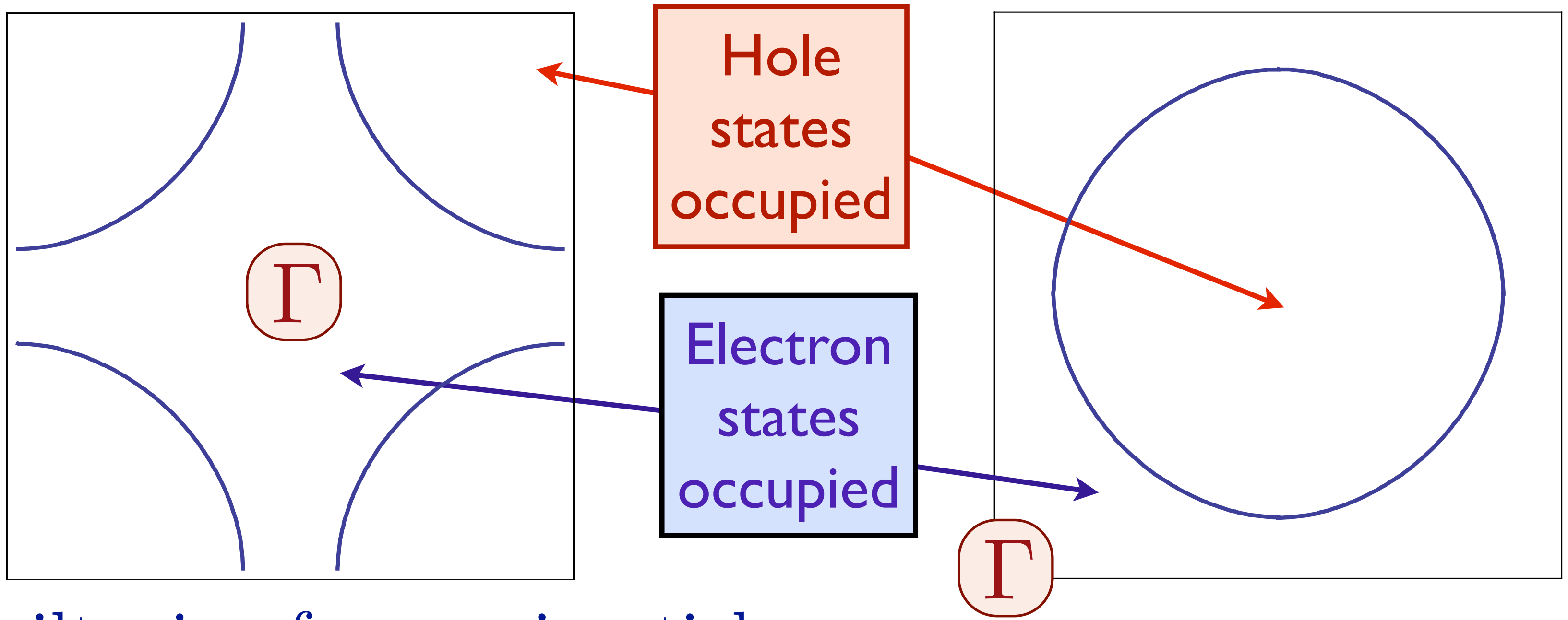
   (a) Schwinger-boson mean-field theory - square lattice
   (b) Gauge theories of perturbative fluctuations
   (c) Non-perturbative effects: Berry phases
   (d) Schwinger-boson mean-field theory - triangular lattice
   (e) Visons and the Kitaev model

3. Cuprate superconductivity

   (a) Review of experiments, old and new
   (b) Fermi surfaces and the spin density wave theory
   (c) Fermi pockets and the underdoped cuprates

Fermi surfaces in electron- and hole-doped cuprates

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-p) & \text{for hole-doping } p \\ 2\pi^2(1+x) & \text{for electron-doping } x \end{cases}$$

The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

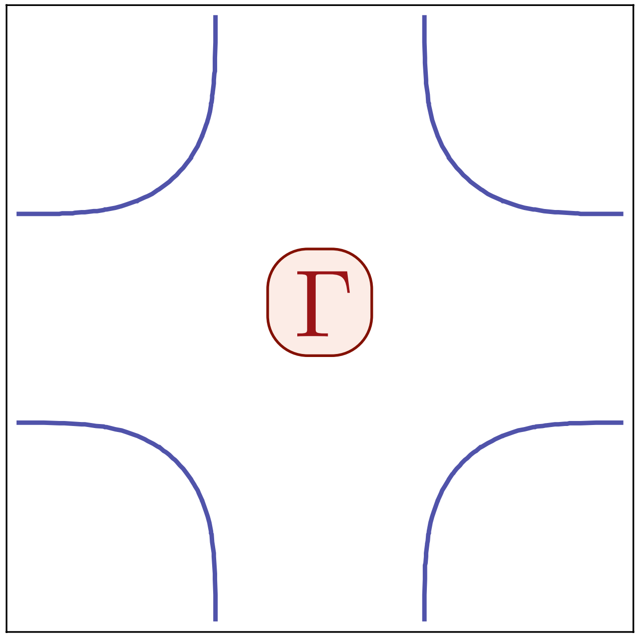
$$H_{\mathrm{sdw}} = -\vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\mathrm{sdw}}$ for $\vec{\varphi} = (0, 0, \varphi)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

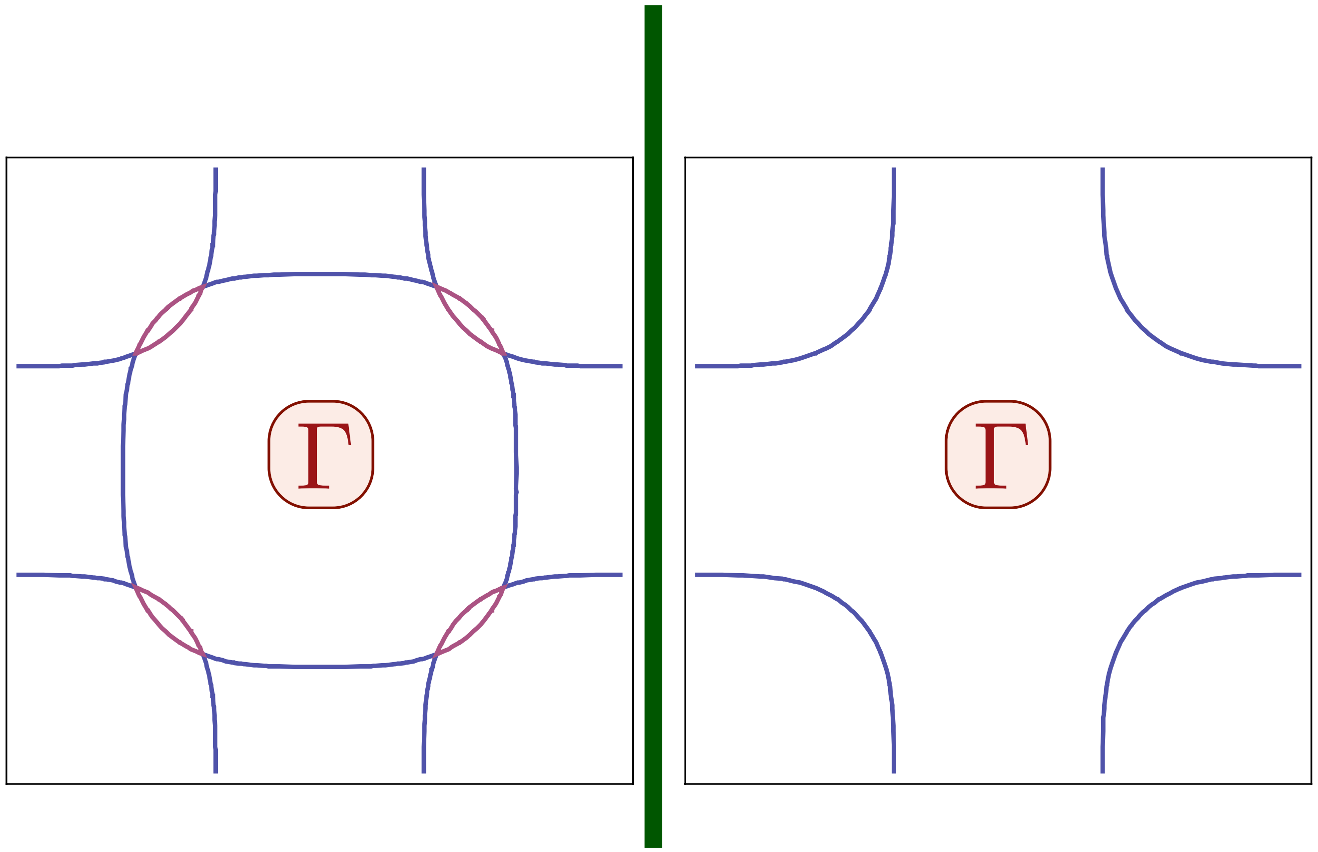
This leads to the Fermi surfaces shown in the following slides for electron and hole doping.

Spin density wave theory in electron-doped cuprates



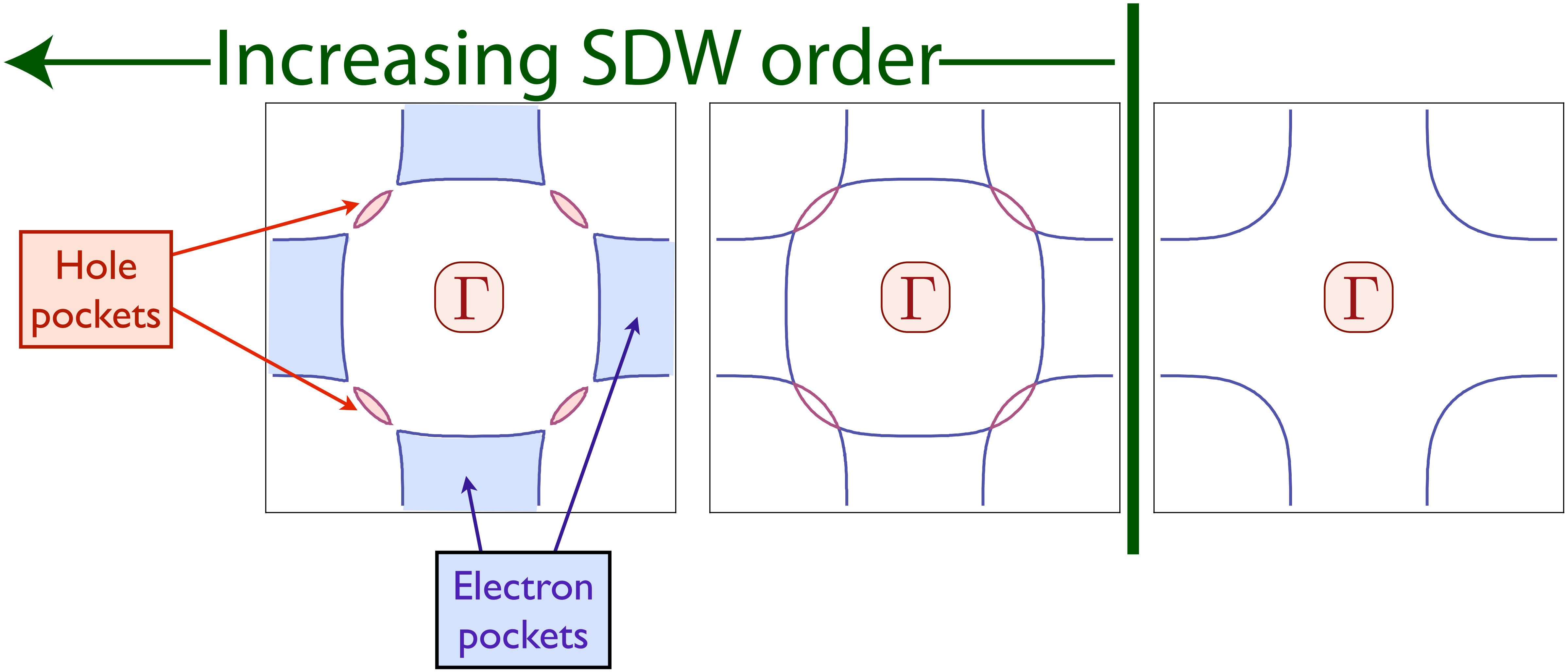
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in electron-doped cuprates



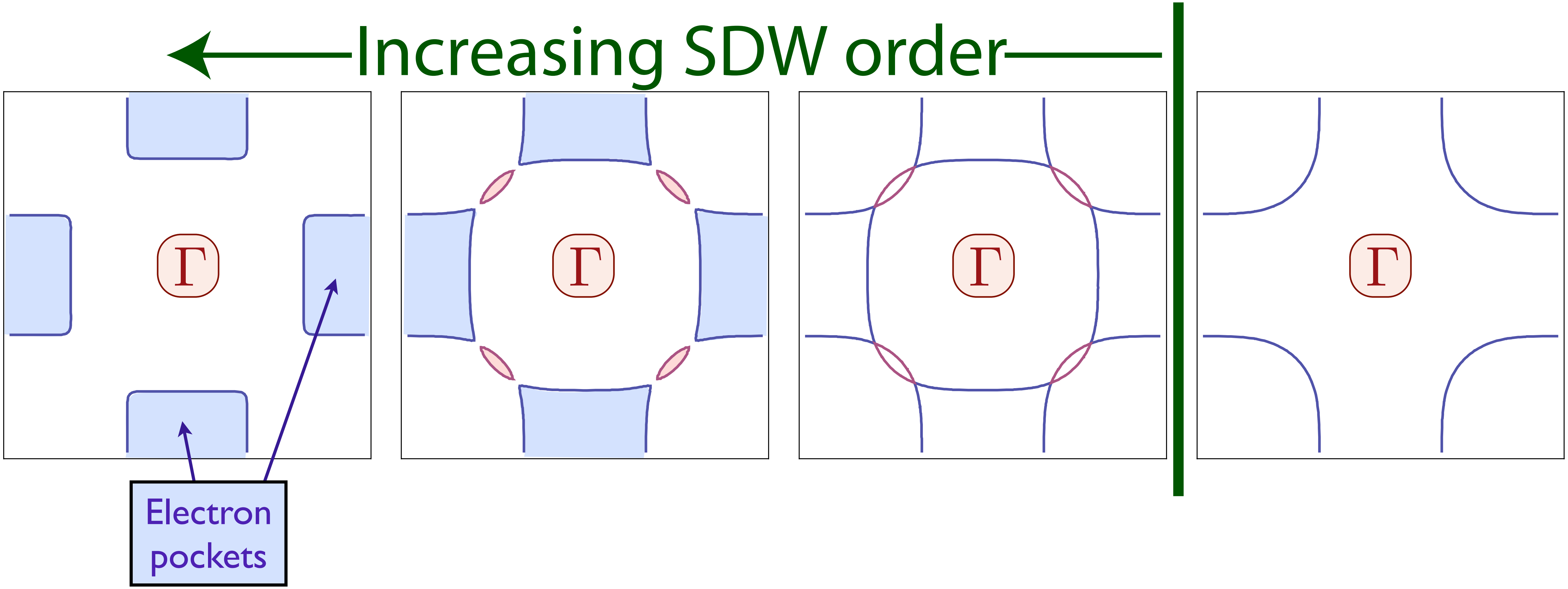
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in electron-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

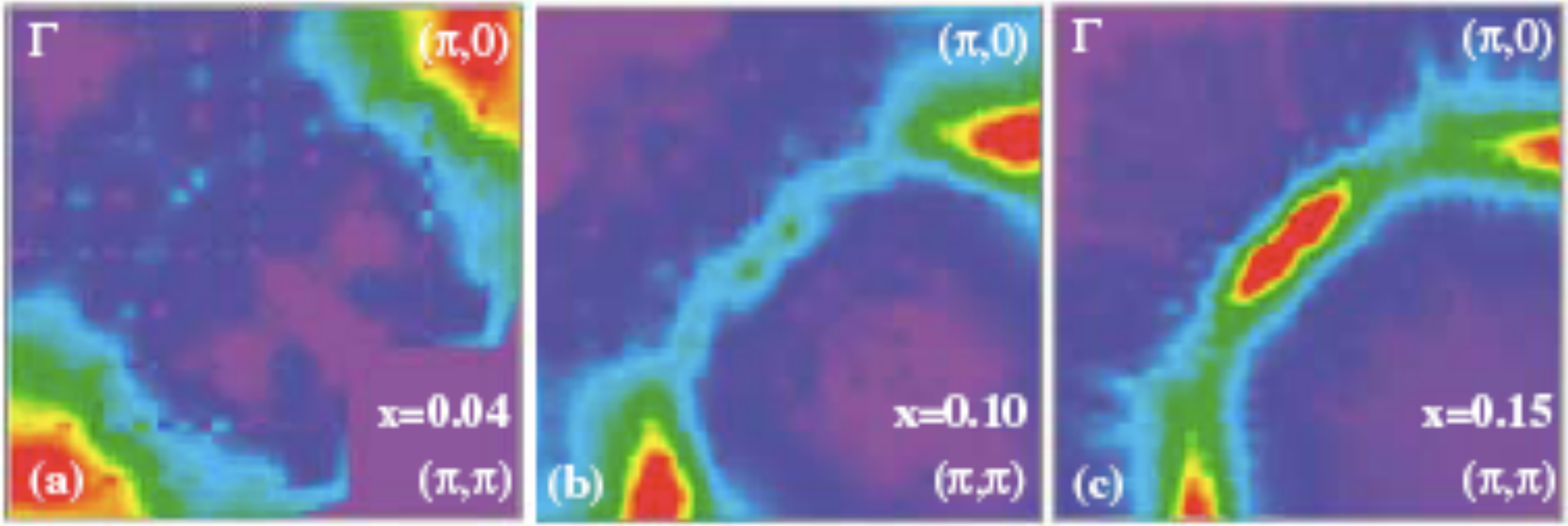
Spin density wave theory in electron-doped cuprates



SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

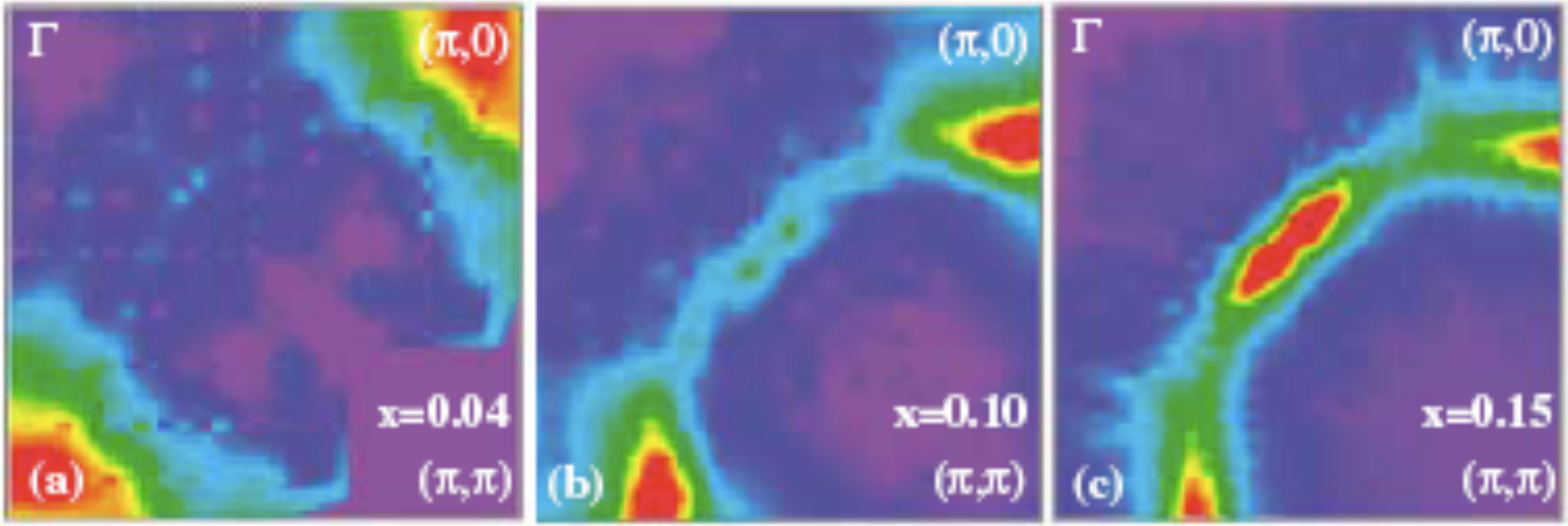
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Photoemission in NCCO



