Outline

1. Coupled dimer antiferromagnets

   Landau-Ginzburg quantum criticality

2. Spin liquids and valence bond solids

   (a) Schwinger-boson mean-field theory - square lattice
   (b) Gauge theories of perturbative fluctuations
   (c) Non-perturbative effects: Berry phases
   (d) Schwinger-boson mean-field theory - triangular lattice
   (e) Visons and the Kitaev model

3. Cuprate superconductivity

   (a) Review of experiments, old and new
   (b) Fermi surfaces and the spin density wave theory
   (c) Fermi pockets and the underdoped cuprates

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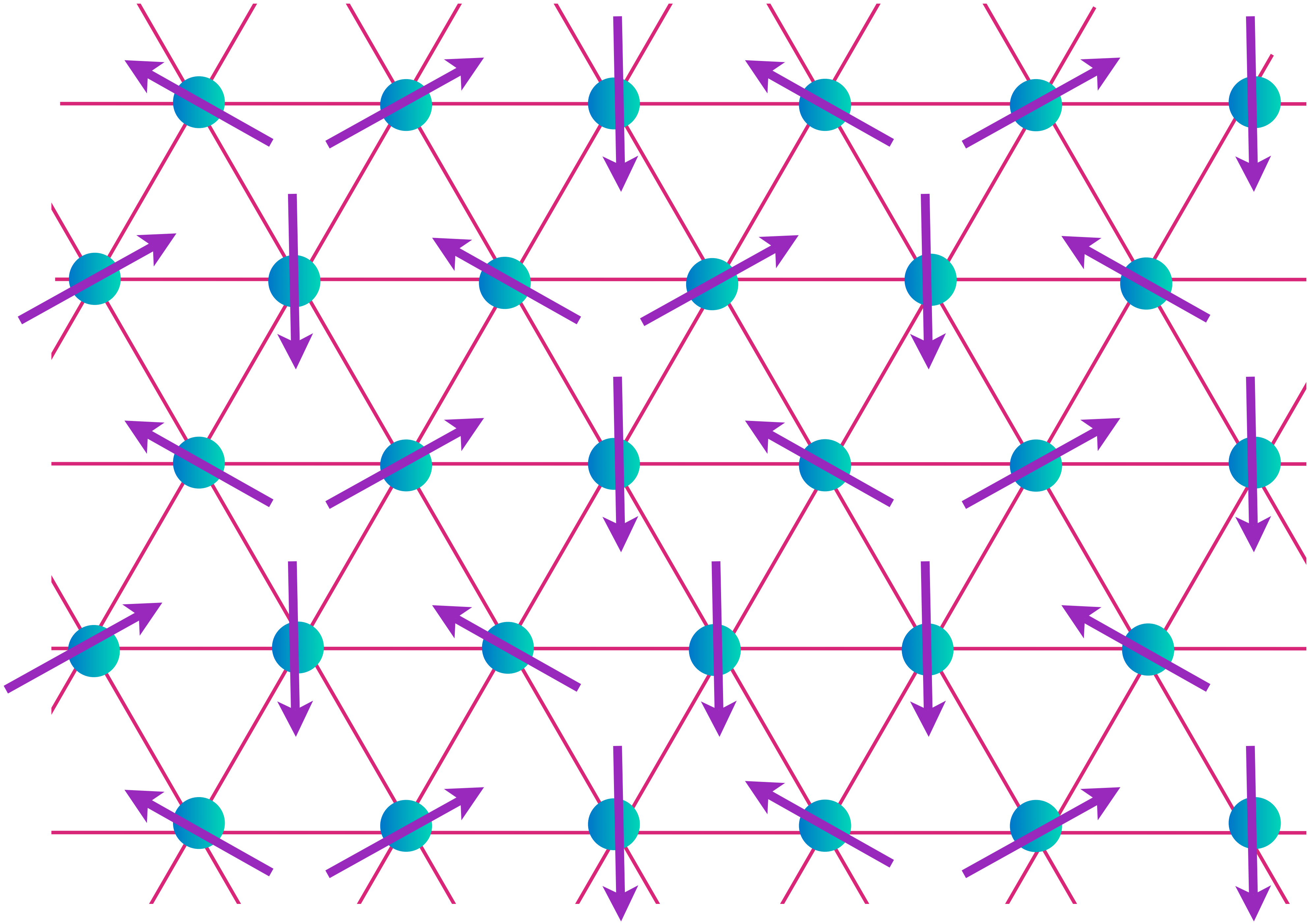
Triangular lattice antiferromagnet