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Photoemission in NCCO

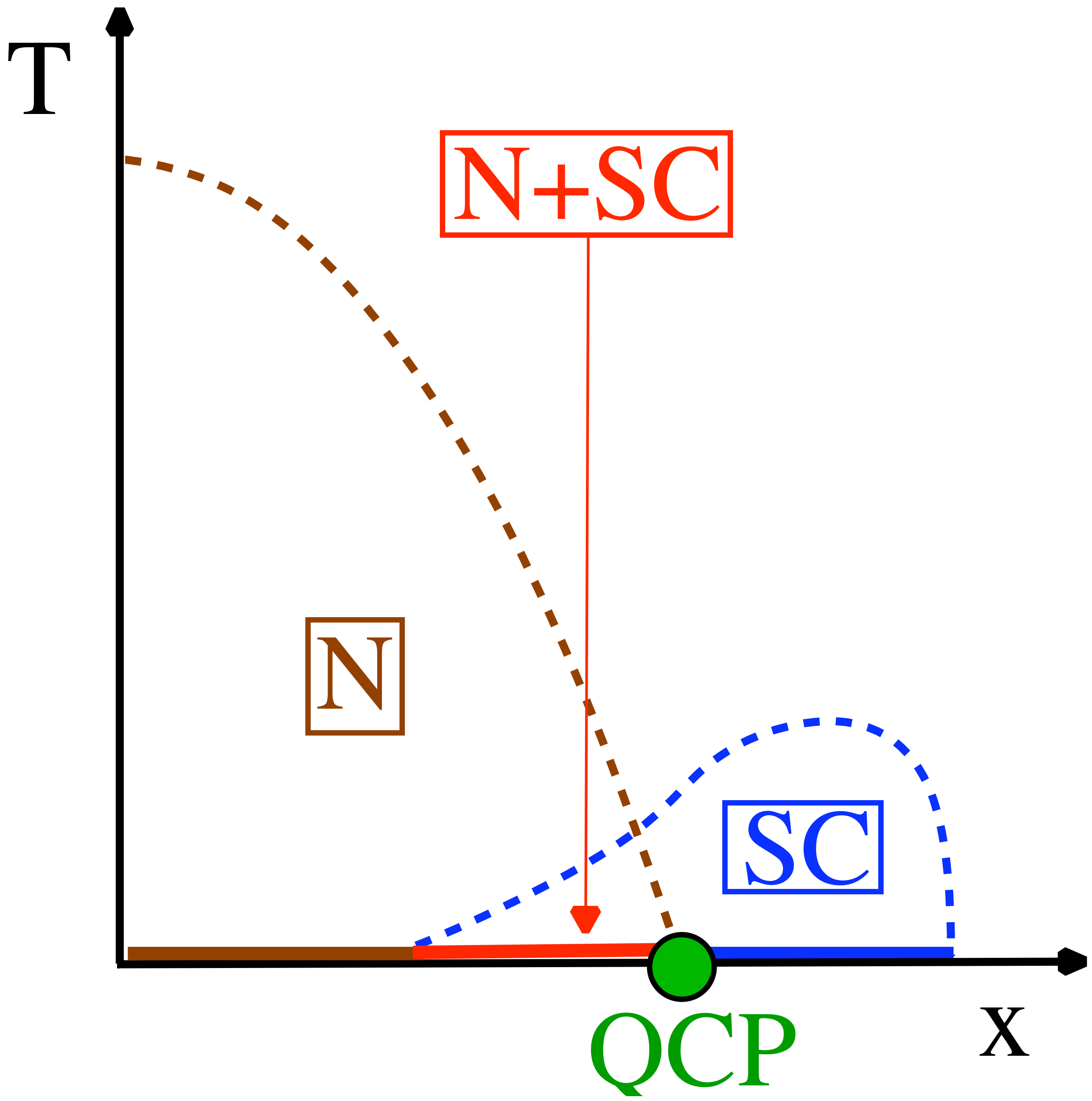


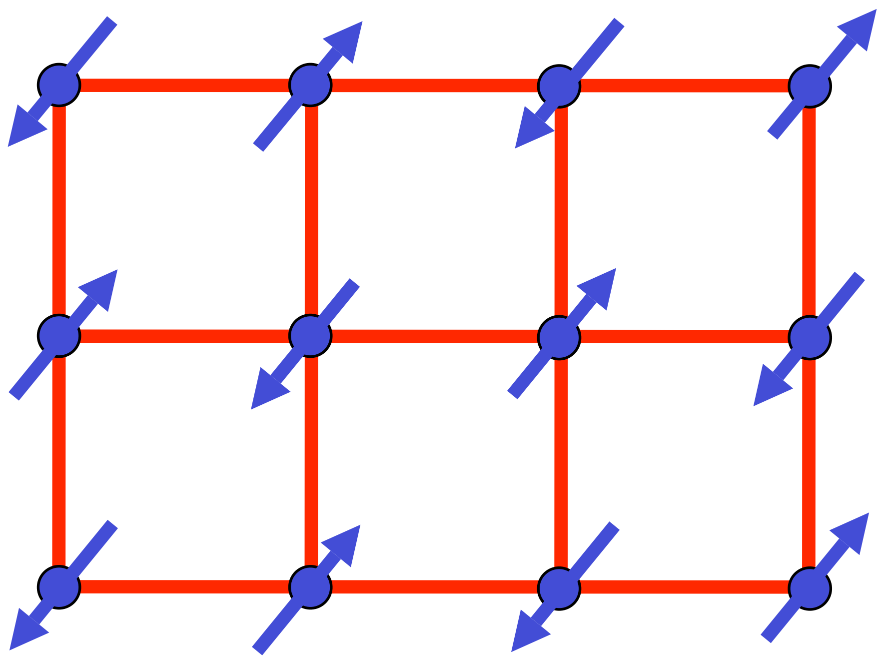
Reasonable agreement with SDW theory

N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Phase diagram of electron-doped superconductors

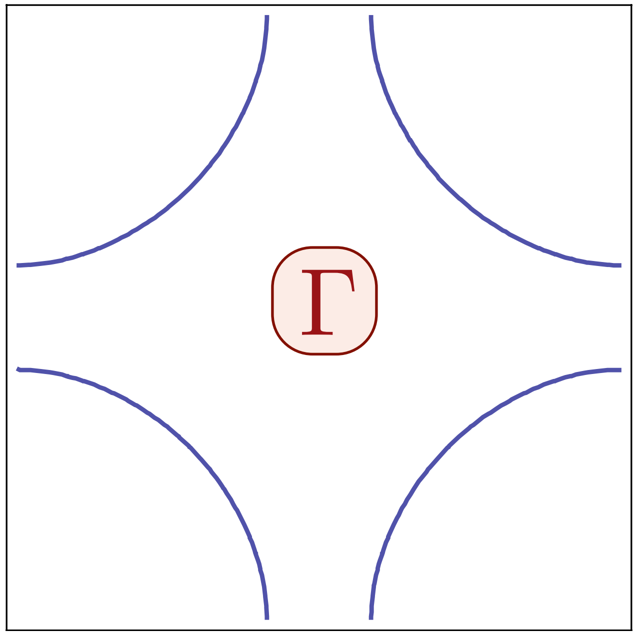
$Nd_{2-x}Ce_xCuO_{4-y}$ and $Pr_{2-x}Ce_xCuO_{4-y}$





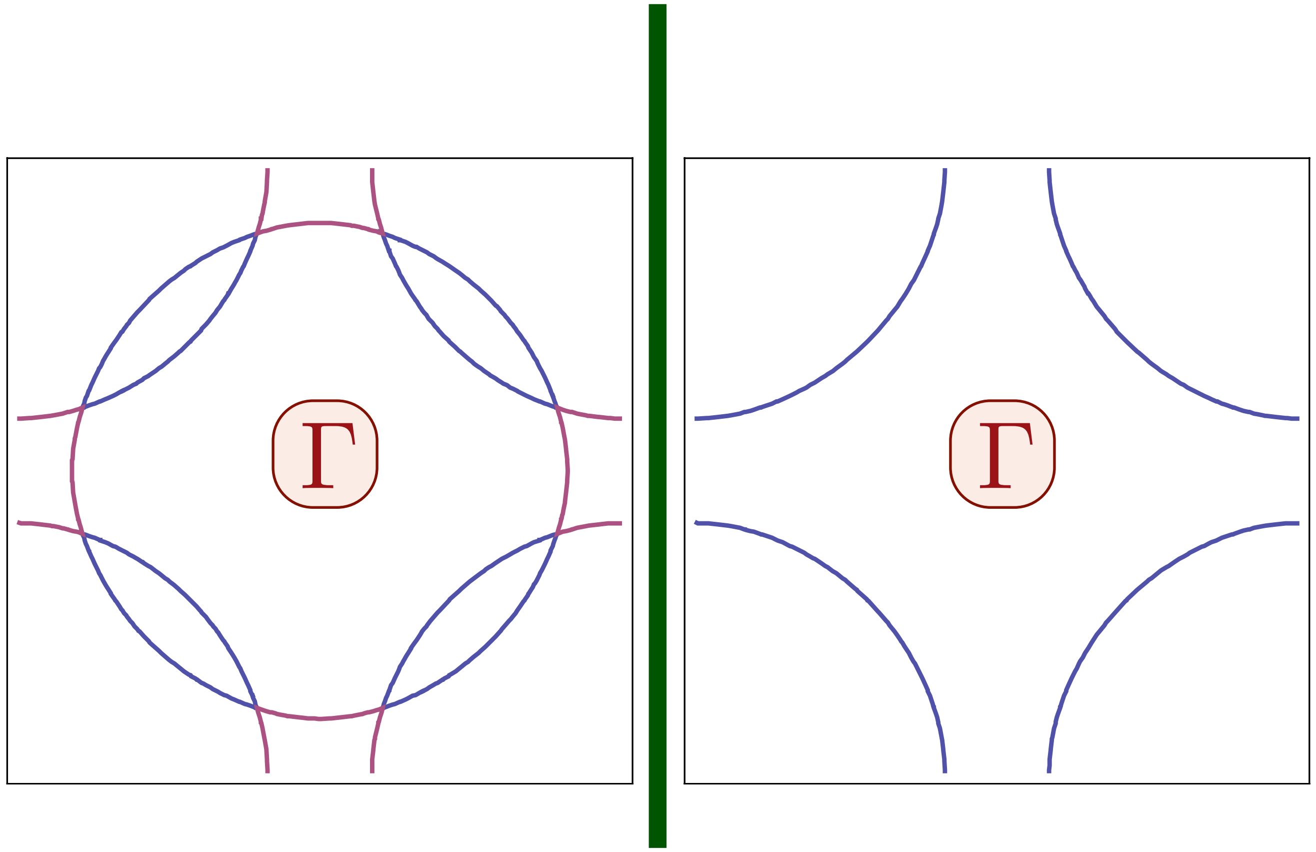
Neel order (N)

Spin density wave theory in hole-doped cuprates



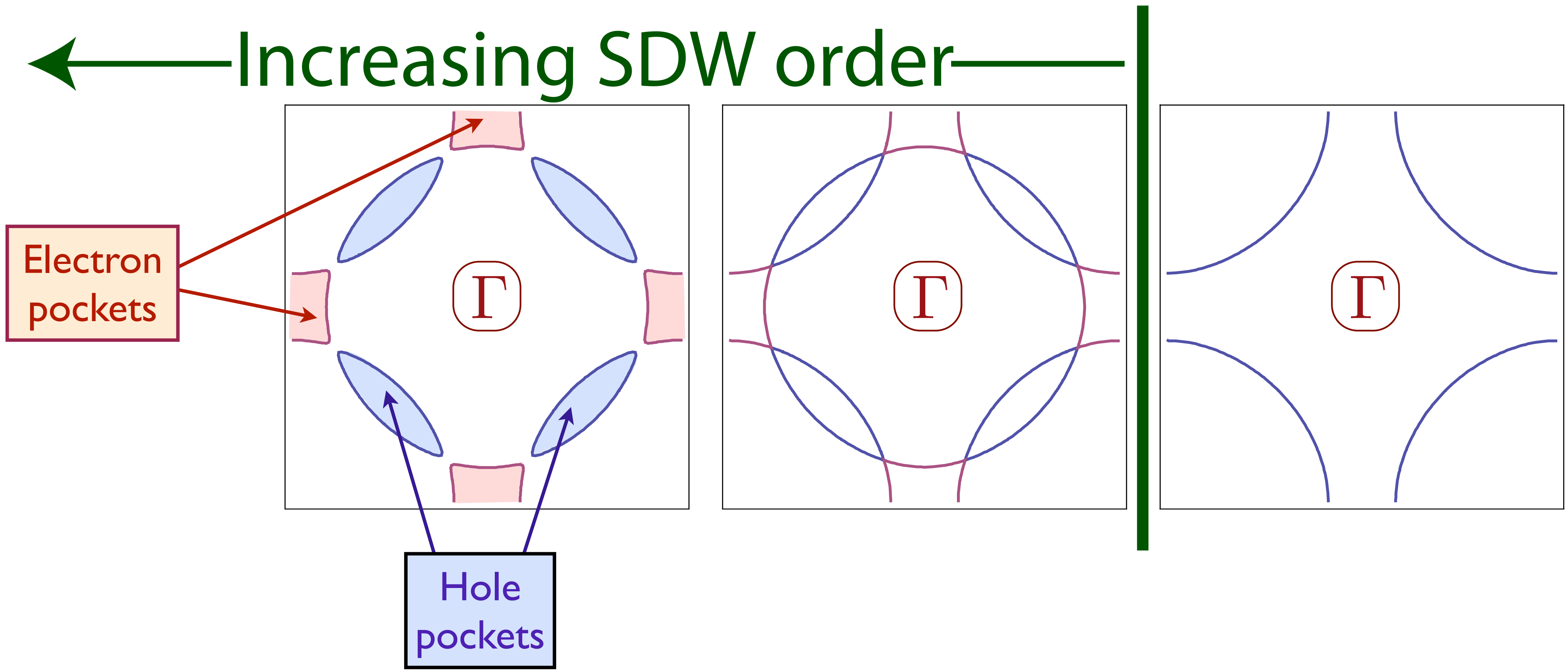
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



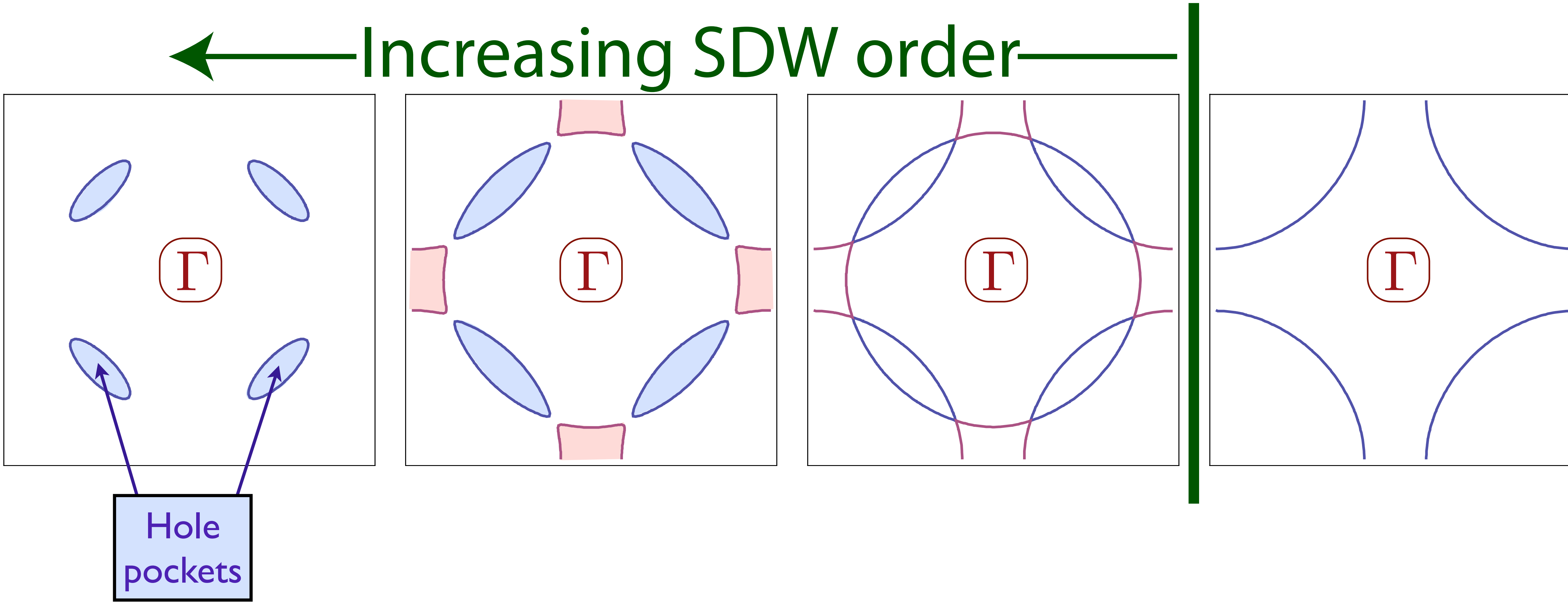
S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

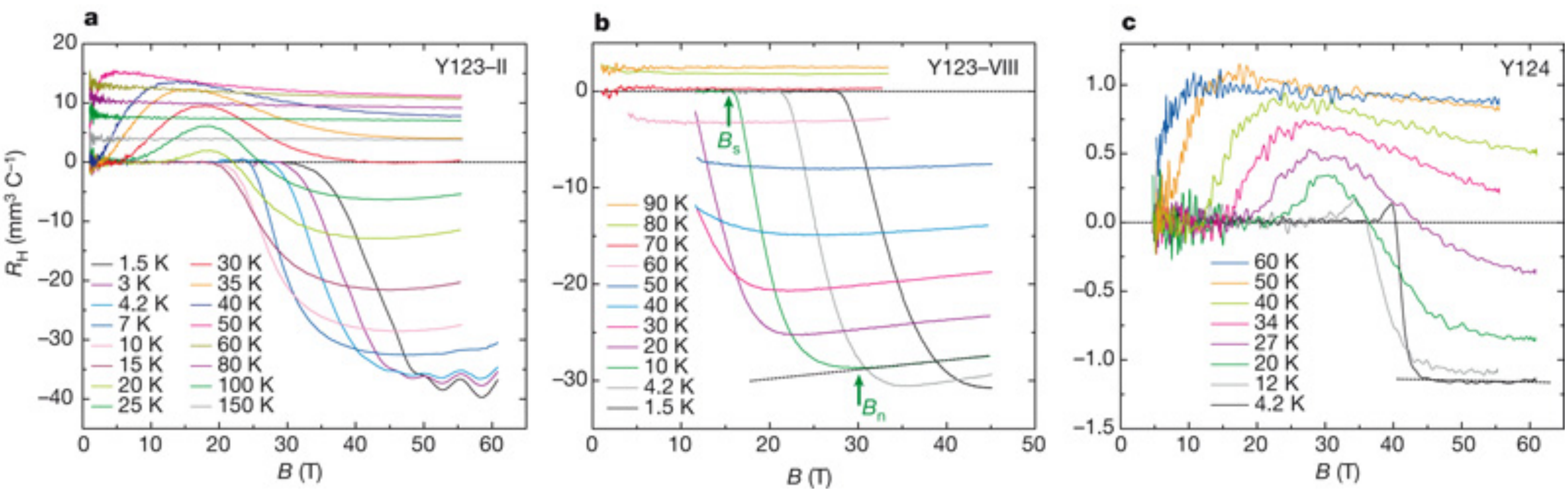
S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)

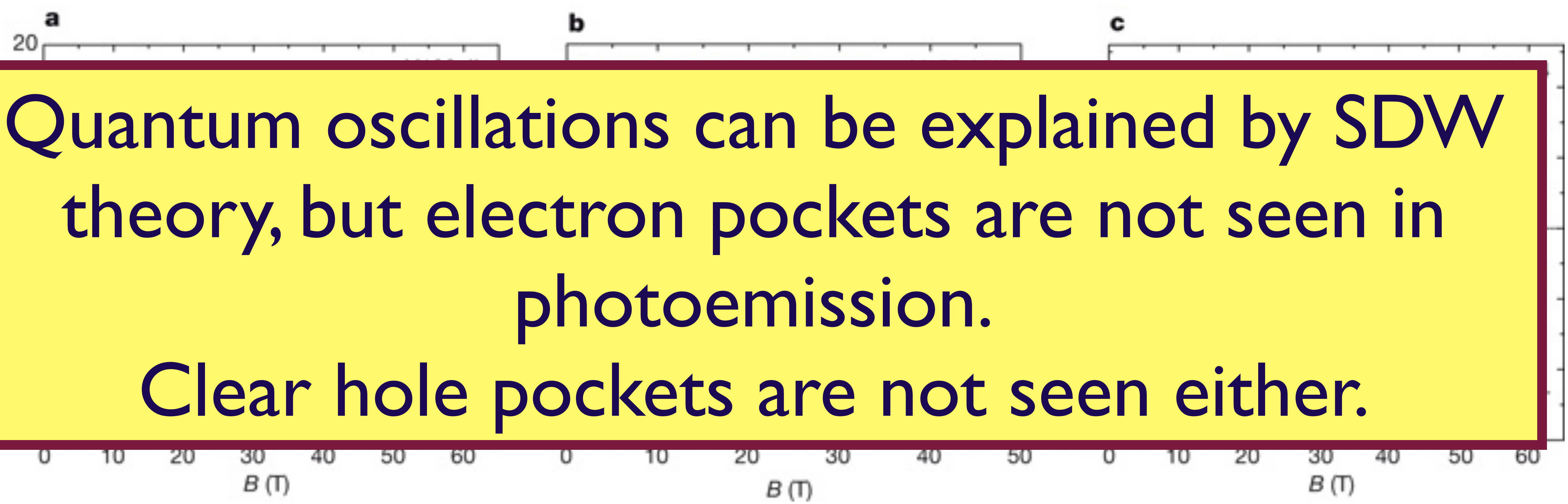


Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)

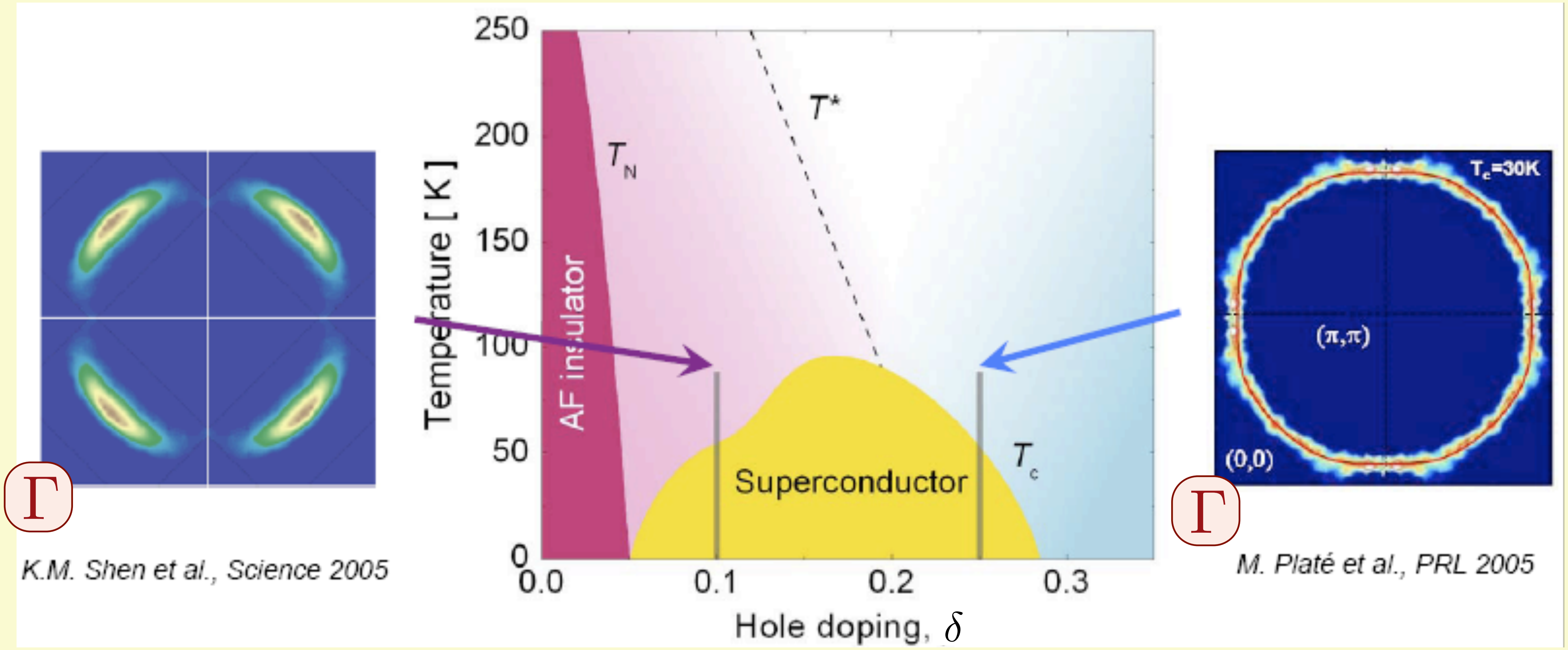


Quantum oscillations can be explained by SDW theory, but electron pockets are not seen in photoemission.

Clear hole pockets are not seen either.

Superconductivity in hole-doped cuprates

Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



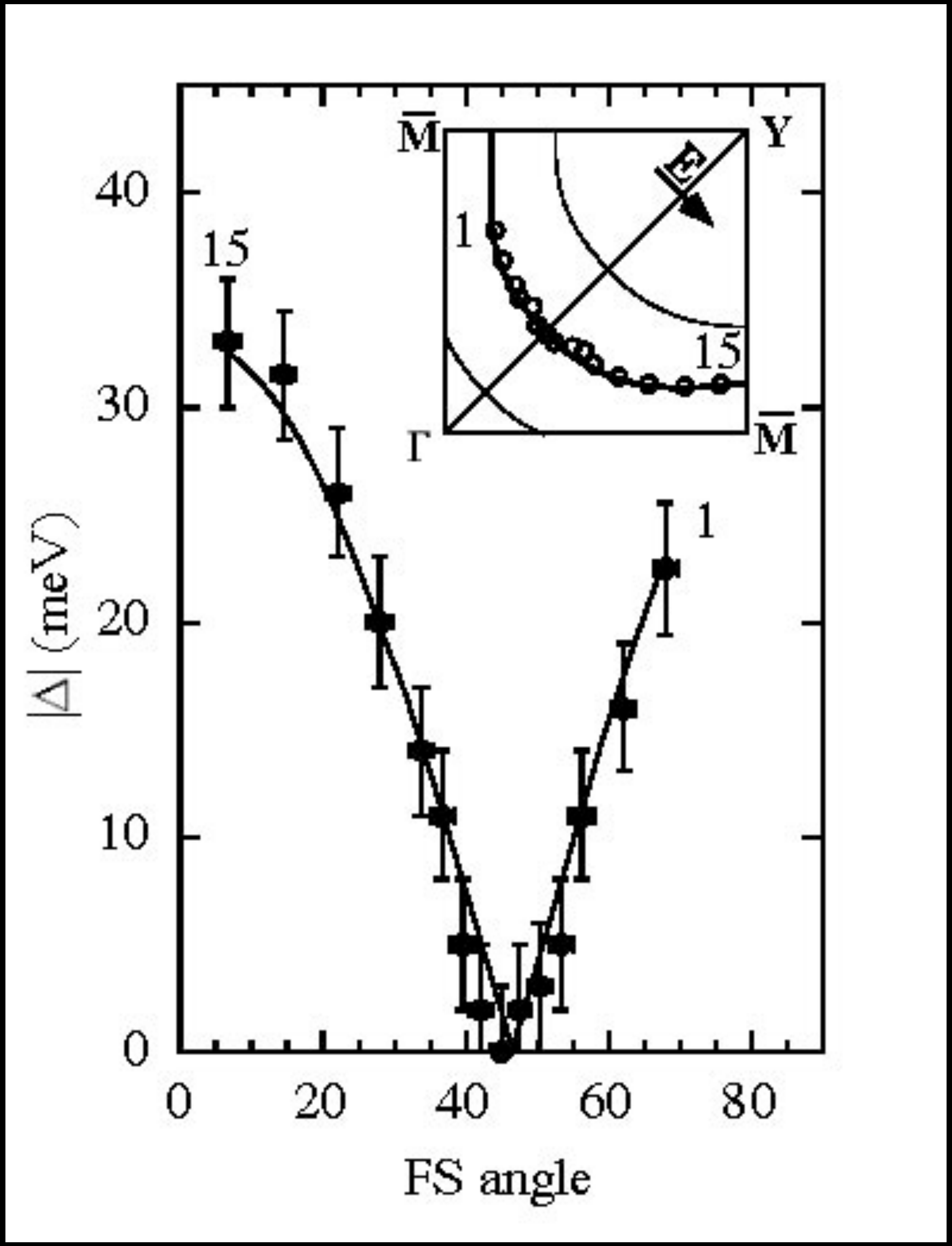
K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

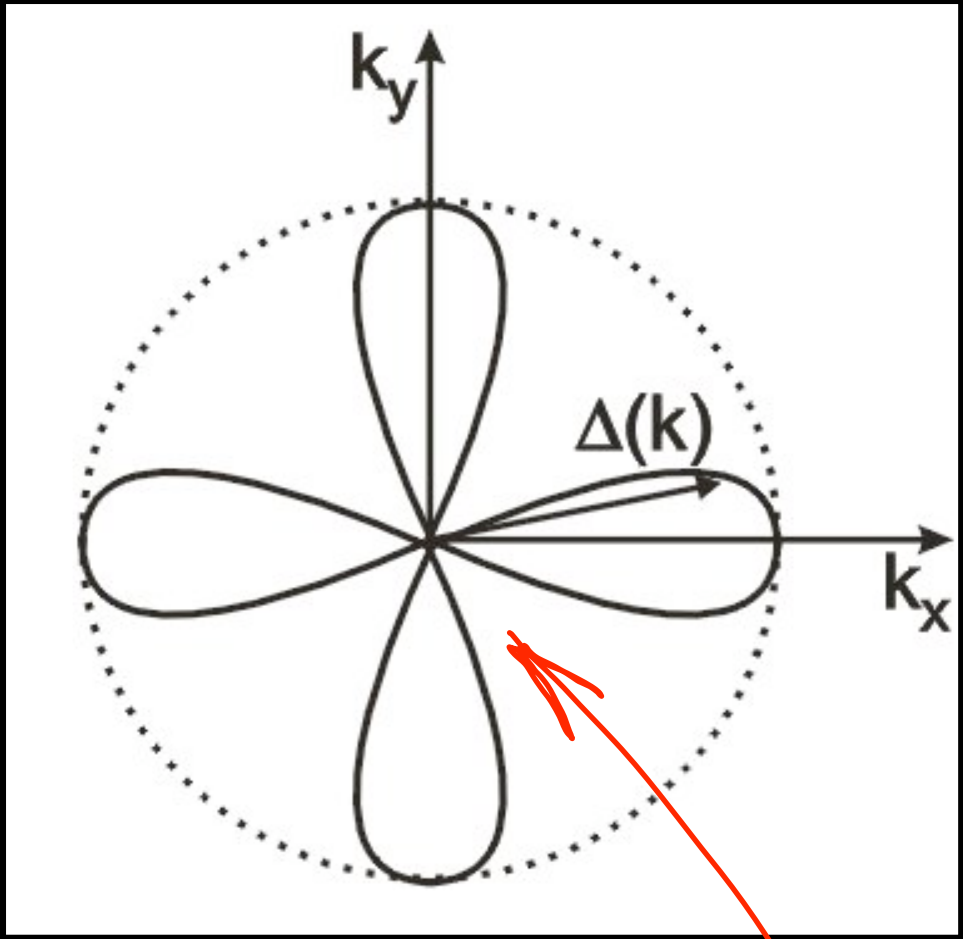
Smaller hole Fermi-pockets

Large hole Fermi surface

Overdoped SC State: Momentum-dependent Pair Energy Gap $\Delta(\vec{k})$

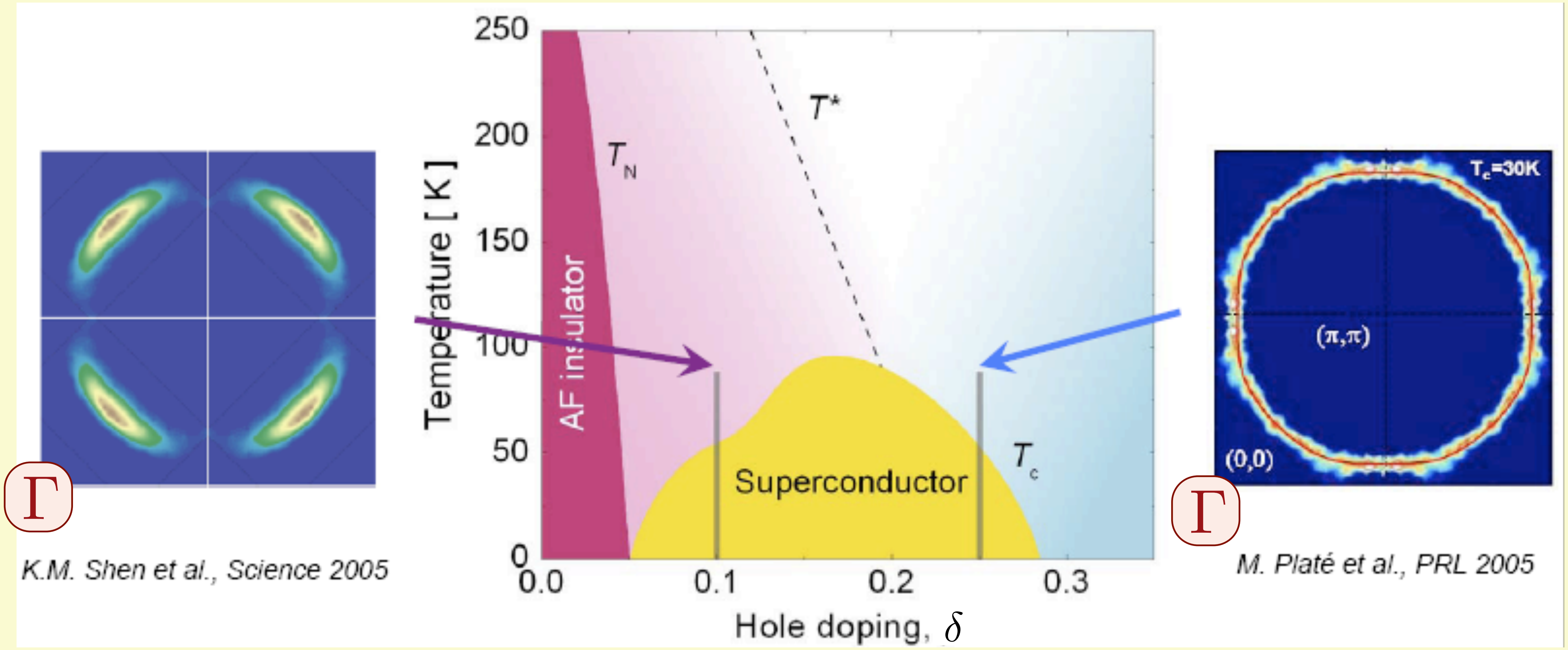


Shen et al PRL **70**, 3999 (1993)
Ding et al PRB **54** 9678 (1996)
Mesot et al PRL **83** 840 (1999)



The SC energy gap $\Delta(\vec{k})$ has four nodes.

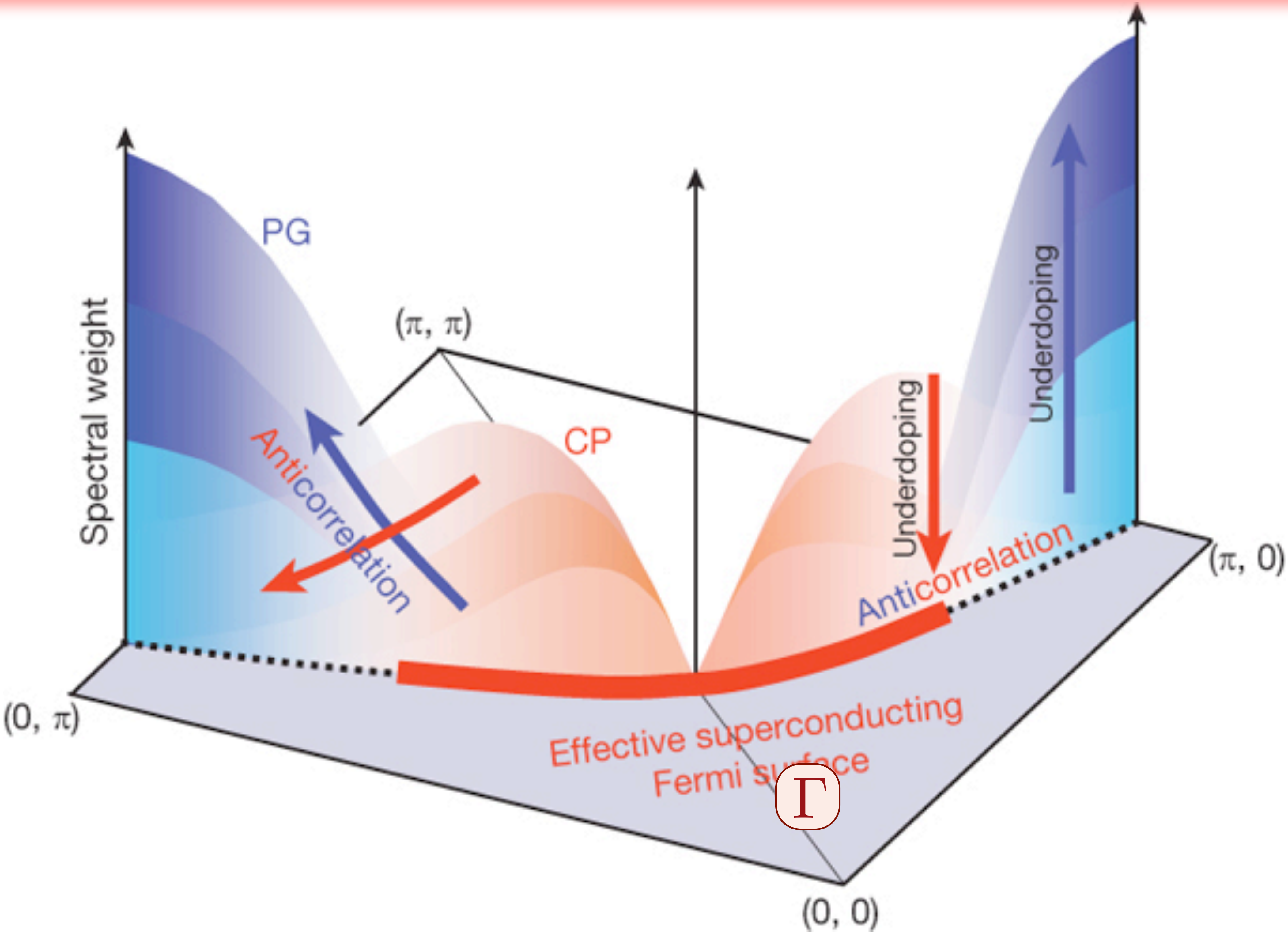
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole Fermi-pockets

Large hole Fermi surface

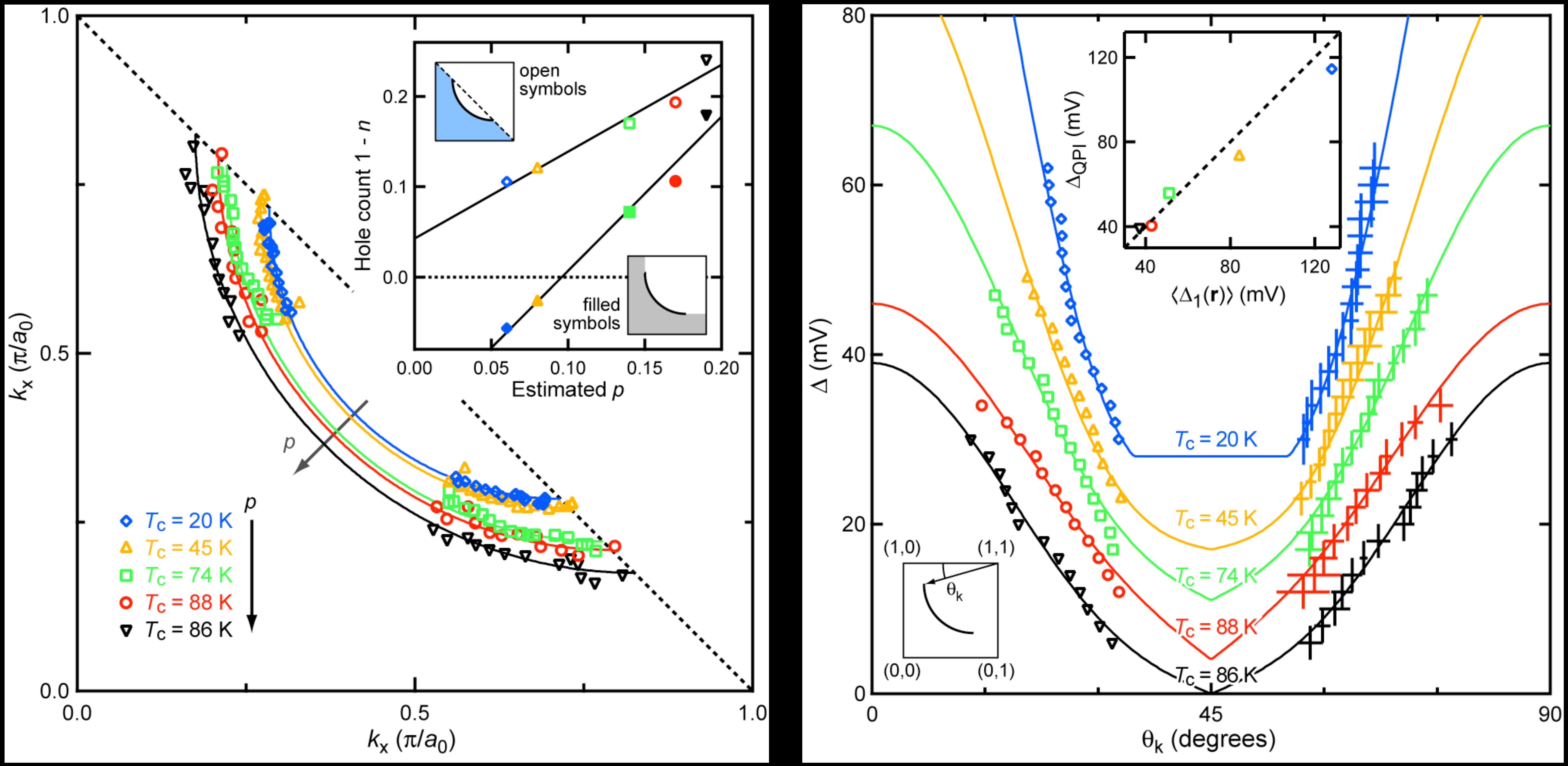
Nodal-anti-nodal dichotomy in the underdoped cuprates



Competition between the pseudogap and superconductivity in the high-Tc copper oxides

T. Kondo, R. Khasanov, T. Takeuchi, J. Schmalian, A. Kaminski, *Nature* **457**, 296 (2009)

Nodal-anti-nodal dichotomy in the underdoped cuprates



Y. Kohsaka et al., Nature 454, 1072, (2008)

Competition between SDW order and superconductivity

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \frac{u}{4} \left(\vec{\varphi}^2 \right)^2 \right.$$

$$\left. + \kappa \, \vec{\varphi}^2 \, |\psi|^2 \right]$$

$$+ \int d^2r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \frac{u}{4} \left(\vec{\varphi}^2 \right)^2 \right.$$
$$\left. + \boxed{\kappa \, \vec{\varphi}^2 \, |\psi|^2} \right]$$
$$+ \int d^2r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\boxed{\kappa > 0 \text{ is the repulsion between the two order parameters}}$, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

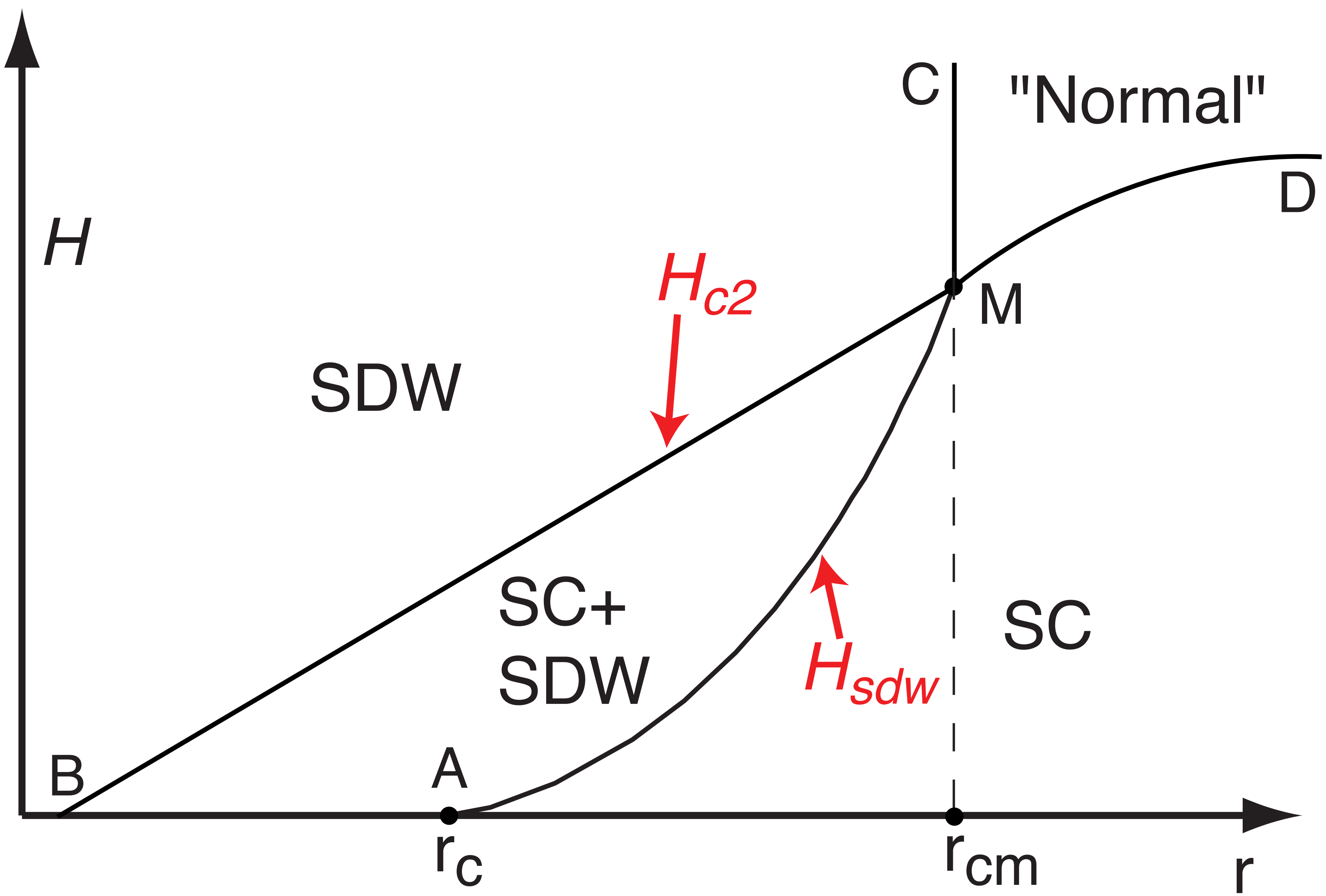
$$\begin{aligned}\mathcal{S} \;=\; & \int d^2r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 \right. \\ & \qquad \left. + \kappa\, \vec{\varphi}^2\, |\psi|^2 \right] \\ & + \int d^2r \left[\boxed{|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2} - |\psi|^2 + \frac{|\psi|^4}{2} \right]\end{aligned}$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\boxed{\nabla \times \mathcal{A} = H \text{ is the applied magnetic field.}}$