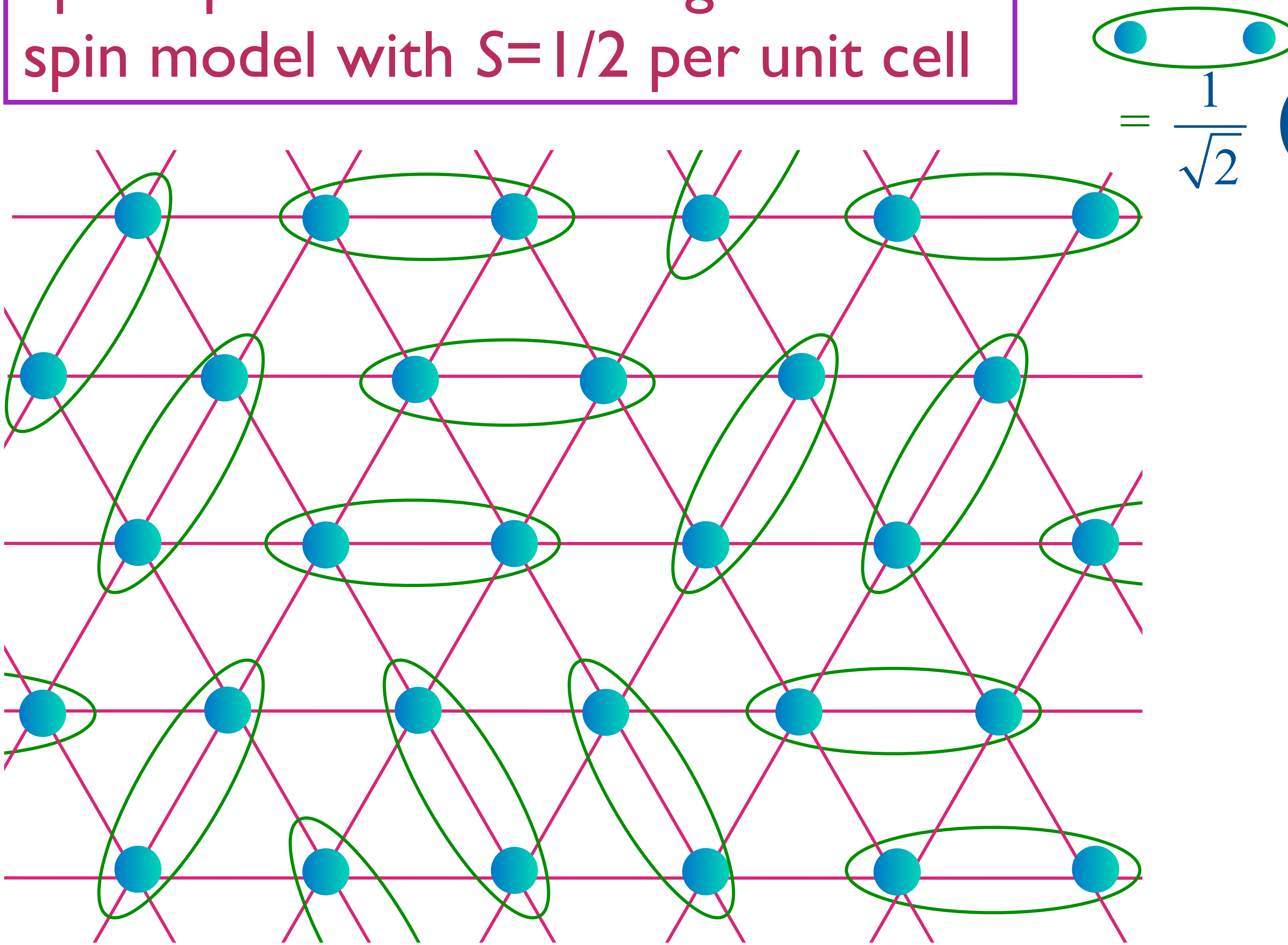
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with S=1/2 per unit cell