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

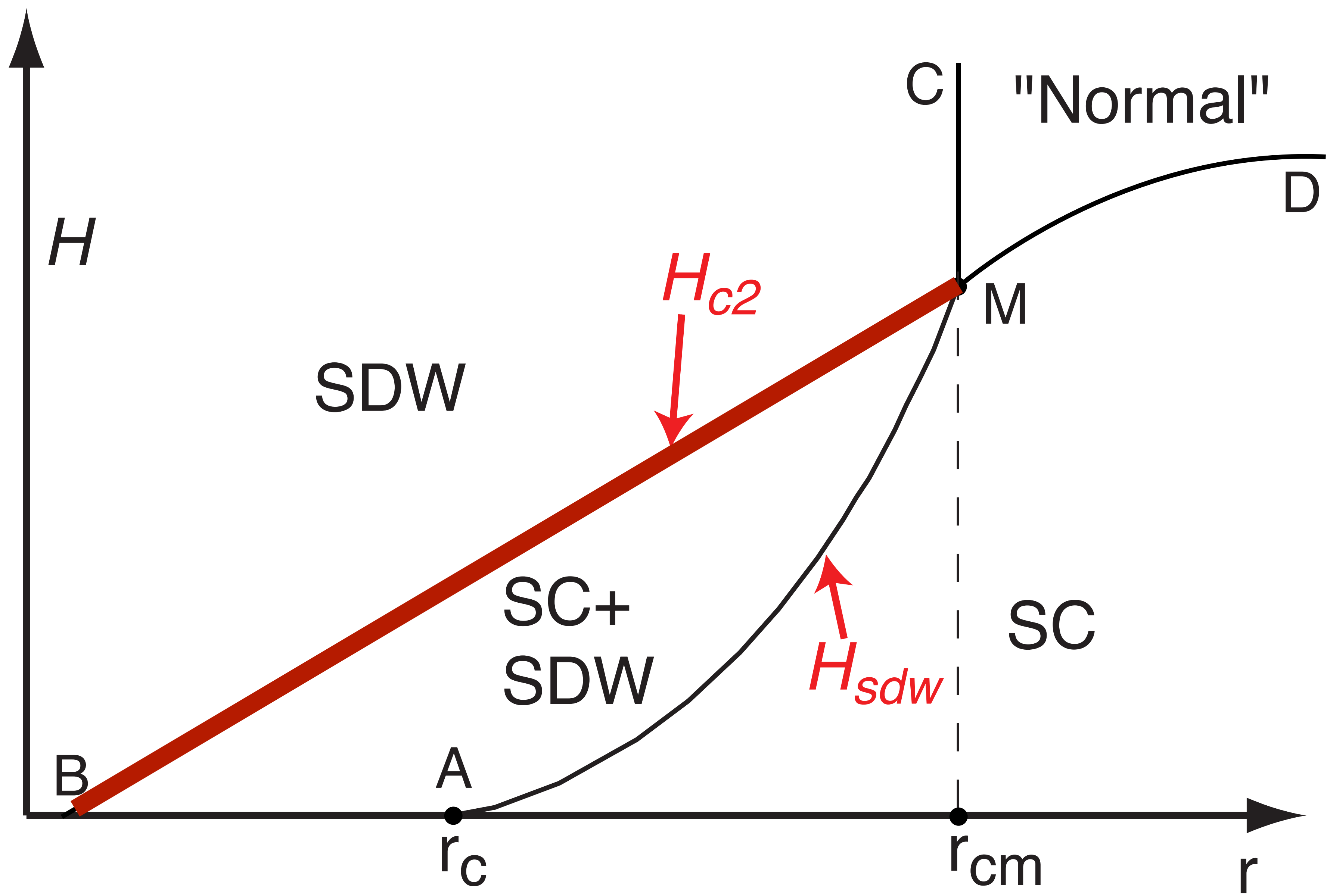
Competition between superconductivity (SC) and spin-density wave (SDW) order



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

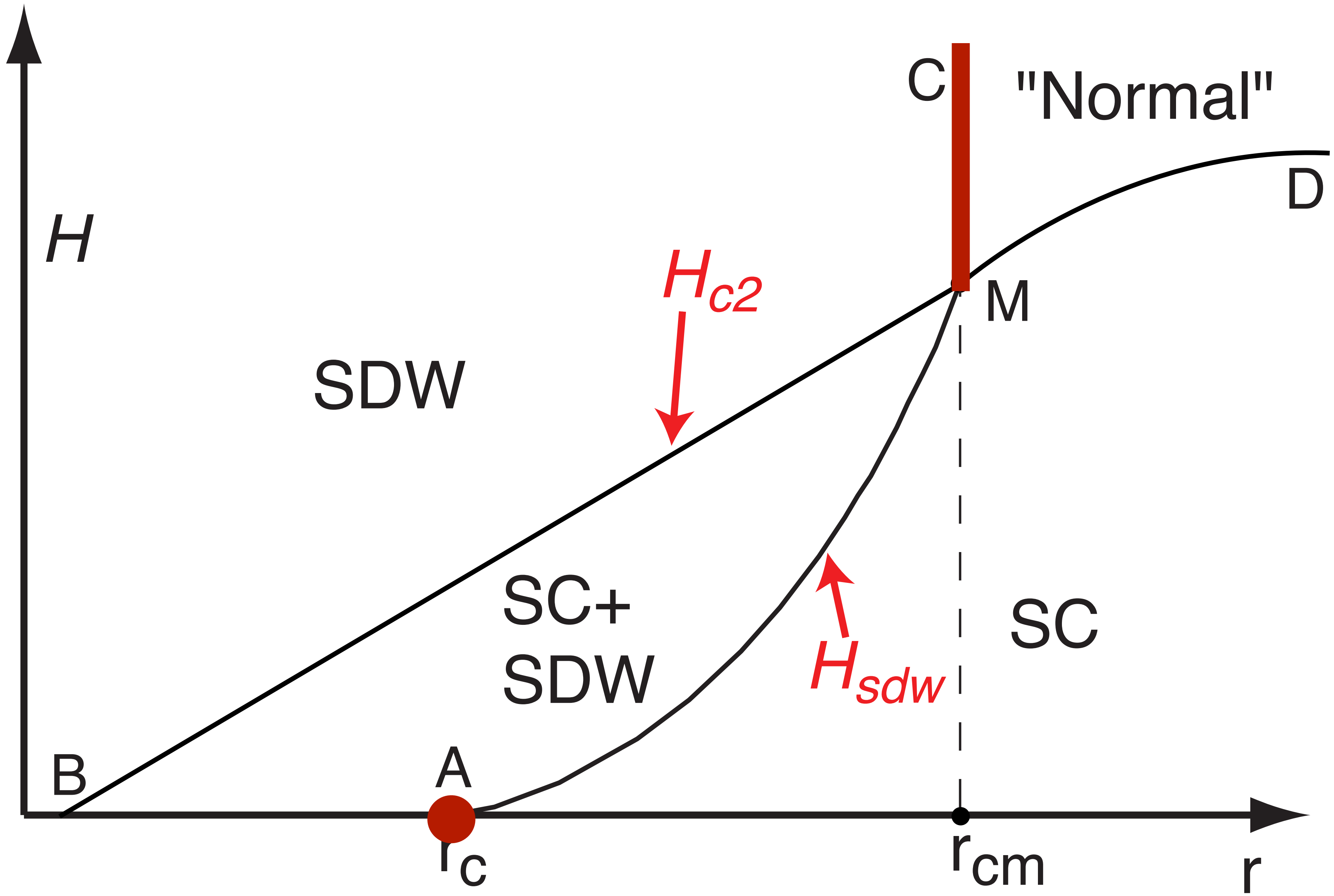


- Upper-critical field, H_{c2}, decreases as SDW is enhanced with decreasing doping (r)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

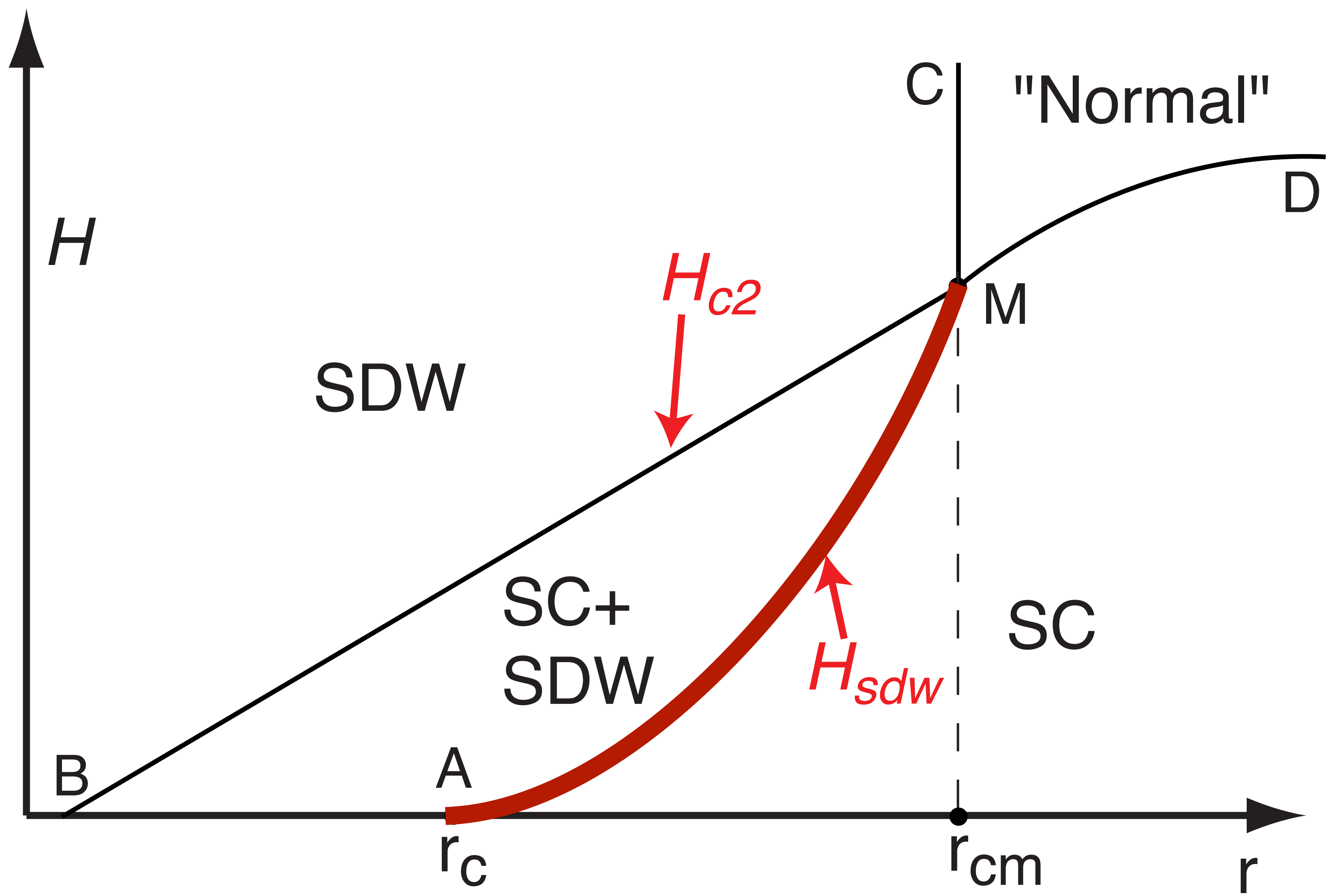


- SDW order is more stable in the metal than in the superconductor: $r_{cm} > r_c$.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

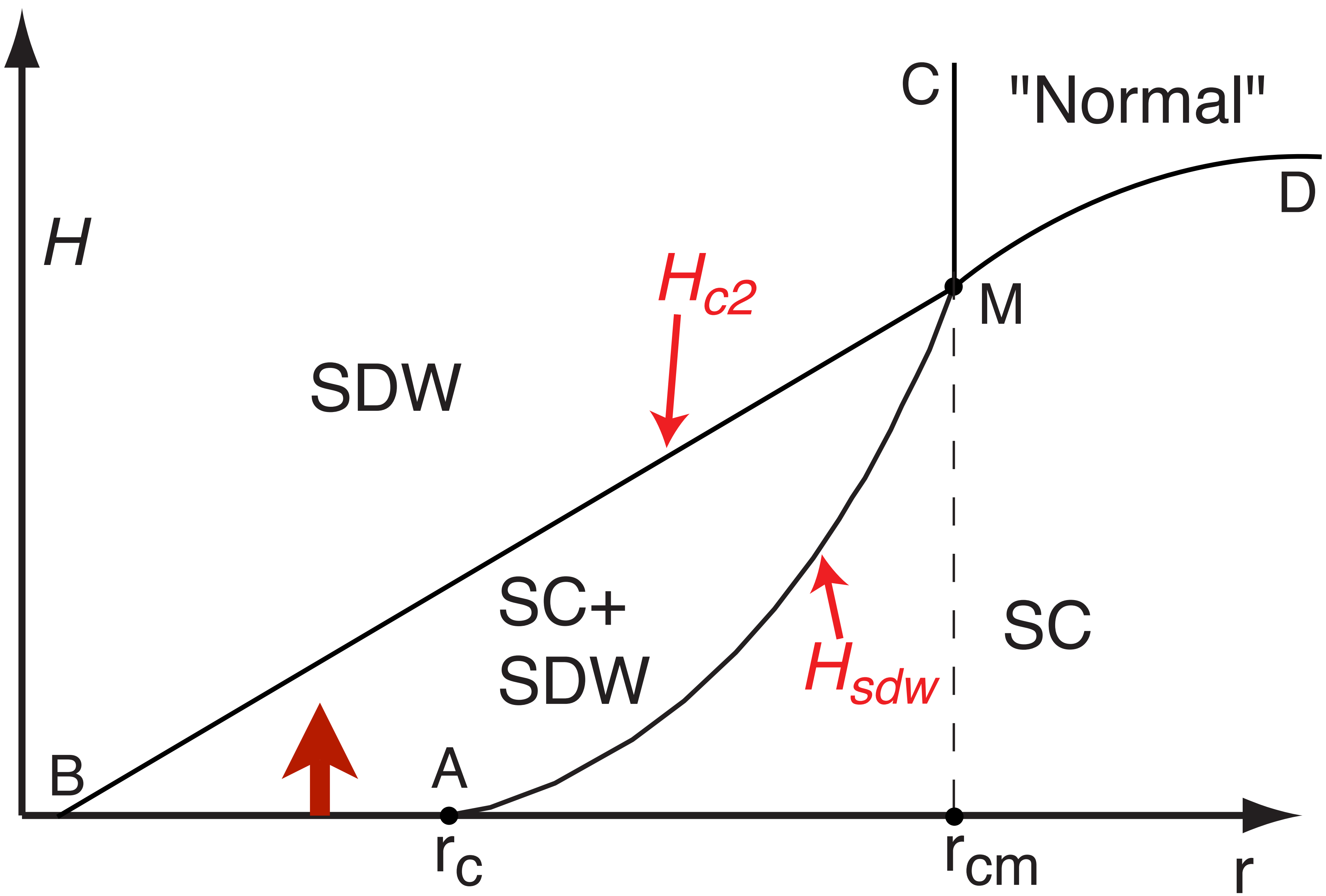


- For doping with $r_c < r < r_{cm}$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



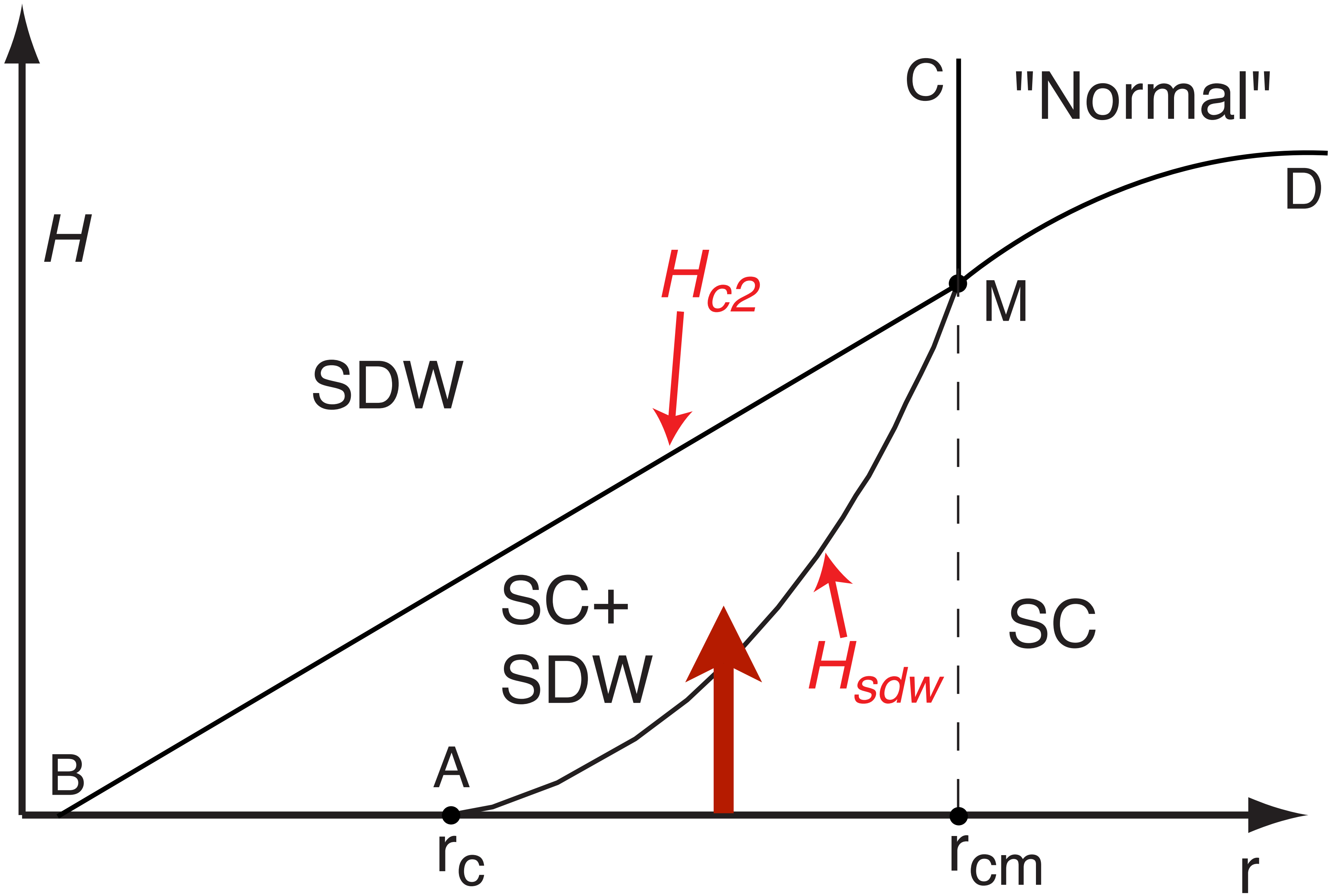
Neutron scattering on $La_{1.9}Sr_{0.1}CuO_4$

B. Lake *et al.*, *Nature* **415**, 299 (2002)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

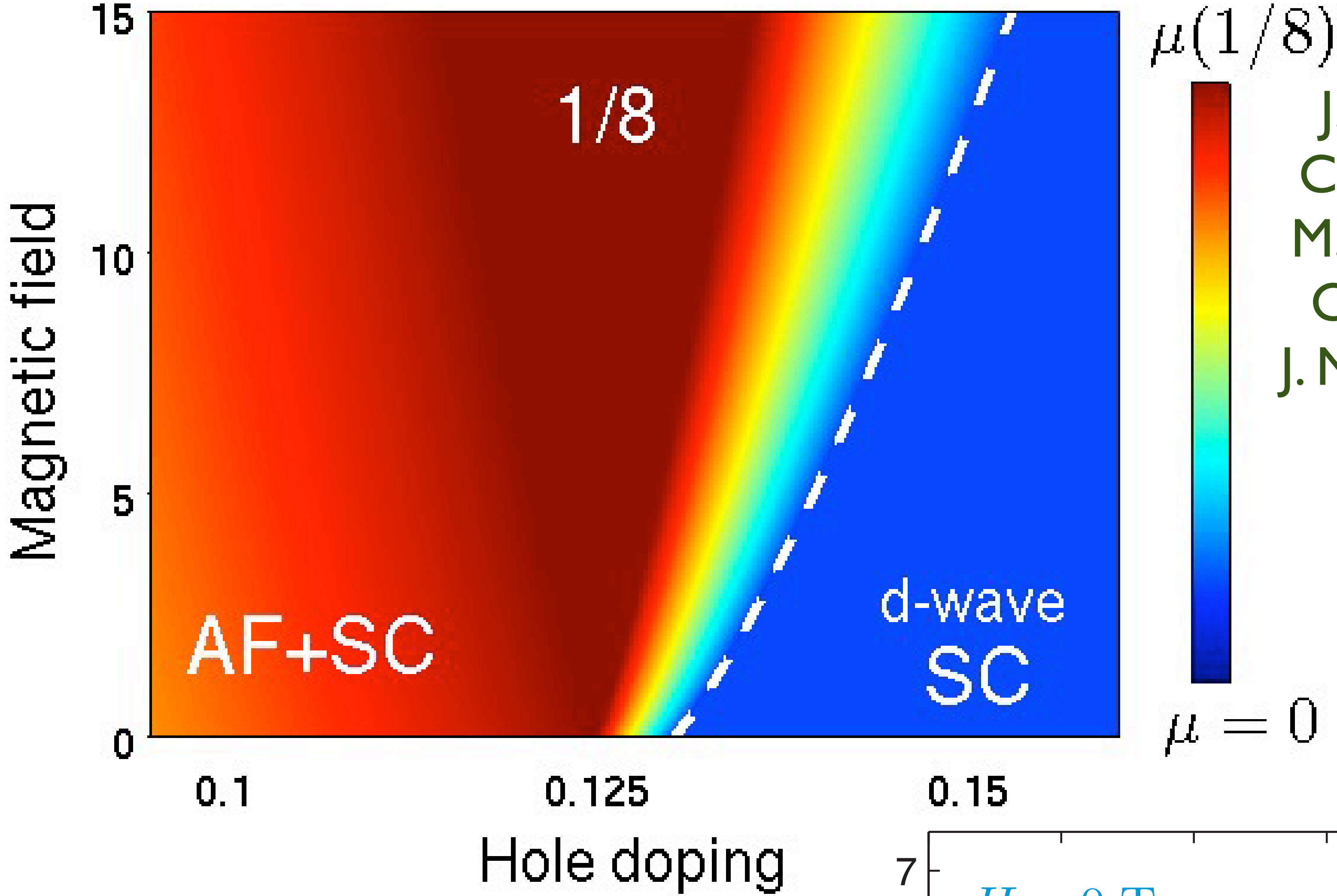
Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

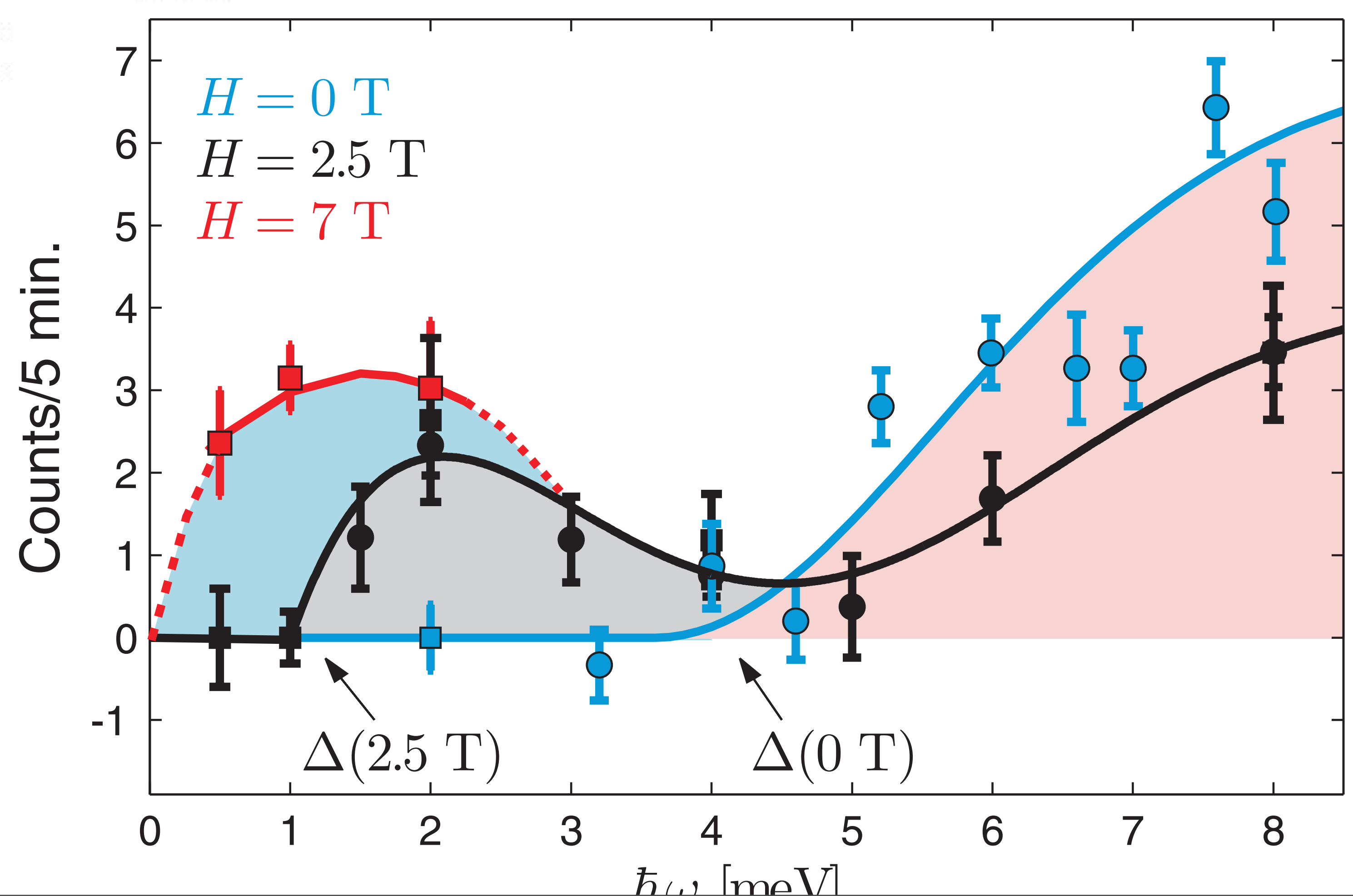


Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
J. Chang *et al.*, arXiv:0902.1191

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



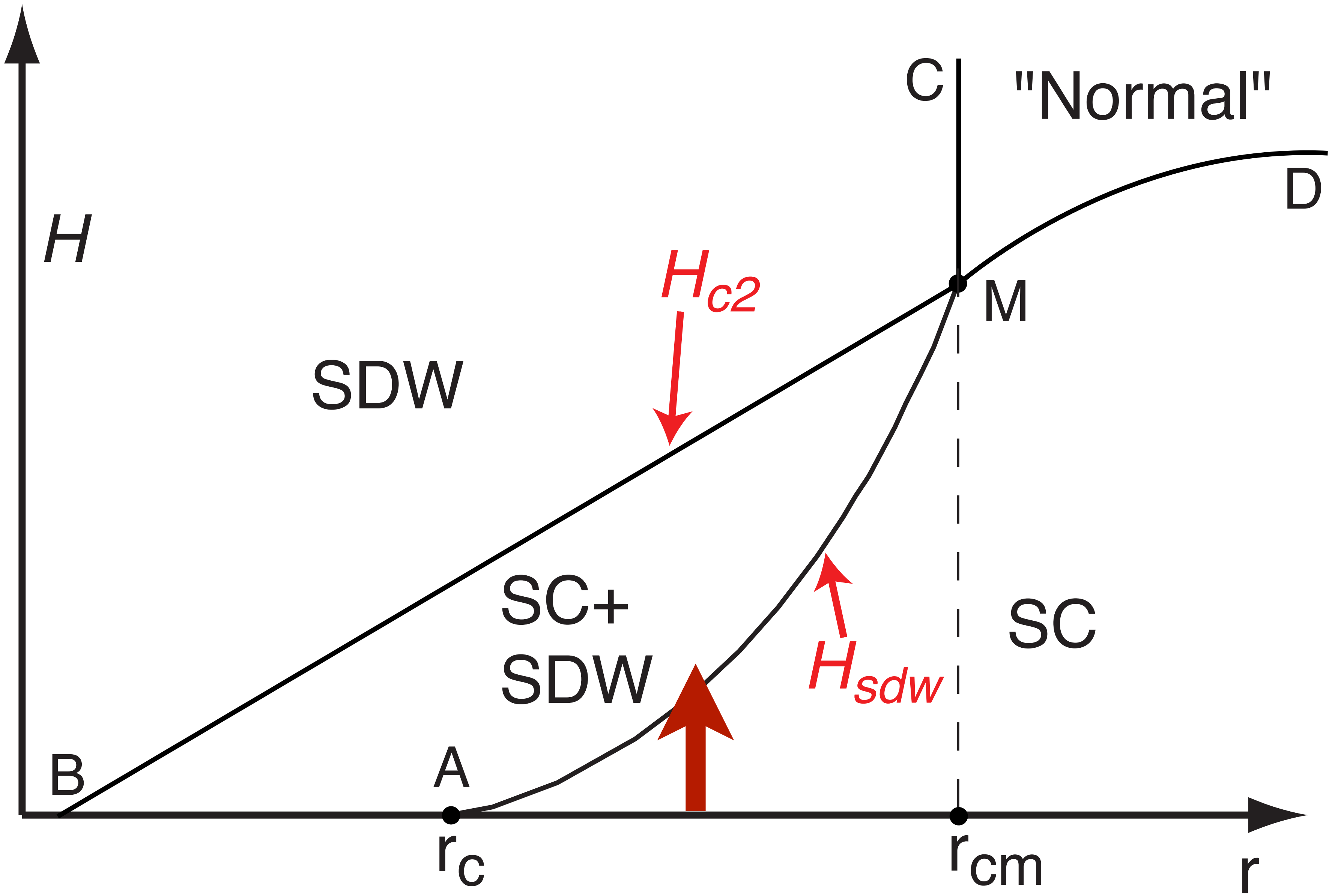
J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *arXiv:0902.1191*

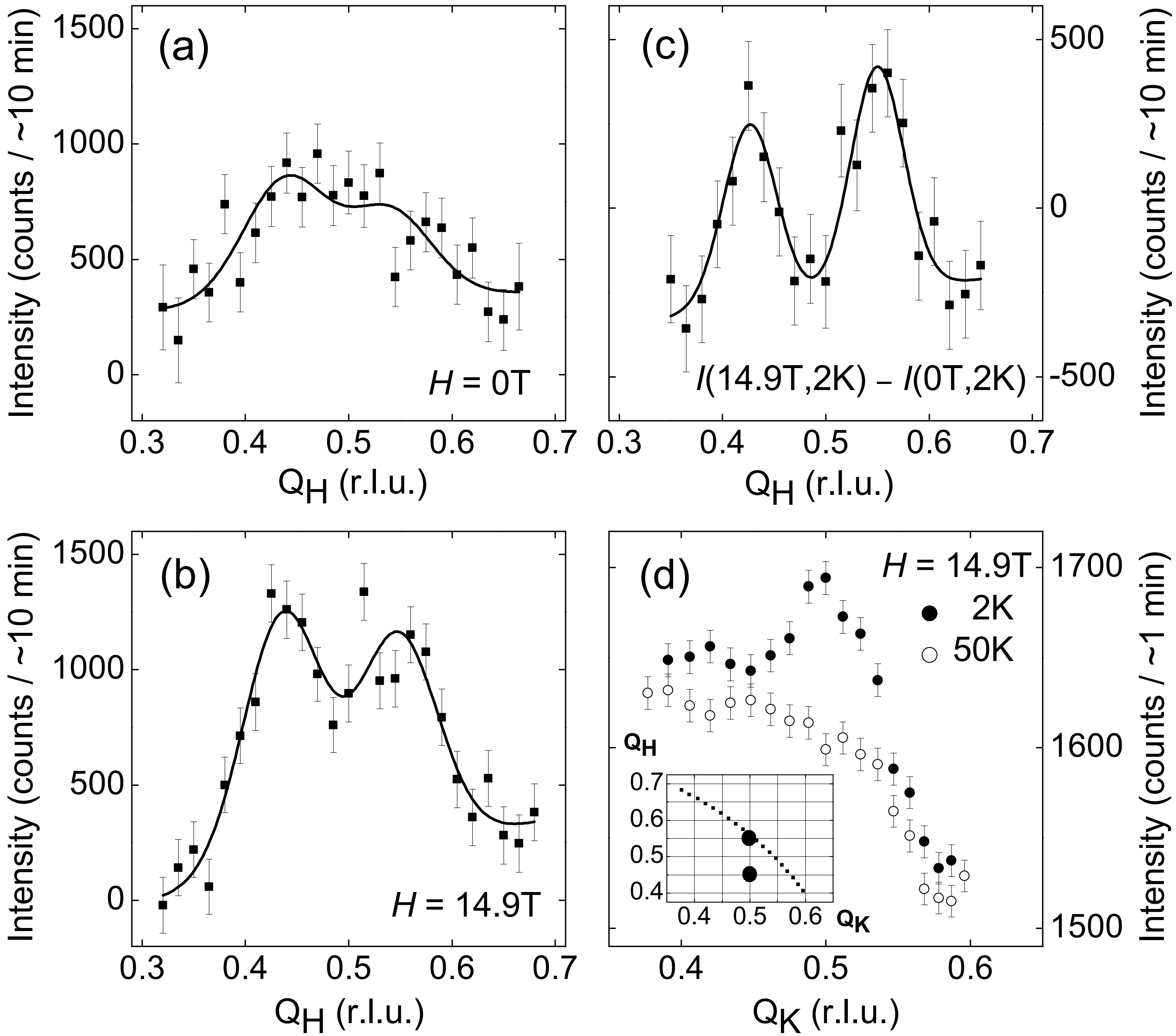
Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, arXiv:0902.3335

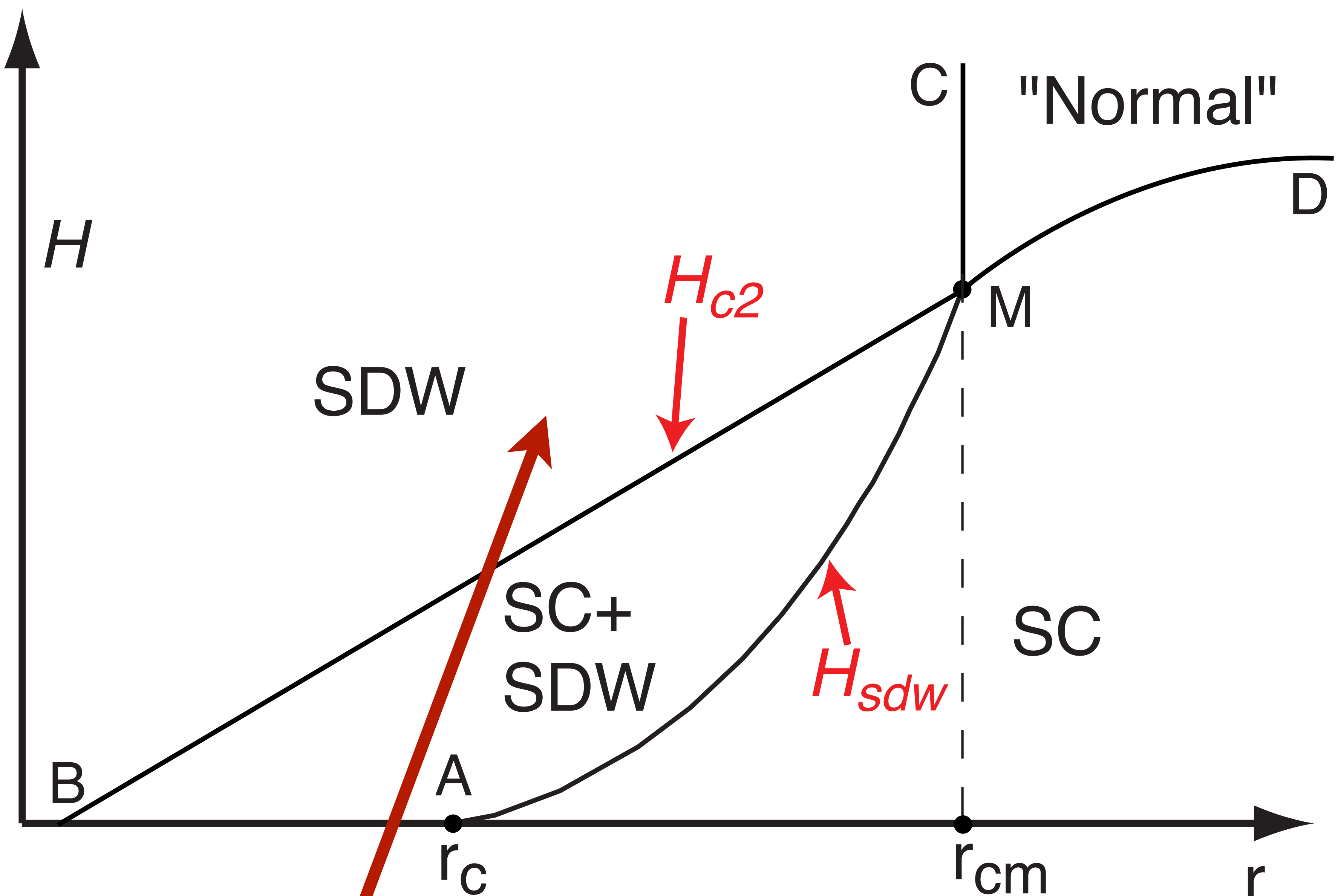
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *arXiv:0902.3335*.

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

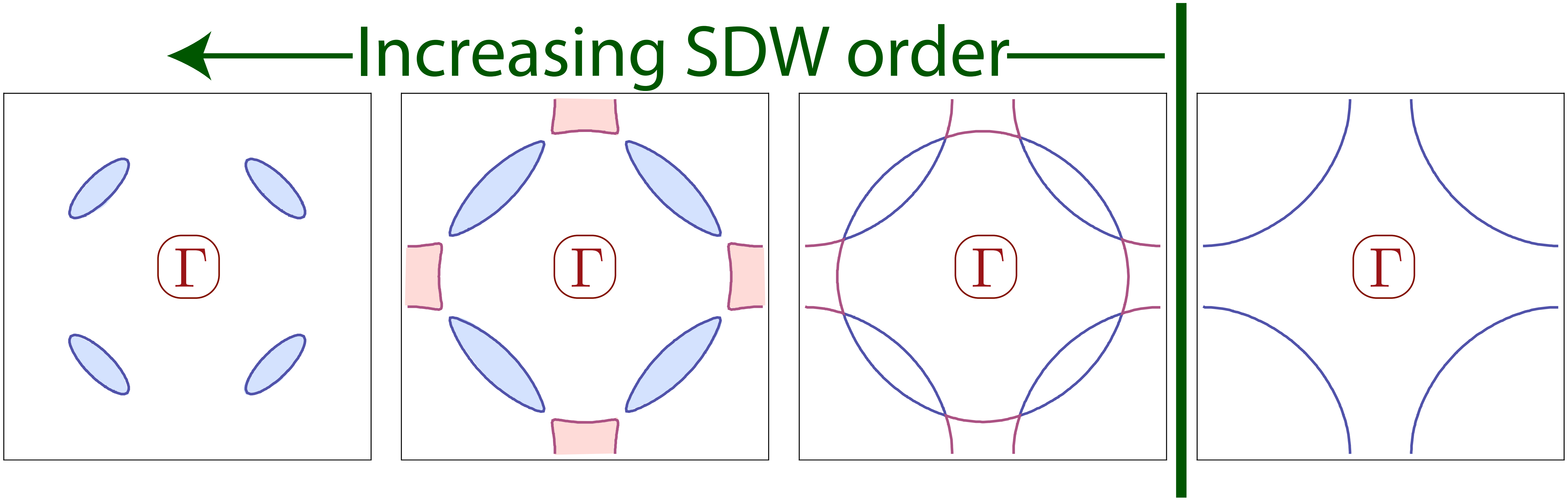


Quantum oscillations without Zeeman splitting

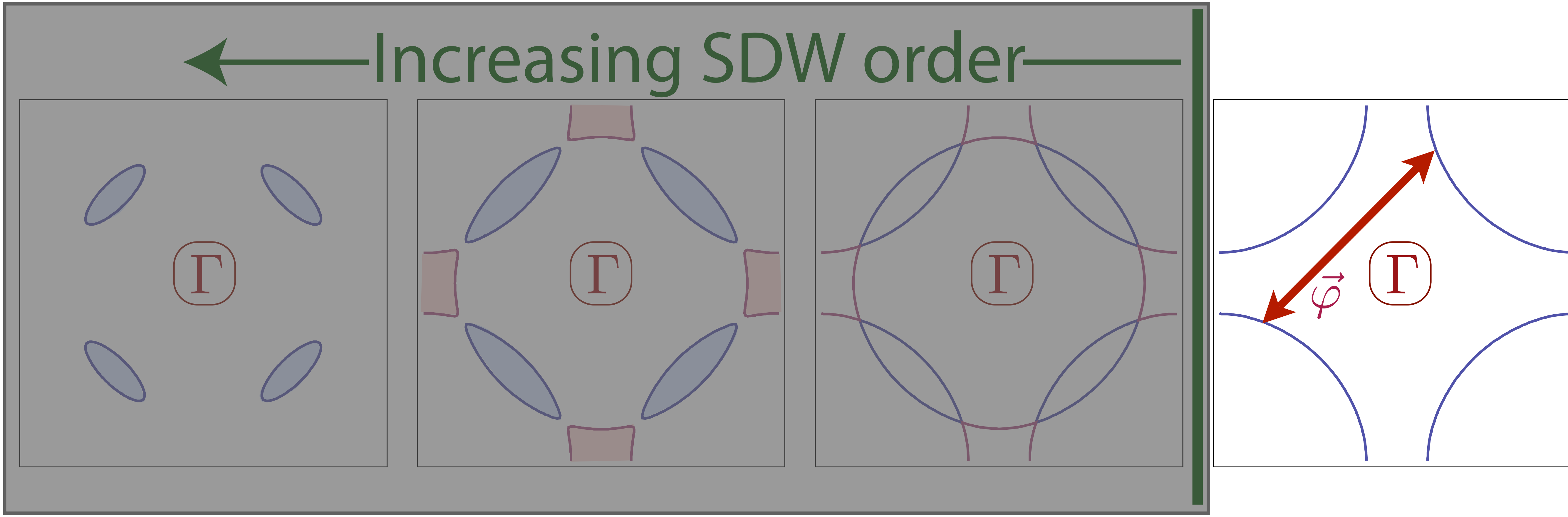
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

Superconductivity by SDW fluctuation exchange

Spin density wave theory in hole-doped cuprates



Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

David Pines, Douglas Scalapino

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^{\dagger}_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q}-\mathbf{K})^2},$$

with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

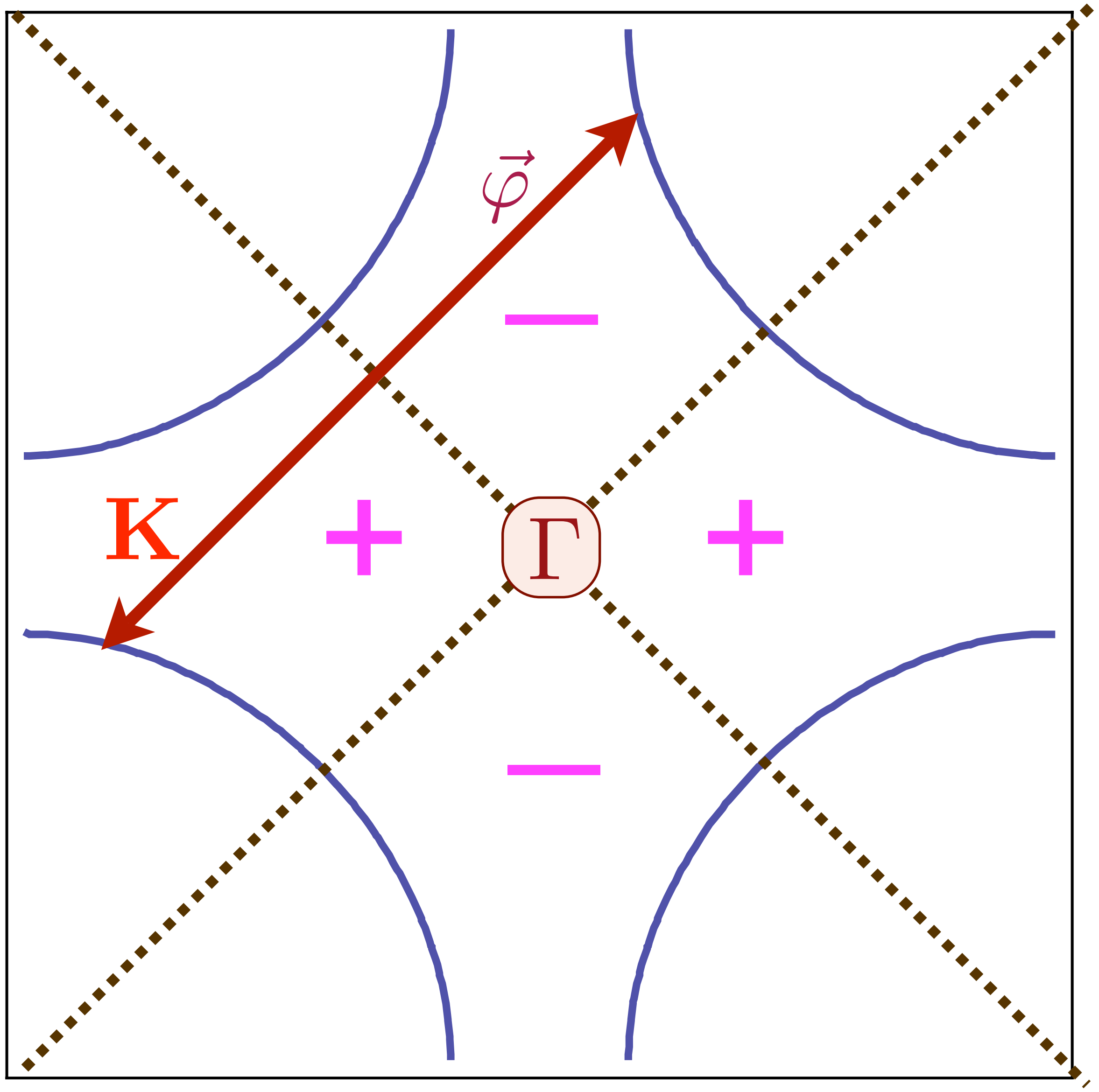
BCS Gap equation

In BCS theory, this interaction leads to the 'gap equation' for the pairing gap $\Delta_{\mathbf{k}} \propto \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$.