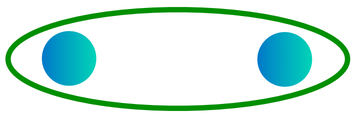


$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



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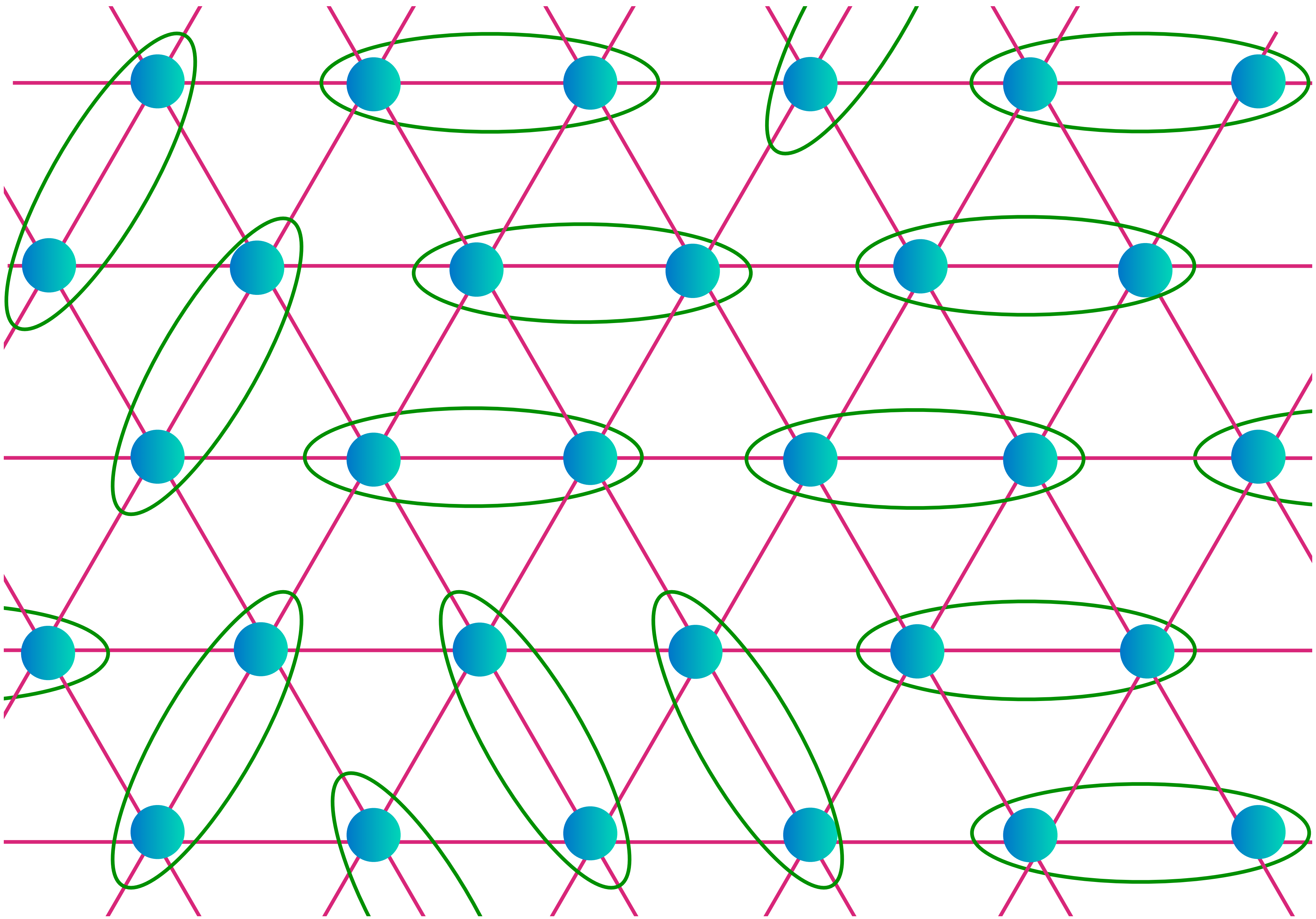