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{p}} \left(\frac{3\chi_0}{\xi^{-2} + (\mathbf{p} - \mathbf{k} - \mathbf{K})^2} \right) \frac{\Delta_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_{\mathbf{p}}^2}}$$

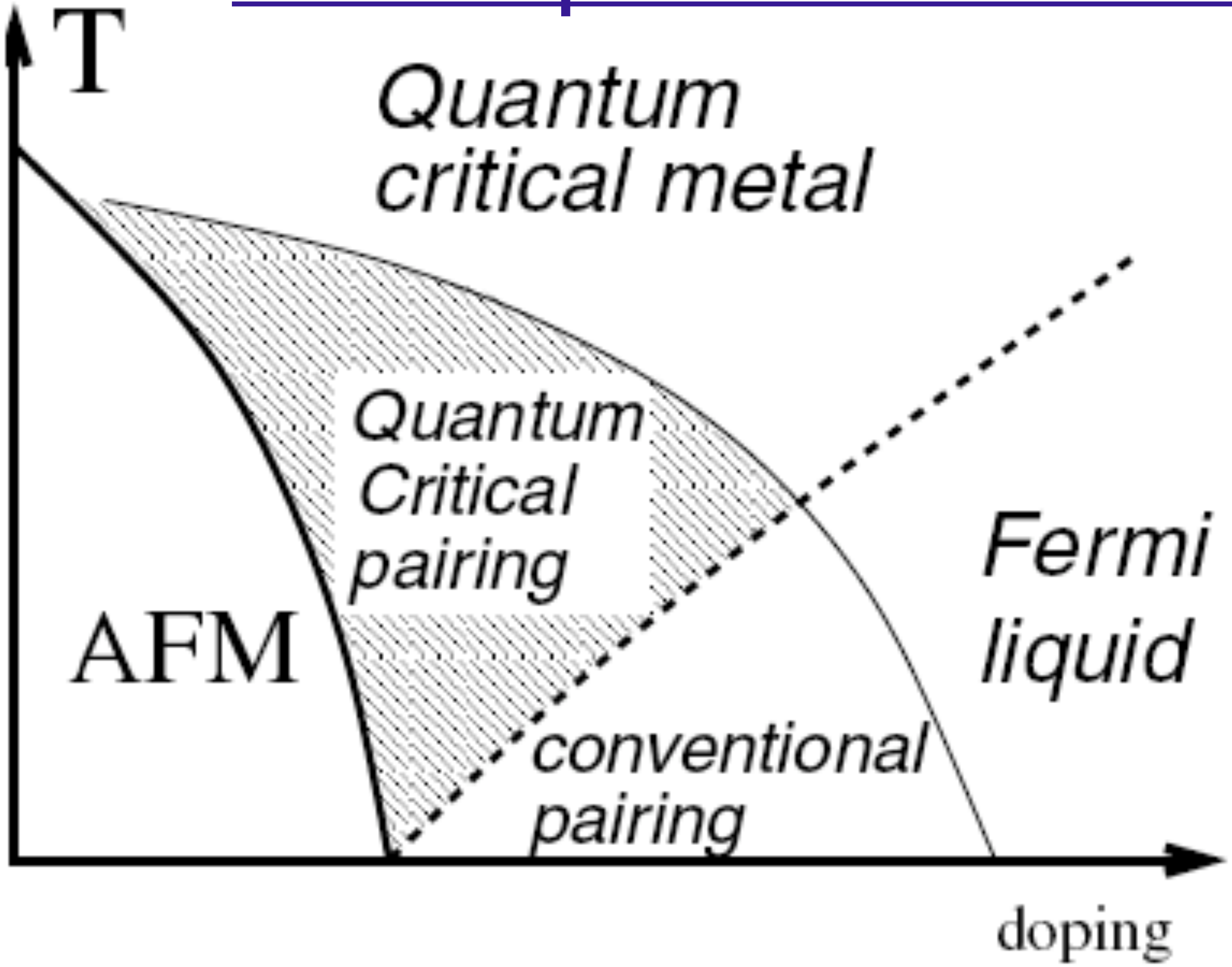
Non-zero solutions of this equation require that $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{p}}$ have opposite signs when $\mathbf{p} - \mathbf{k} \approx \mathbf{K}$.

d-wave pairing of the large Fermi surface



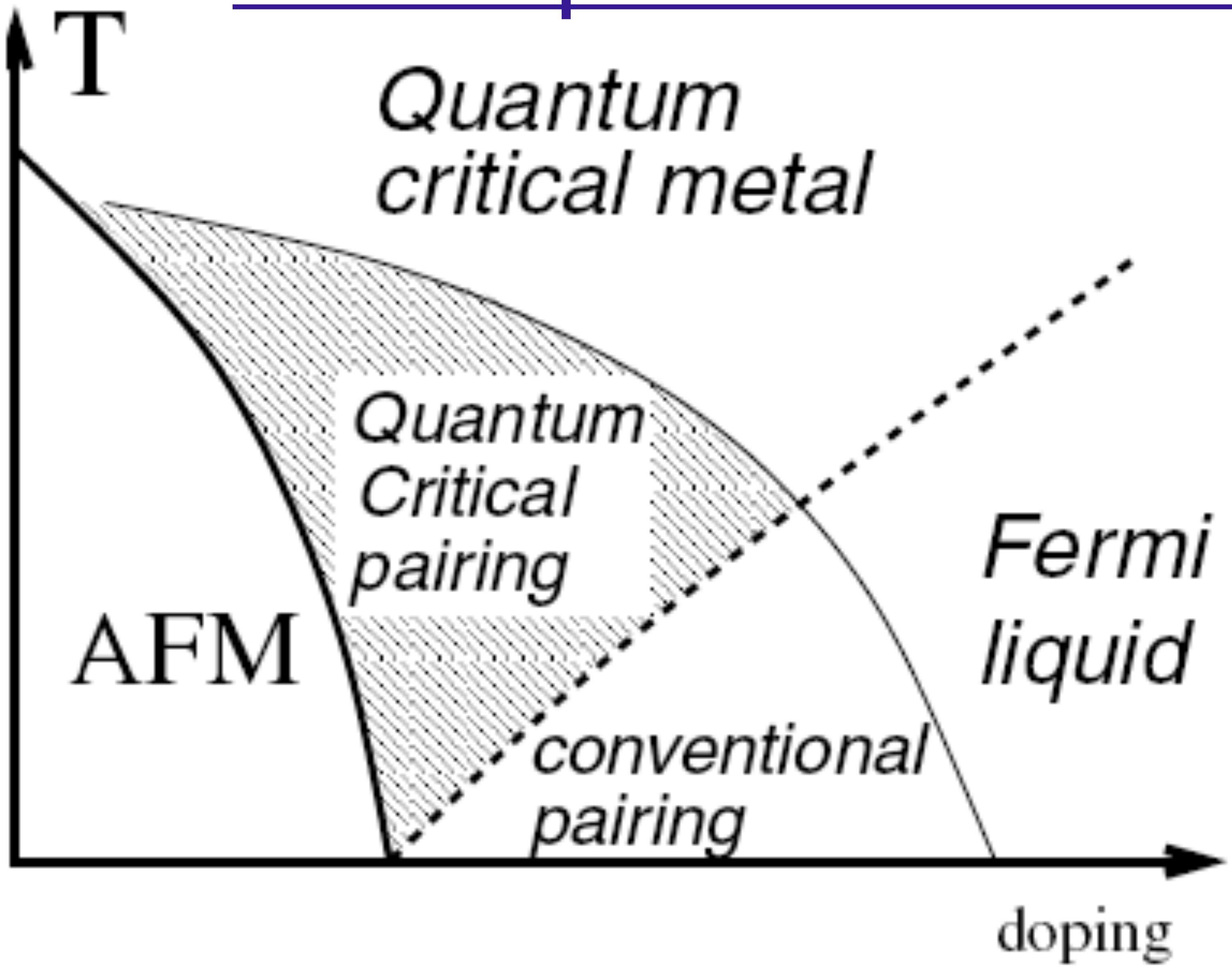
$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



- T_c *increases* upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.
- No simple mechanism for nodal-anti-nodal dichotomy.

Ar. Abanov, A. V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

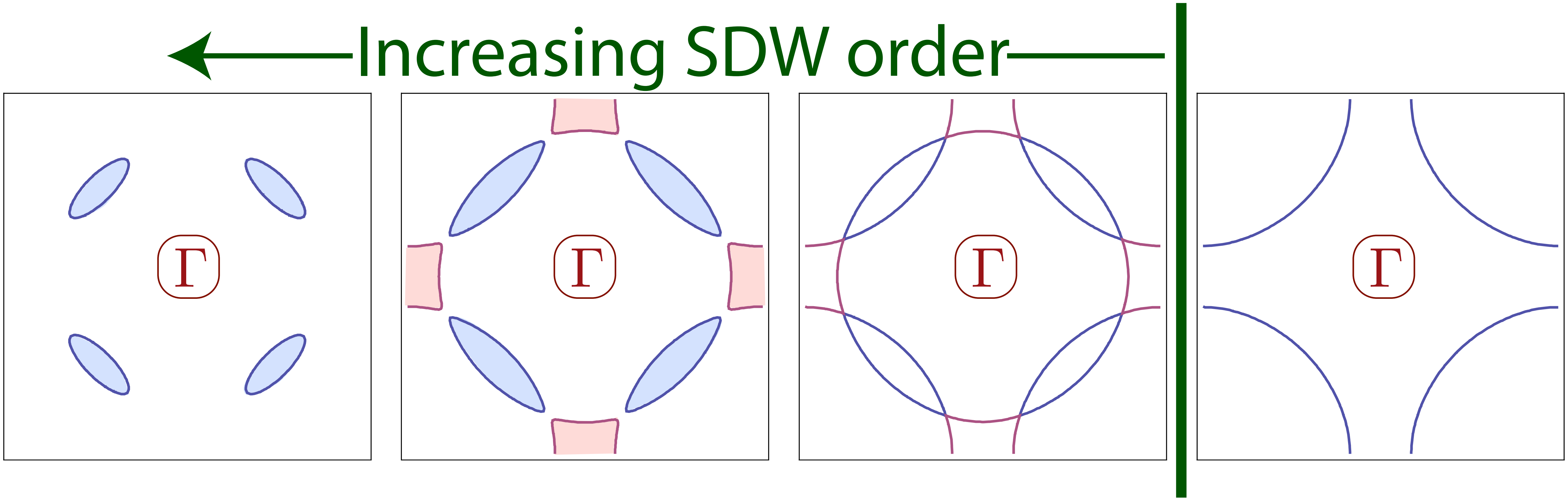
Superconductivity of fluctuating Fermi pockets in the underdoped cuprates

R. K. Kaul, M. Metlitksi, S. Sachdev, and C. Xu, *Physical Review B* **78**, 045110 (2008).

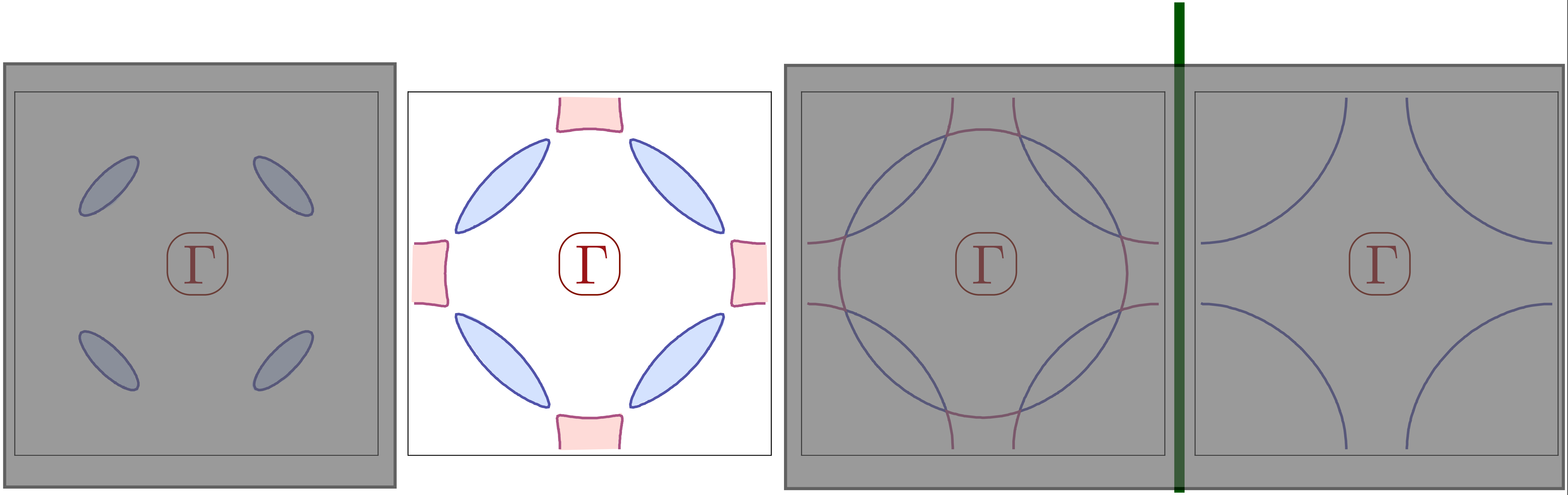
V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

Eun Gook Moon and S. Sachdev, *arXiv:0905.2608*

Spin density wave theory in hole-doped cuprates



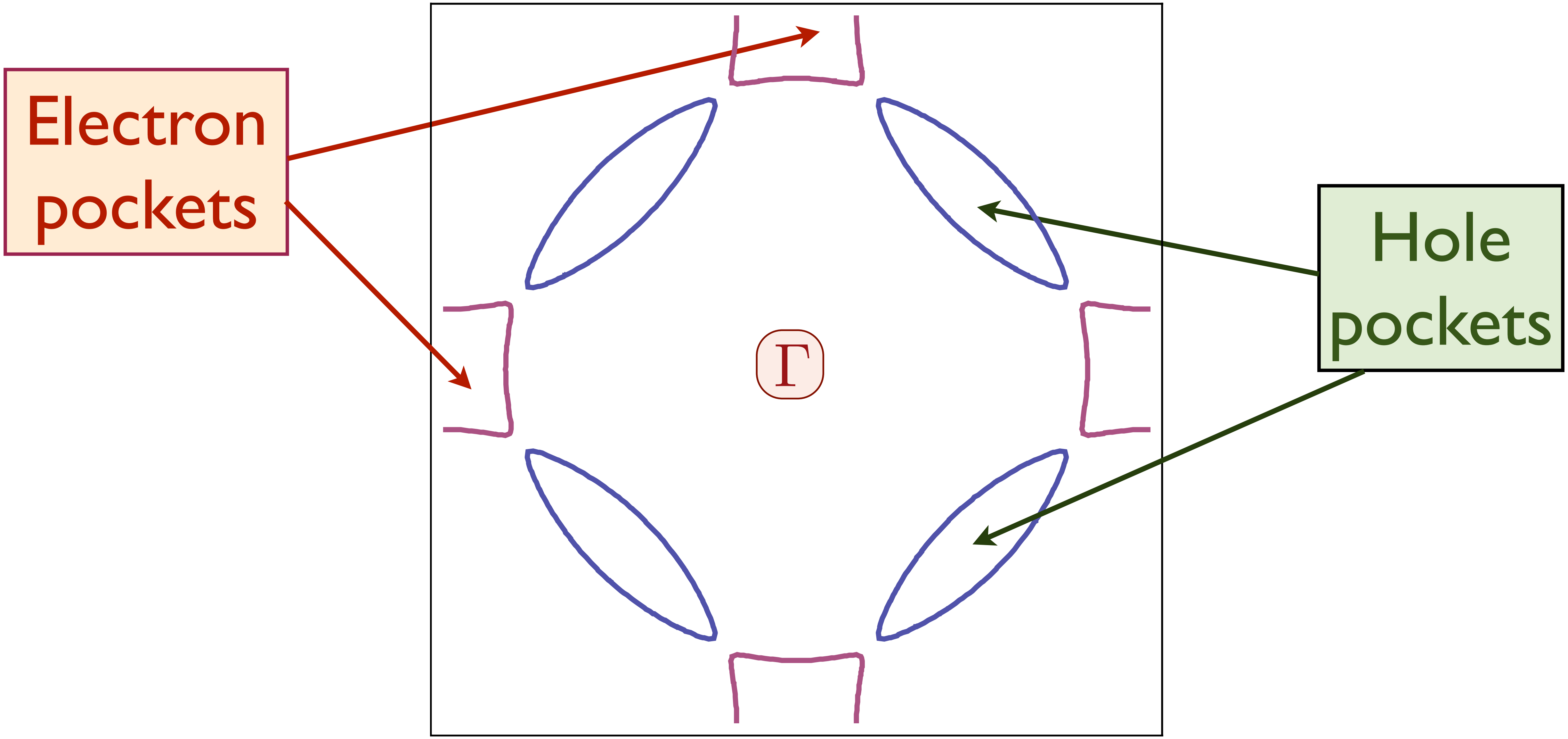
Fermi pockets in hole-doped cuprates



Begin with SDW ordered state, and focus on fluctuations in the *orientation* of $\vec{\varphi}$, by using a unit-length bosonic spinor z_α

$$\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

Charge carriers in the lightly-doped cuprates with Neel order



Spin density wave theory for electrons near $(0,\pi)$ and $(\pi,0)$

Let us write $c_{(0,\pi)\alpha} = c_{1\alpha}$, $c_{(\pi,0)\alpha} = c_{2\alpha}$ and $\varepsilon_{(0,\pi)} = \varepsilon_{(\pi,0)} = \varepsilon_0$. Then the Hamiltonian for $\vec{\varphi} = (0,0,\varphi)$ with $\varphi > 0$ is

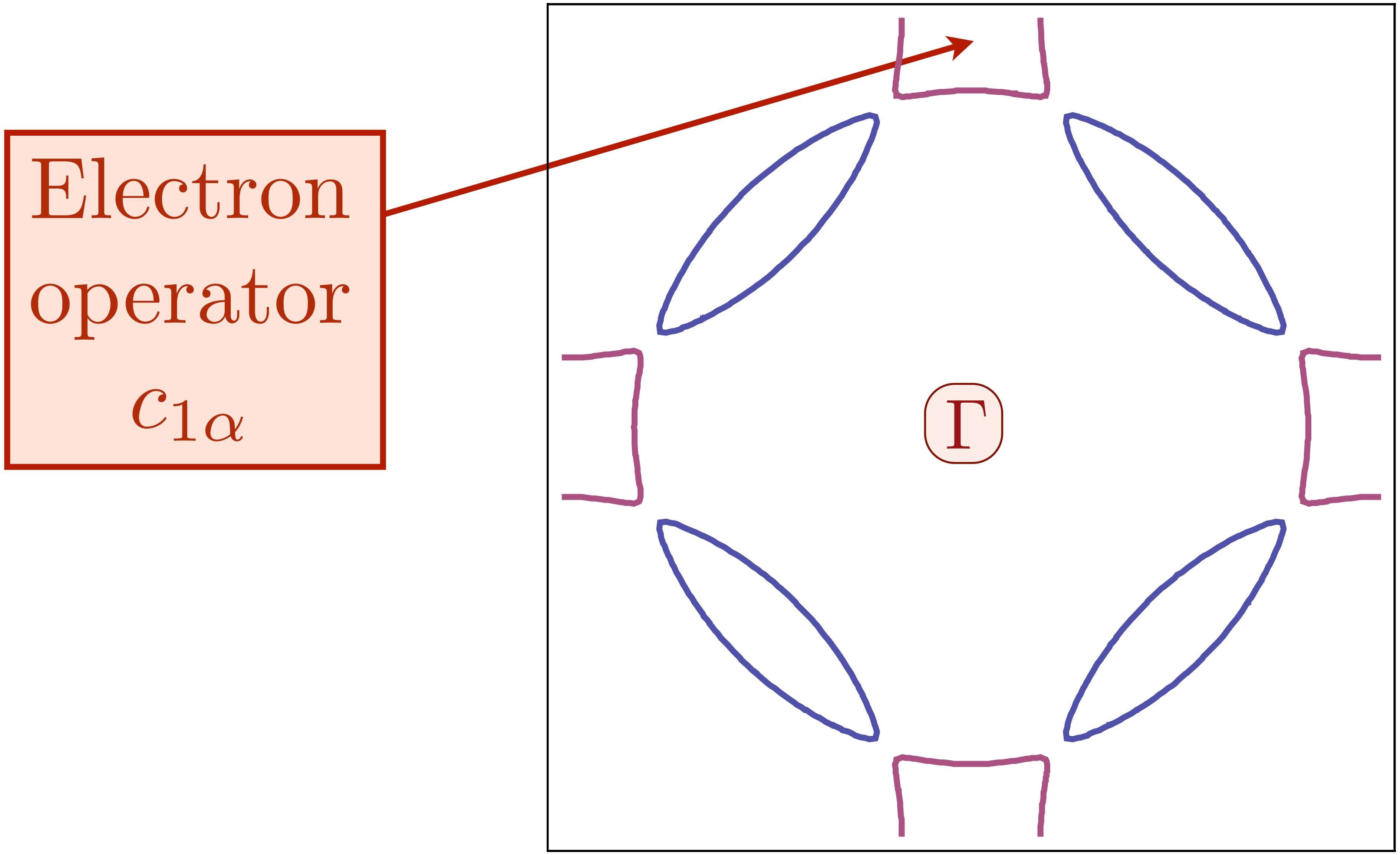
$$
\begin{aligned}
H_0 + H_{\rm sdw} &= \varepsilon_0 \left(c_{1\alpha}^\dagger c_{1\alpha} + c_{2\alpha}^\dagger c_{2\alpha} \right) \\
&\quad - \varphi \left(c_{1\uparrow}^\dagger c_{2\uparrow} - c_{1\downarrow}^\dagger c_{2\downarrow} + c_{2\uparrow}^\dagger c_{1\uparrow} - c_{2\downarrow}^\dagger c_{1\downarrow} \right)
\end{aligned}
$$

We diagonalize this by writing

$$
H_0 + H_{\rm sdw} = (\varepsilon_0 - \varphi) \left(g_+^\dagger g_+ + g_-^\dagger g_- \right) + (\varepsilon_0 + \varphi) \left(h_+^\dagger h_+ + h_-^\dagger h_- \right)
$$

where

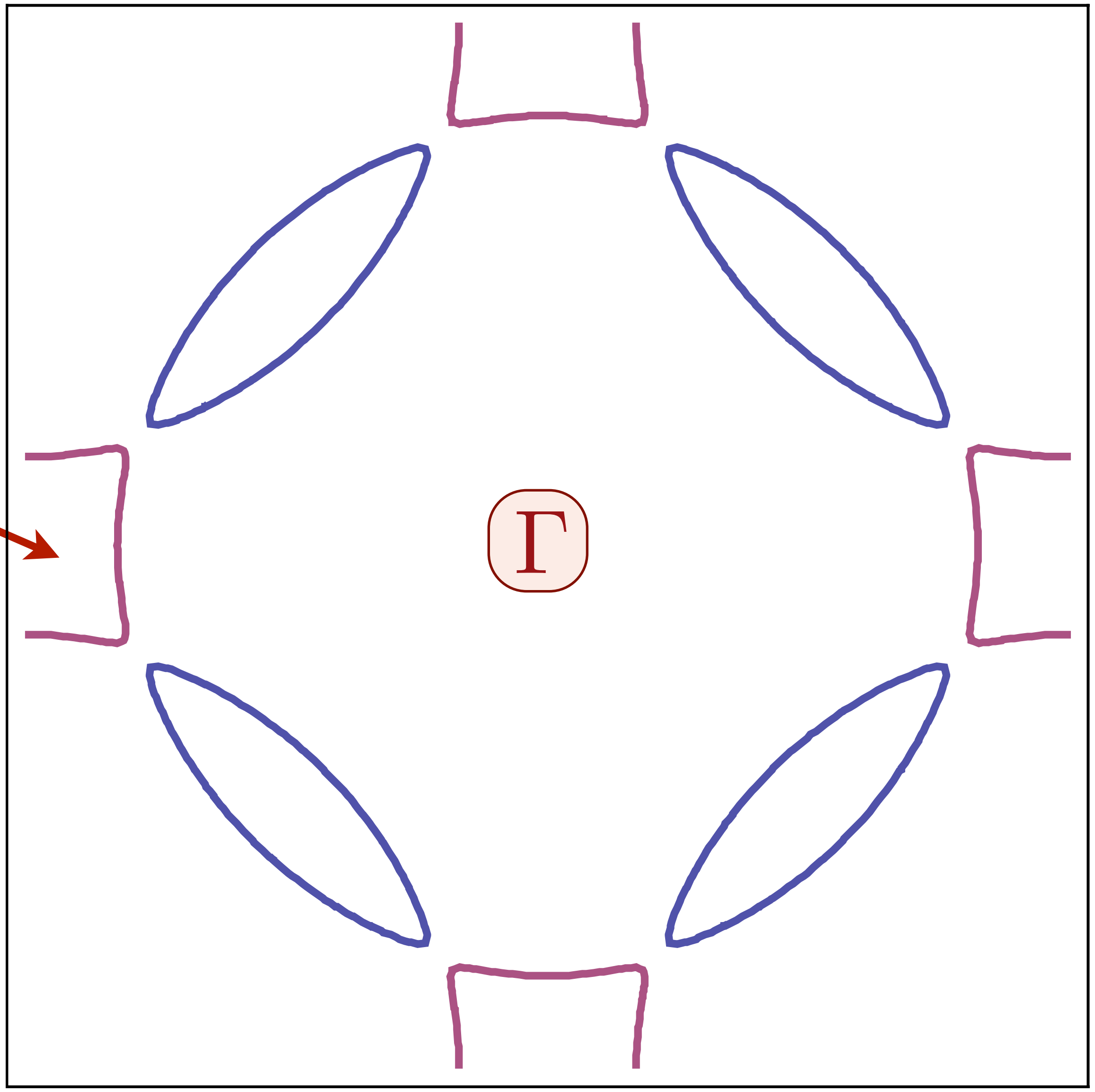
$$
\begin{aligned}
c_{1\uparrow} &= (g_+ + h_+)/\sqrt{2} \\
c_{2\uparrow} &= (g_+ - h_+)/\sqrt{2} \\
c_{1\downarrow} &= (g_- + h_-)/\sqrt{2} \\
c_{2\downarrow} &= (-g_- + h_-)/\sqrt{2}
\end{aligned}
$$



For a uniform SDW order with $\vec{\varphi} = (0, 0, \varphi)$, write

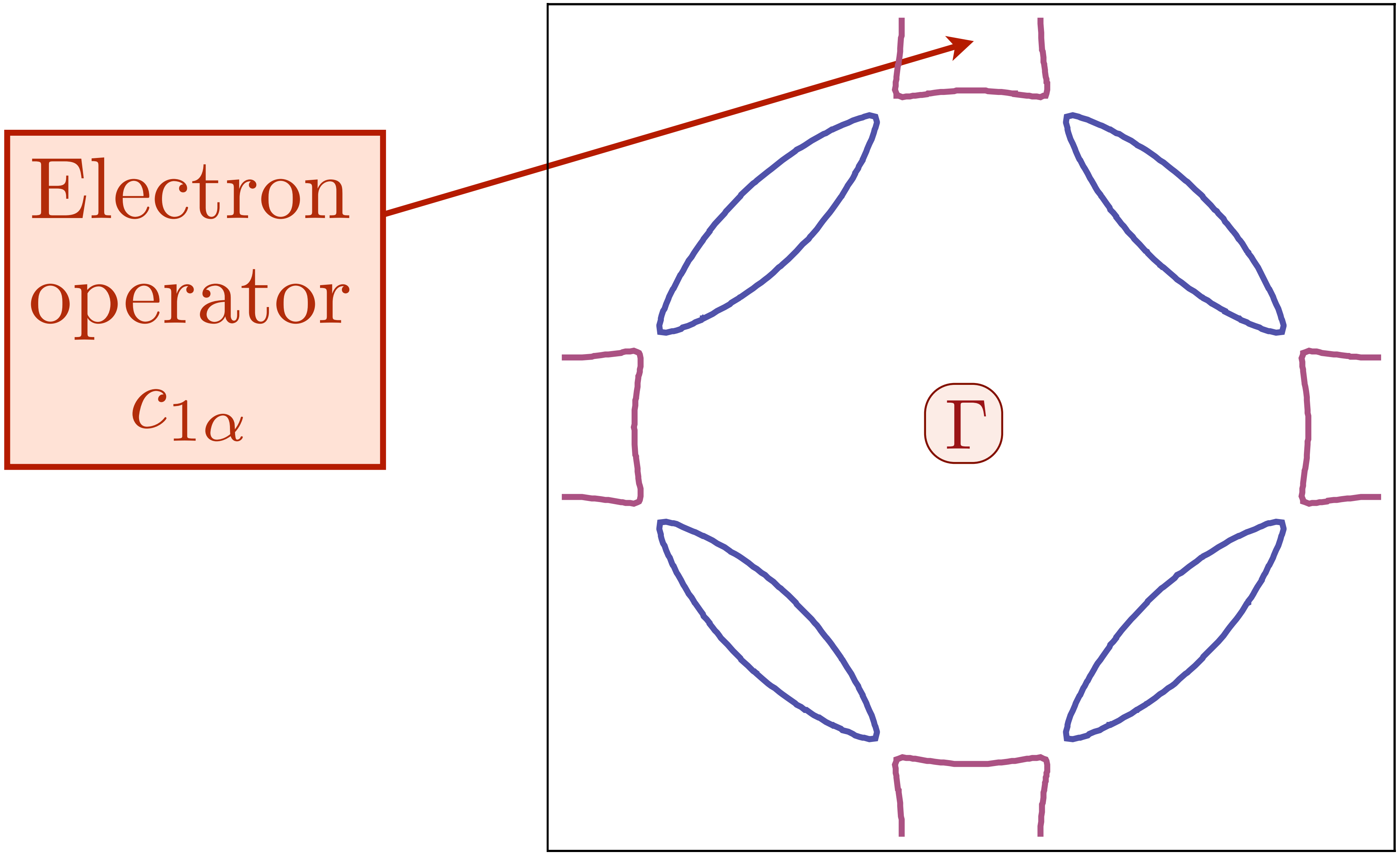
$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \begin{pmatrix} g_{+} \\ g_{-} \end{pmatrix}$$

Electron operator $c_{2\alpha}$



SDW theory also specifies electrons at second pocket for $\vec{\varphi} = (0, 0, \varphi)$

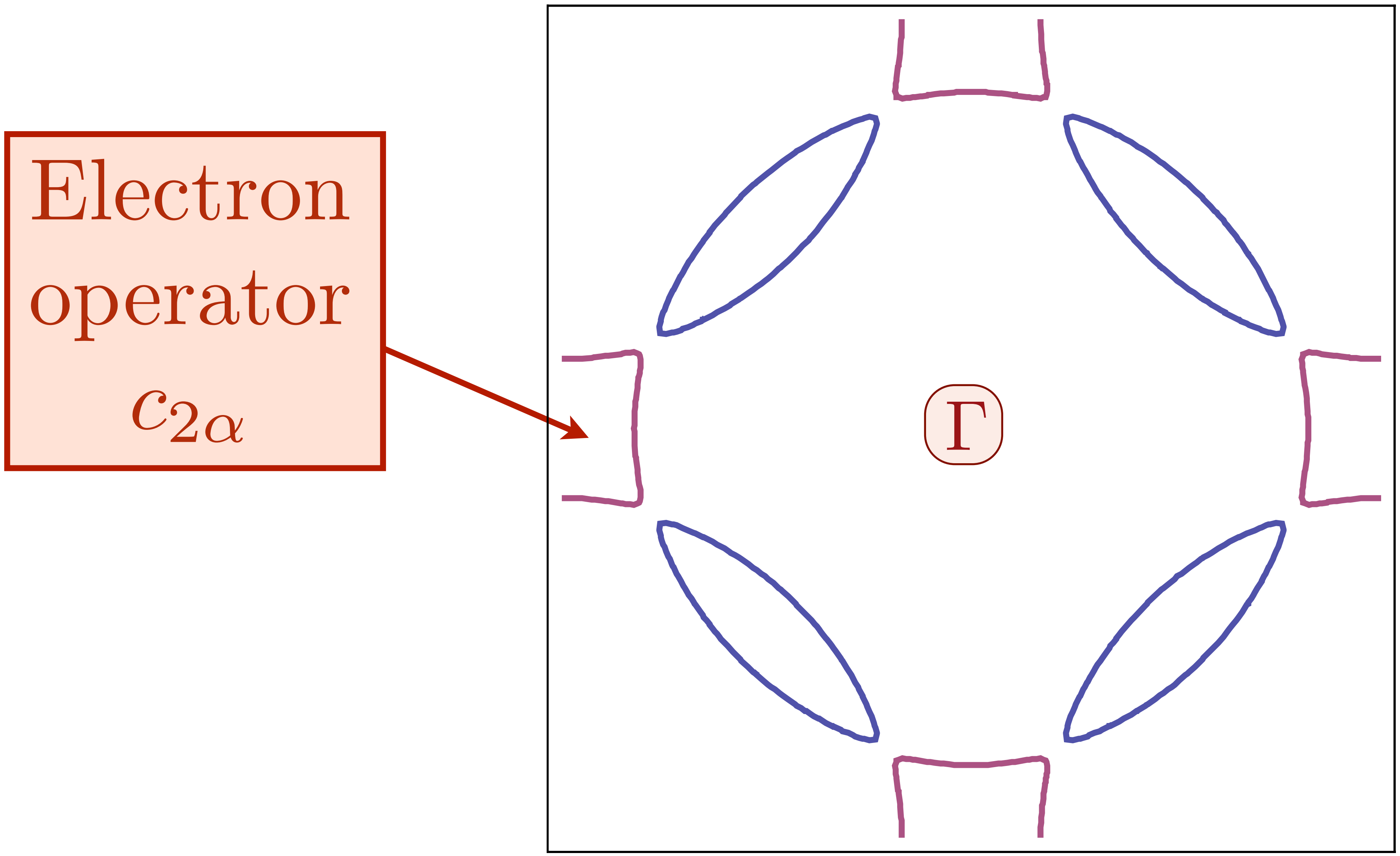
$$\begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \begin{pmatrix} g_+ \\ -g_- \end{pmatrix} ,$$



For a spacetime dependent SDW order, $\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

So $g_\pm$ are the "up/down" electron operators
in a rotating reference frame defined by the local SDW order



For a spacetime dependent SDW order, $\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\left(\begin{array}{c} c_{2\uparrow} \\ c_{2\downarrow} \end{array} \right) = \mathcal{R}_z \left(\begin{array}{c} g_+ \\ -g_- \end{array} \right) \quad ; \quad \mathcal{R}_z \equiv \left(\begin{array}{cc} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{array} \right).$$

Same SU(2) matrix also rotates electrons in second pocket.

Fluctuating pocket theory for electrons near $(0, \pi)$ and $(\pi, 0)$

Summarizing, in the low energy theory, the $c_{1,2\alpha}$ are expressed in terms of the $g_\pm$ fermions and the z_α by

$$
\begin{aligned}
c_{1\uparrow} &= z_\uparrow g_+ - z_\downarrow^* g_- \\
c_{2\uparrow} &= z_\uparrow g_+ + z_\downarrow^* g_- \\
c_{1\downarrow} &= z_\downarrow g_+ + z_\uparrow^* g_- \\
c_{2\downarrow} &= z_\downarrow g_+ - z_\uparrow^* g_-
\end{aligned}
$$

Note that this invariant under the U(1) gauge transformation

$$
z_\alpha \rightarrow e^{i\phi} z_\alpha \quad ; \quad g_+ \rightarrow e^{-i\phi} g_+ \quad ; \quad g_- \rightarrow e^{i\phi} g_-,
$$

which must be obeyed by the effective action for z_α and $g_\pm$.

Fluctuating pocket theory for electrons near $(0,\pi)$ and $(\pi,0)$

We will show that in the resulting theory, the $g_\pm$ are unstable to a simple s-wave pairing with

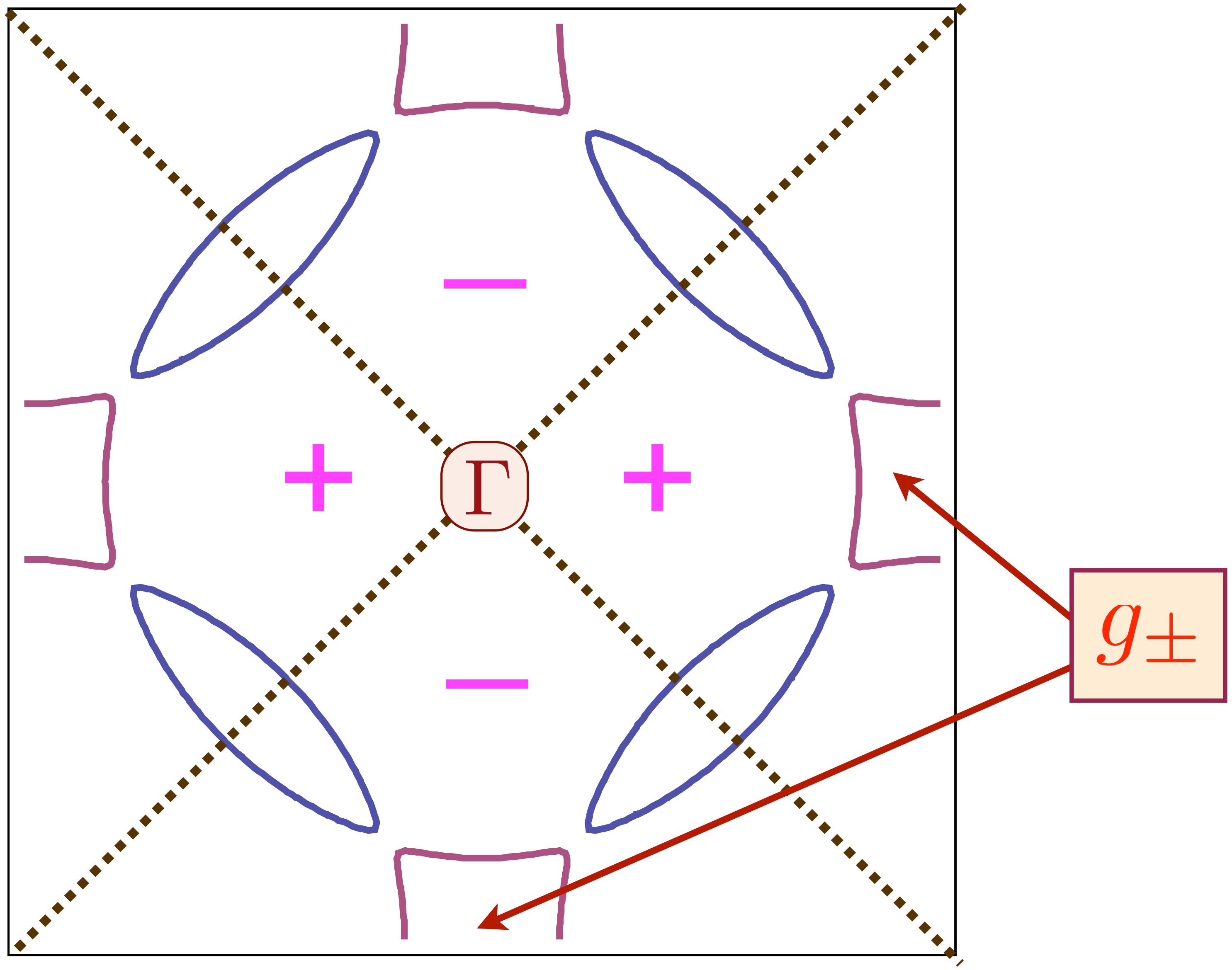
$$\langle g_+ g_- \rangle = \Delta$$

For the physical electron operators, this pairing implies

$$\begin{aligned} \langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \left\langle |z_\alpha|^2 \right\rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \left\langle |z_\alpha|^2 \right\rangle \end{aligned}$$

i.e. d-wave pairing !

Strong pairing of the $g_\pm$ electron pockets



$$\langle g_+ g_- \rangle = \Delta$$

Low energy theory for spinless, charge $-e$ fermions $g_{\pm}$, and spinful, charge 0 bosons z_α:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_z + \mathcal{L}_g \\
\mathcal{L}_z &= \frac{1}{t}\left[|(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2|\nabla - i\mathbf{A})z_\alpha|^2\right] \\
&+ \text{ Berry phases of monopoles in } A_\mu .
\end{aligned}
$$

CP^1 field theory for z_α and an emergent U(1) gauge field A_μ. Coupling t tunes the strength of SDW orientation fluctuations.

Low energy theory for spinless, charge $-e$ fermions $g_{\pm}$, and spinful, charge 0 bosons z_α:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_z + \mathcal{L}_g \\
\mathcal{L}_z &= \frac{1}{t}\left[|(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2|\nabla - i\mathbf{A})z_\alpha|^2\right] \\
&+ \text{Berry phases of monopoles in } A_\mu.
\end{aligned}
$$

CP^1 field theory for z_α and an emergent U(1) gauge field A_μ. Coupling t tunes the strength of SDW orientation fluctuations.