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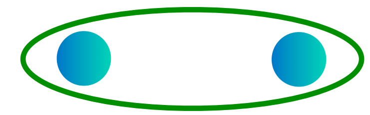

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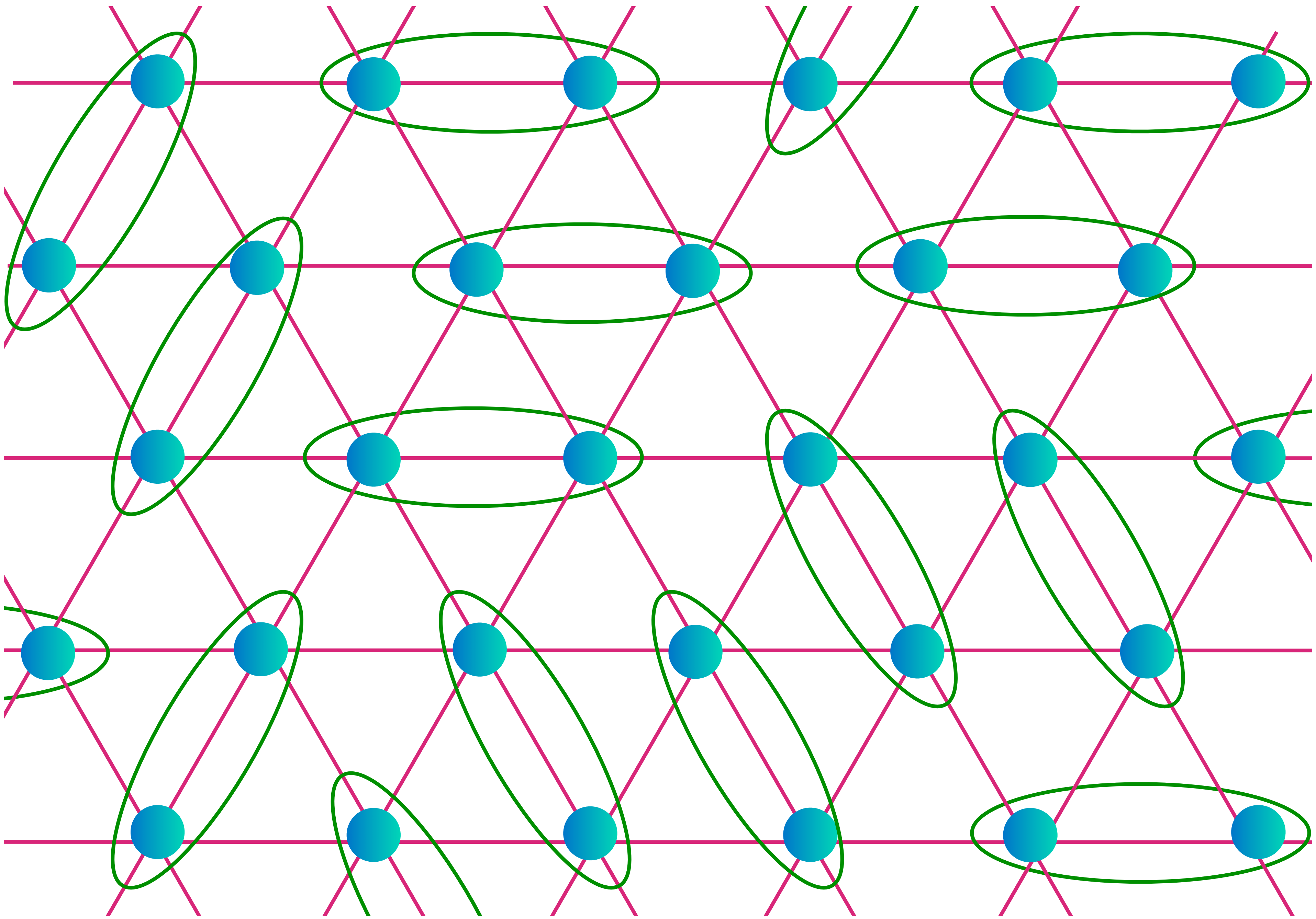