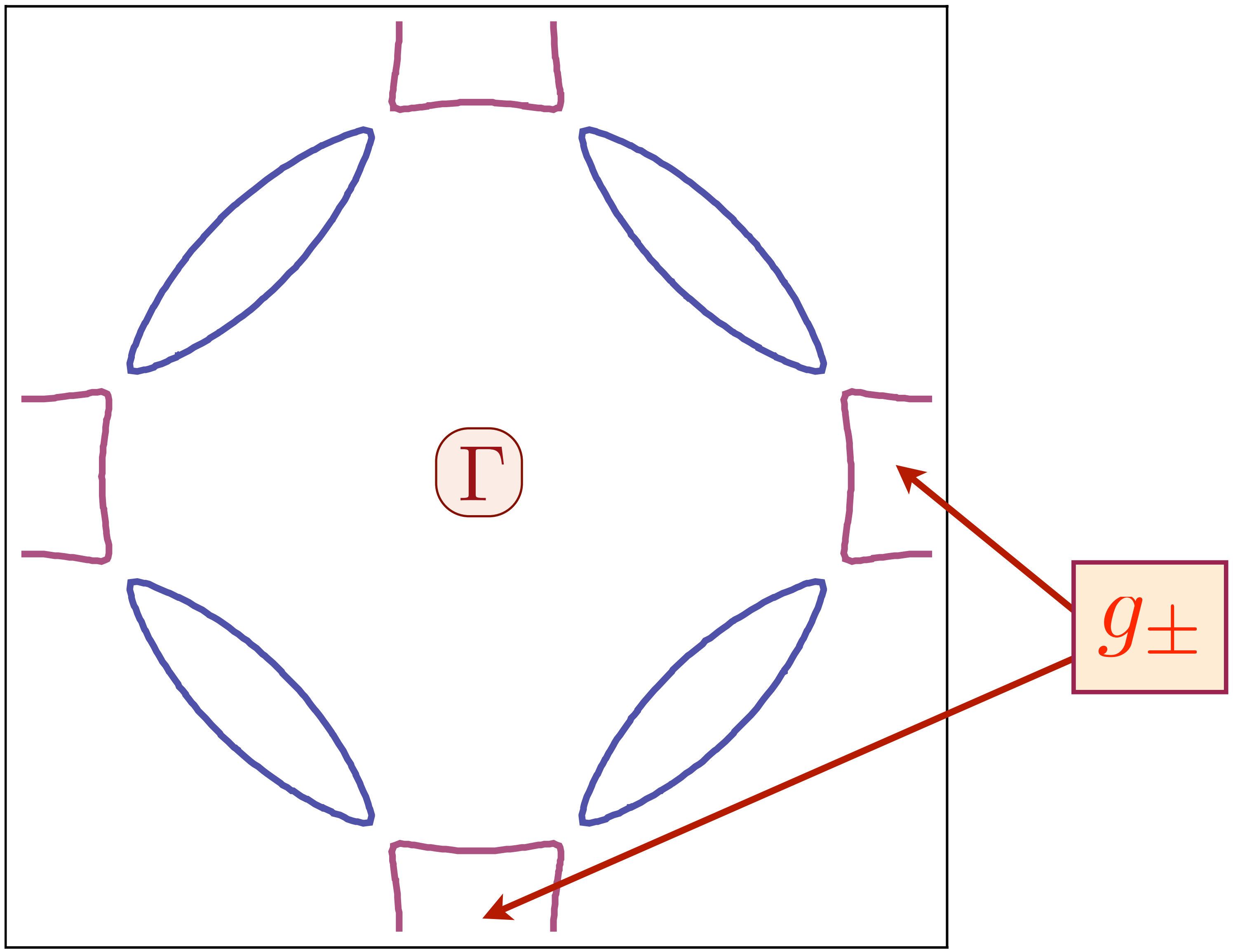
$$
\begin{aligned}
\mathcal{L}_g &= g_+^\dagger \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu\right] g_+ \\
&\quad + g_-^\dagger \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu\right] g_-
\end{aligned}
$$

Two Fermi surfaces coupled to the emergent U(1) gauge field A_μ with opposite charges

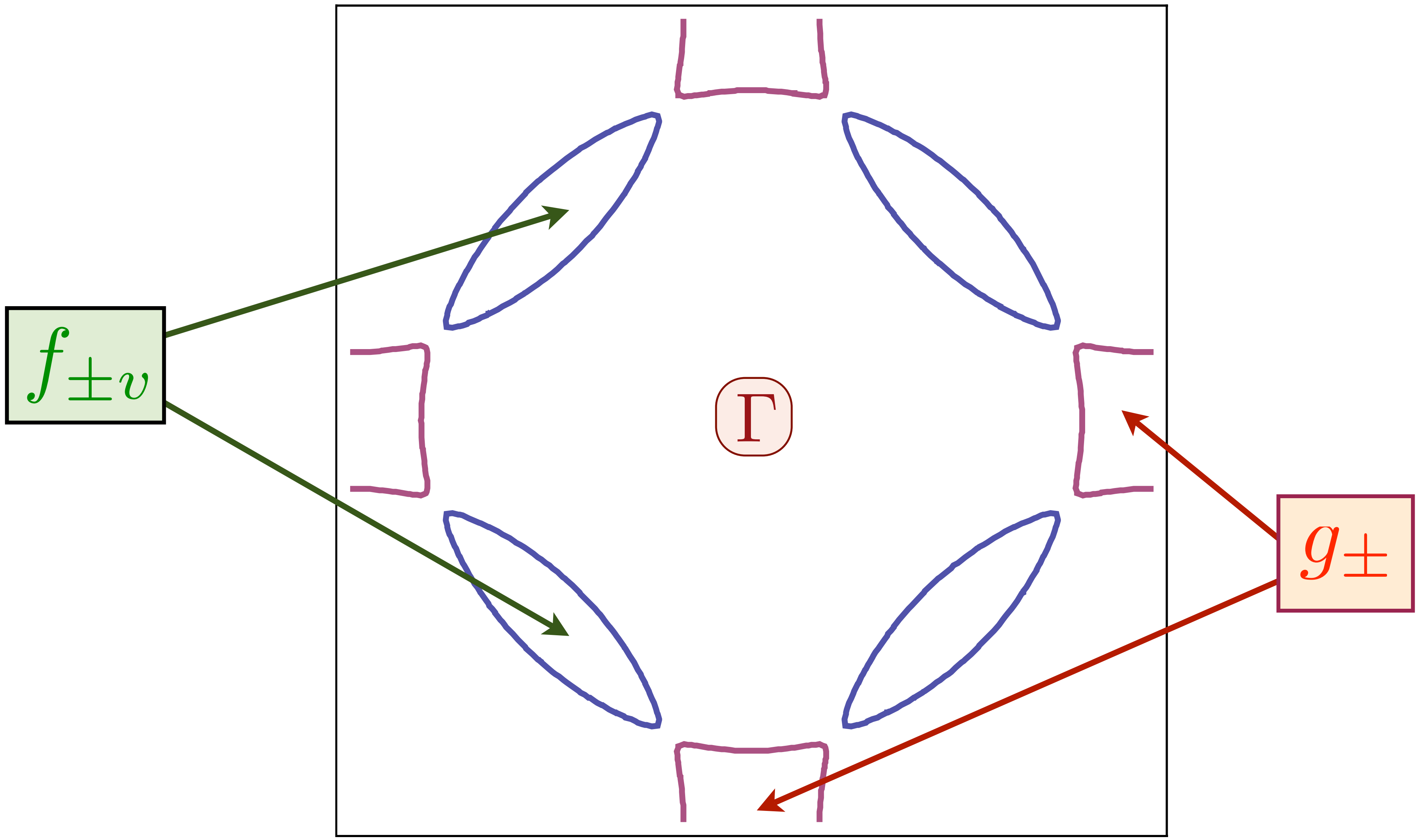
Strong pairing of the $g_\pm$ electron pockets

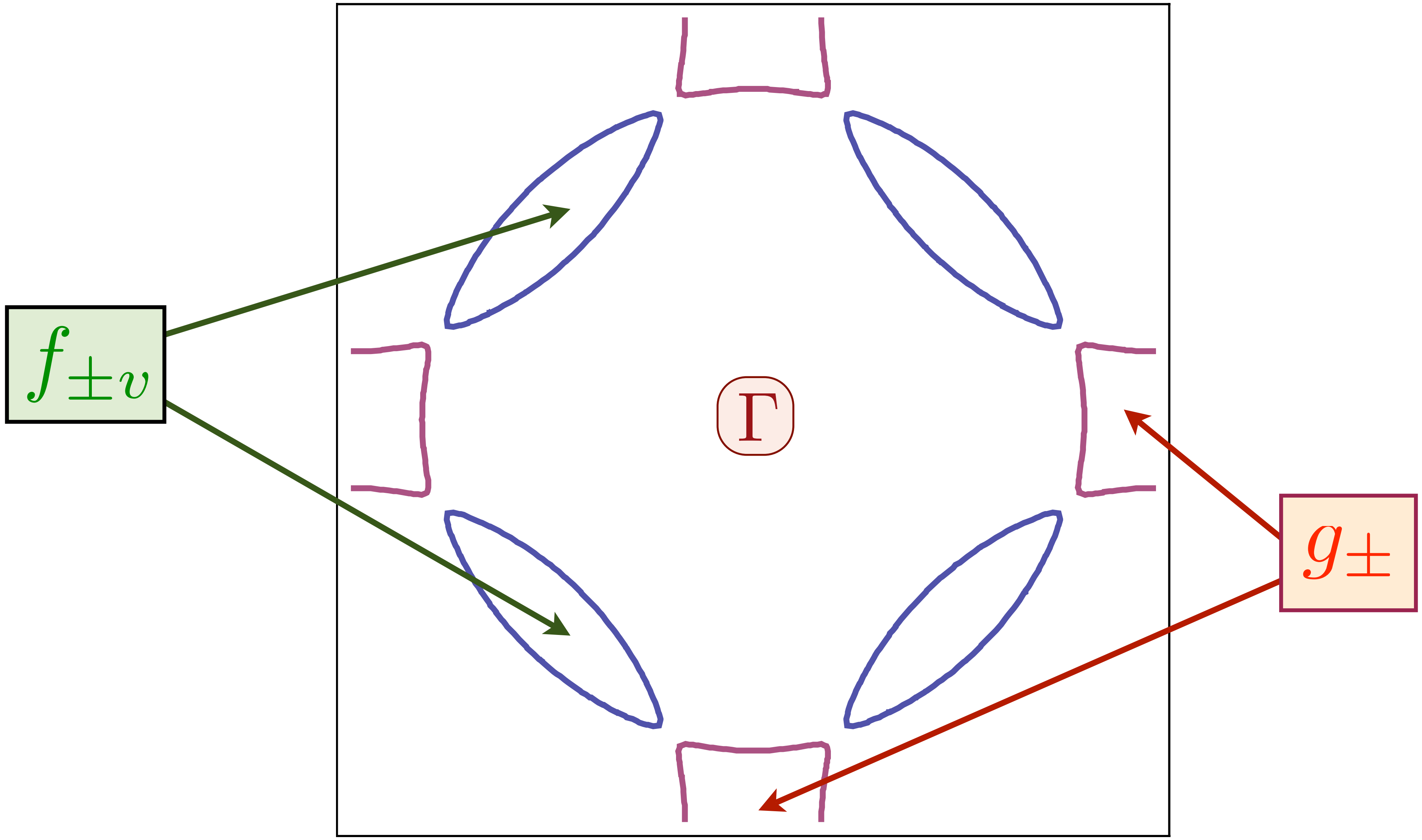
- Gauge forces lead to a s-wave paired state with a T_c of order the Fermi energy of the pockets. Inelastic scattering from low energy gauge modes lead to very singular $g_\pm$ self energy, but is *not* pair-breaking.

$$\langle g_+ g_- \rangle = \Delta$$



Γ
$g_\pm$


$f_{\pm v}$
Γ
$g_{\pm}$



Low energy theory for spinless, charge $+e$ fermions $f_{\pm v}$:

$$
\begin{aligned}
\mathcal{L}_f &= \sum_{v=1,2} \Bigg\{ f_{+v}^{\dagger} \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu \right] f_{+v} \\
&\qquad + f_{-v}^{\dagger} \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu \right] f_{-v} \Bigg\}
\end{aligned}
$$

Weak pairing of the $f_\pm$ hole pockets

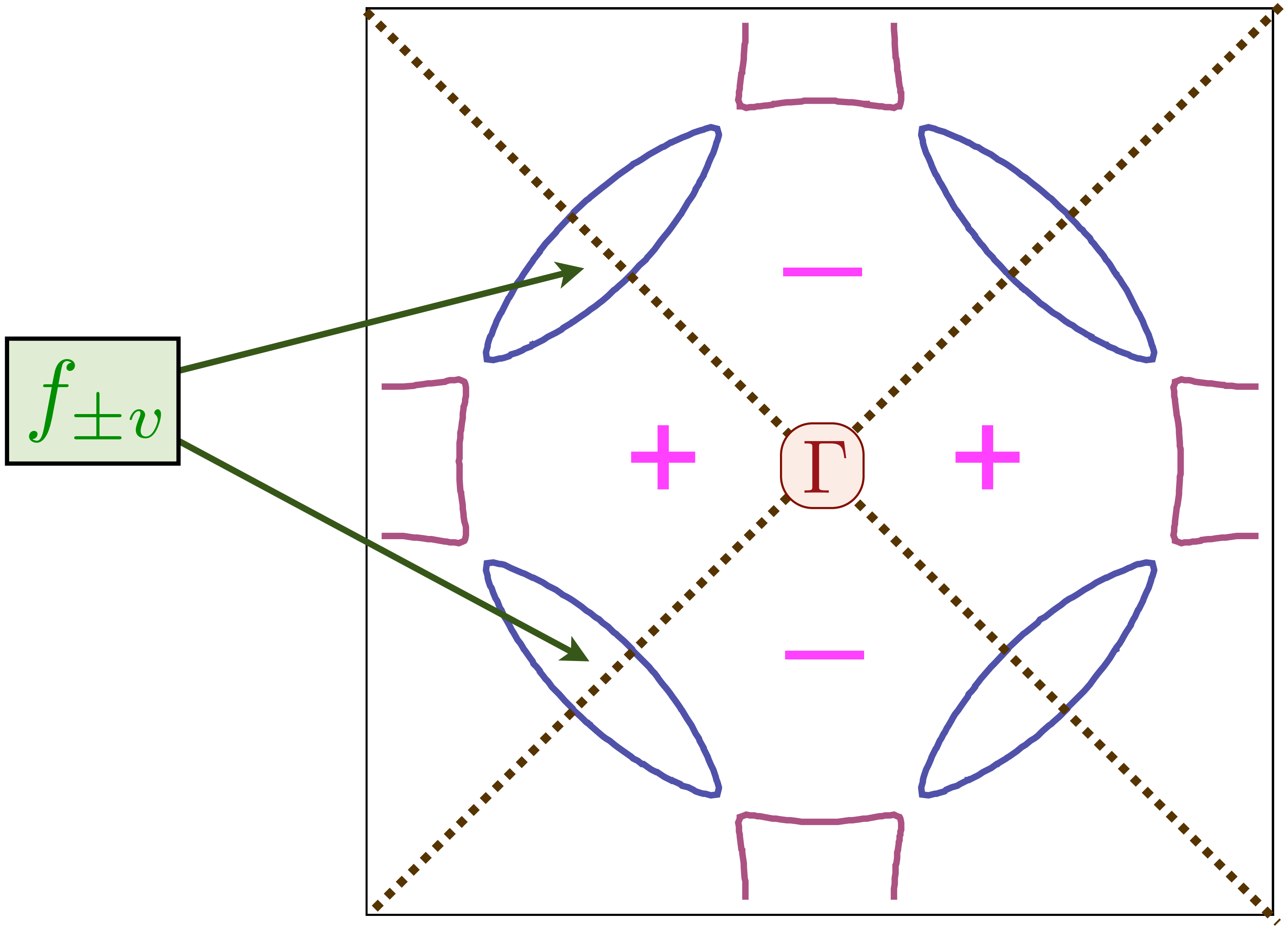
$$\mathcal{L}_{\text{Josephson}} = iJ\left[g_+ g_-\right]\left[f_{+1} \overleftrightarrow{\partial}_x f_{-1} - f_{+1} \overleftrightarrow{\partial}_y f_{-1} \right.$$
$$\left. + f_{+2} \overleftrightarrow{\partial}_x f_{-2} + f_{+2} \overleftrightarrow{\partial}_y f_{-2}\right] + \text{H.c.}$$

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, Phys. Rev. B **55**, 3173 (1997).

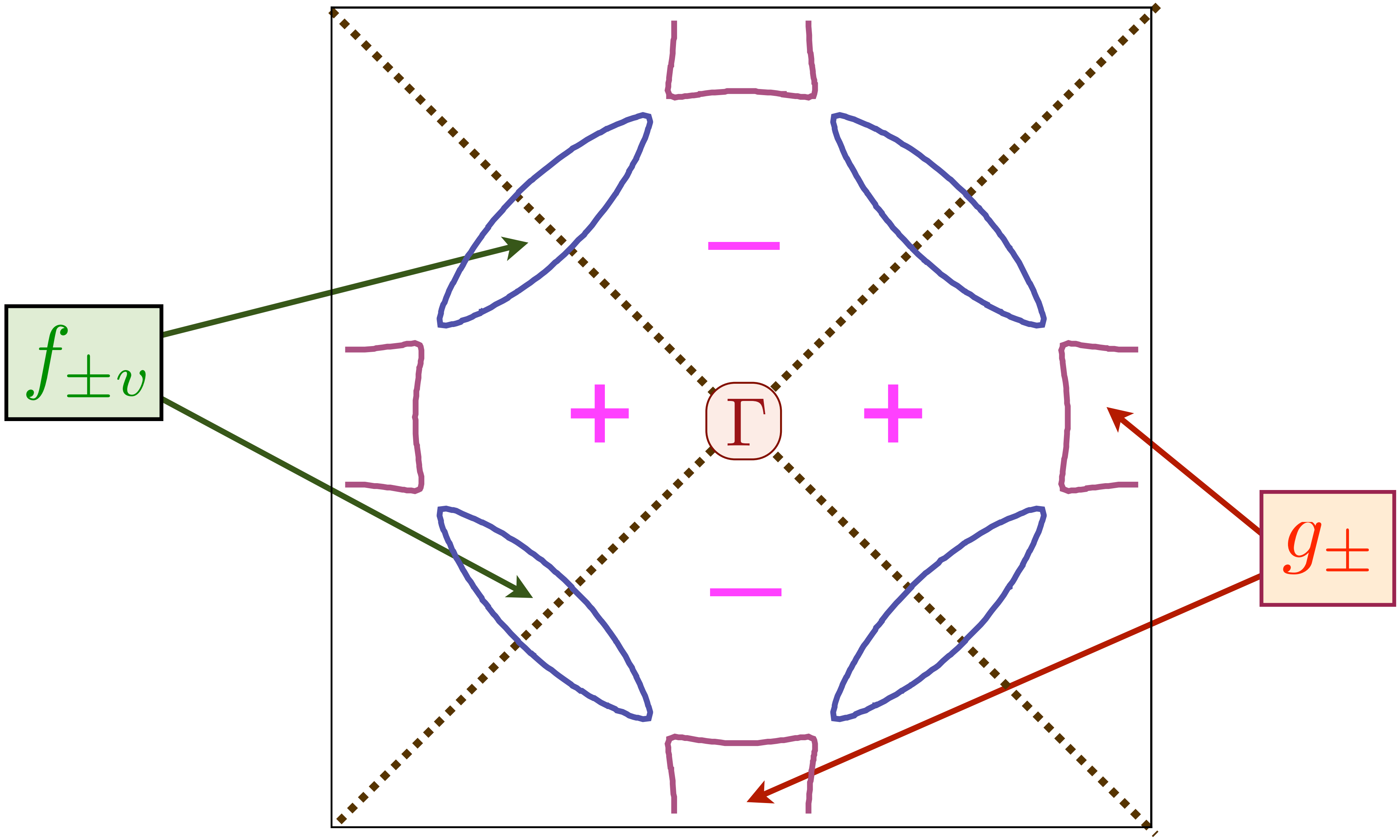
Proximity Josephson coupling J to $g_\pm$ fermions leads to p-wave pairing of the $f_{\pm v}$ fermions. The A_μ gauge forces are pair-breaking, and so the pairing is weak.

$$\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle &\sim (k_x - k_y) J \langle g_+ g_- \rangle; \\
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &\sim (k_x + k_y) J \langle g_+ g_- \rangle; \\
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &= 0,
\end{aligned}$$

$\underline{\underline{\text{Weak}}}$ pairing of the $f_\pm$ hole pockets



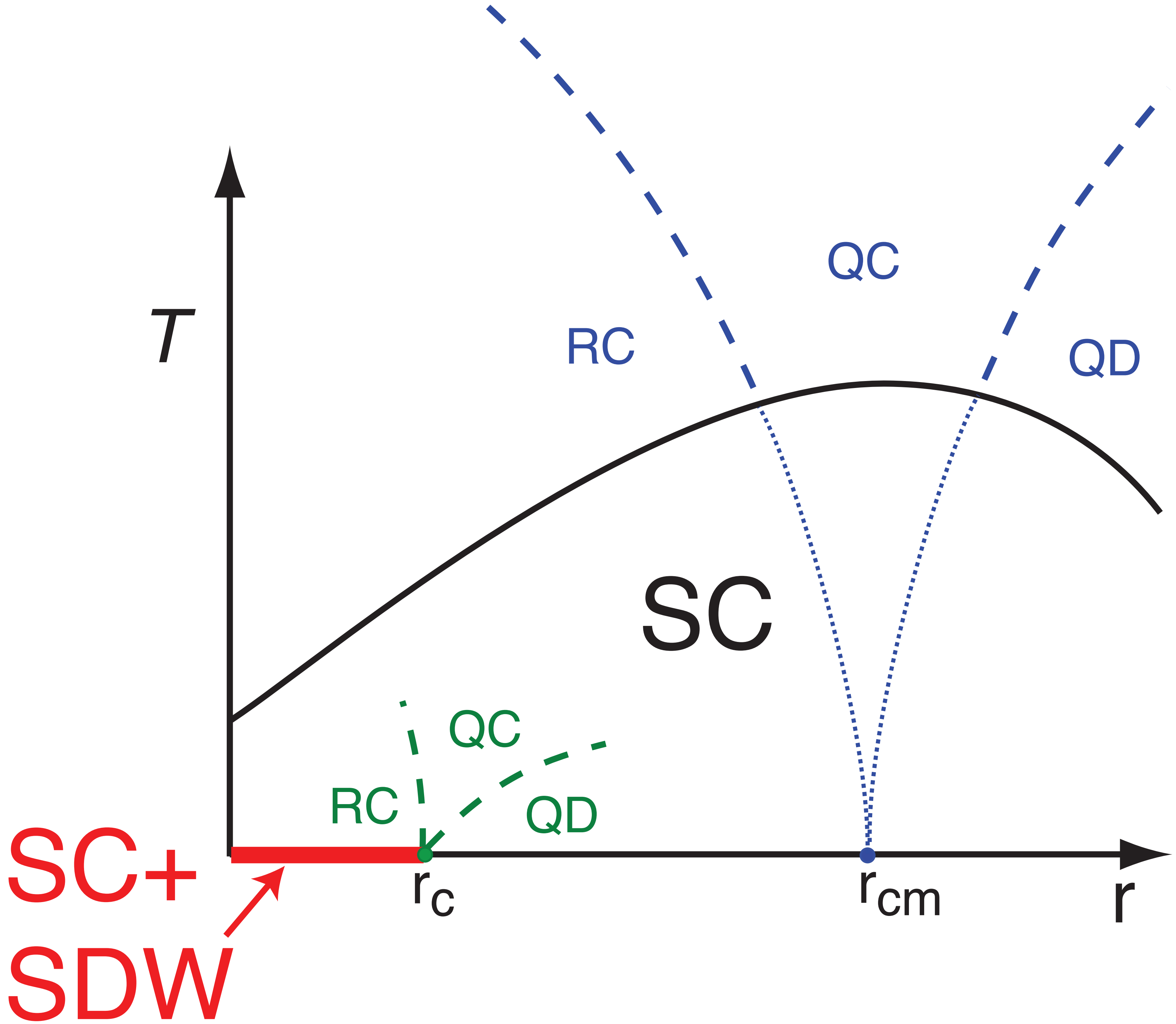
$$
\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k}) \rangle &\sim (k_x - k_y) J \langle g_+ g_- \rangle; \\
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle &\sim (k_x + k_y) J \langle g_+ g_- \rangle; \\
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle &= 0,
\end{aligned}
$$



$\boxed{d\text{-wave}}$ pairing of the electrons is associated with

- $\boxed{\text{Strong } s\text{-wave}}$ pairing of $g_{\pm}$
- $\boxed{\text{Weak } p\text{-wave}}$ pairing of $f_{\pm v}$.

Finite temperature "pseudogap"



- Because $r_{cm} > r_c$, for $T > T_c$ there is local SDW order which is disordered by thermal fluctuations.

Conclusions

- ★ Gauge theory for pairing in the underdoped cuprates, describing "angular" fluctuations of spin-density-wave order
- ★ Natural route to *d*-wave pairing with strong pairing at the antinodes and weak pairing at the nodes
- ★ Explains characteristic "competing order" features of field-doping phase diagram: SDW order is more stable in the metal than in the superconductor.