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Triangular lattice antiferromagnet

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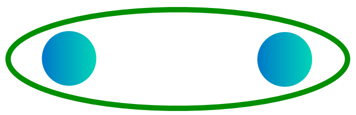

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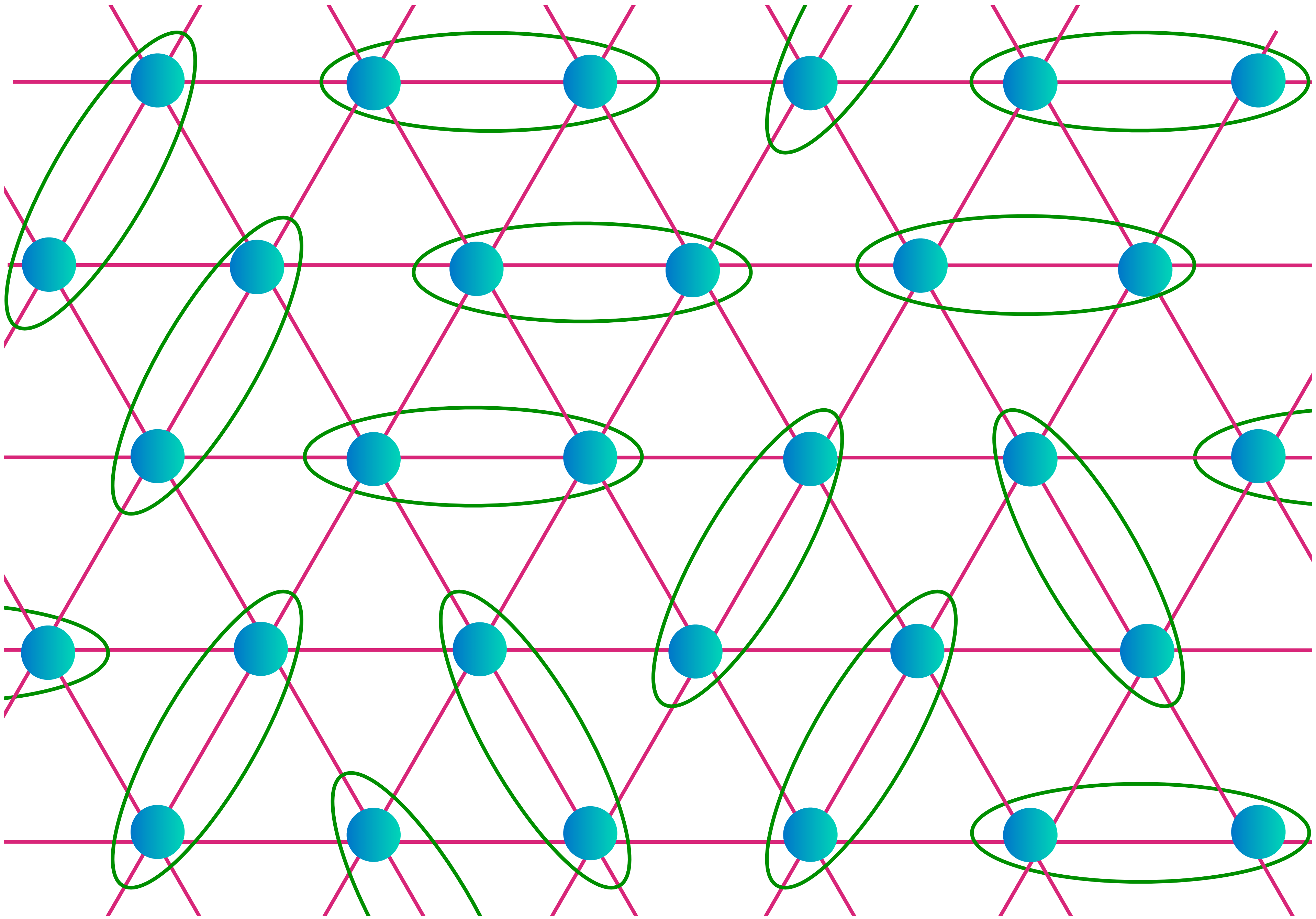

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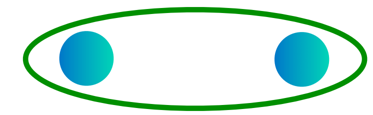

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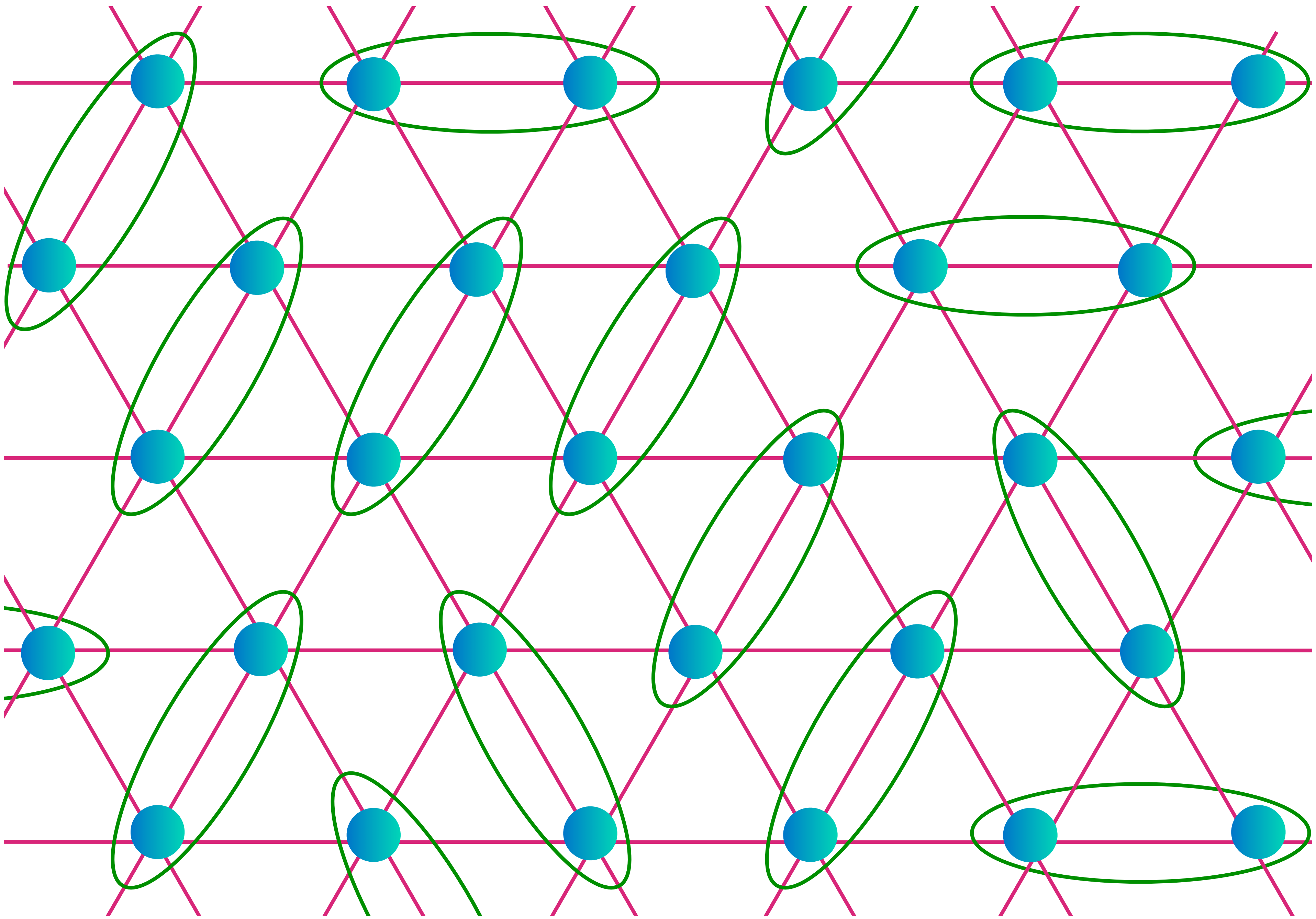

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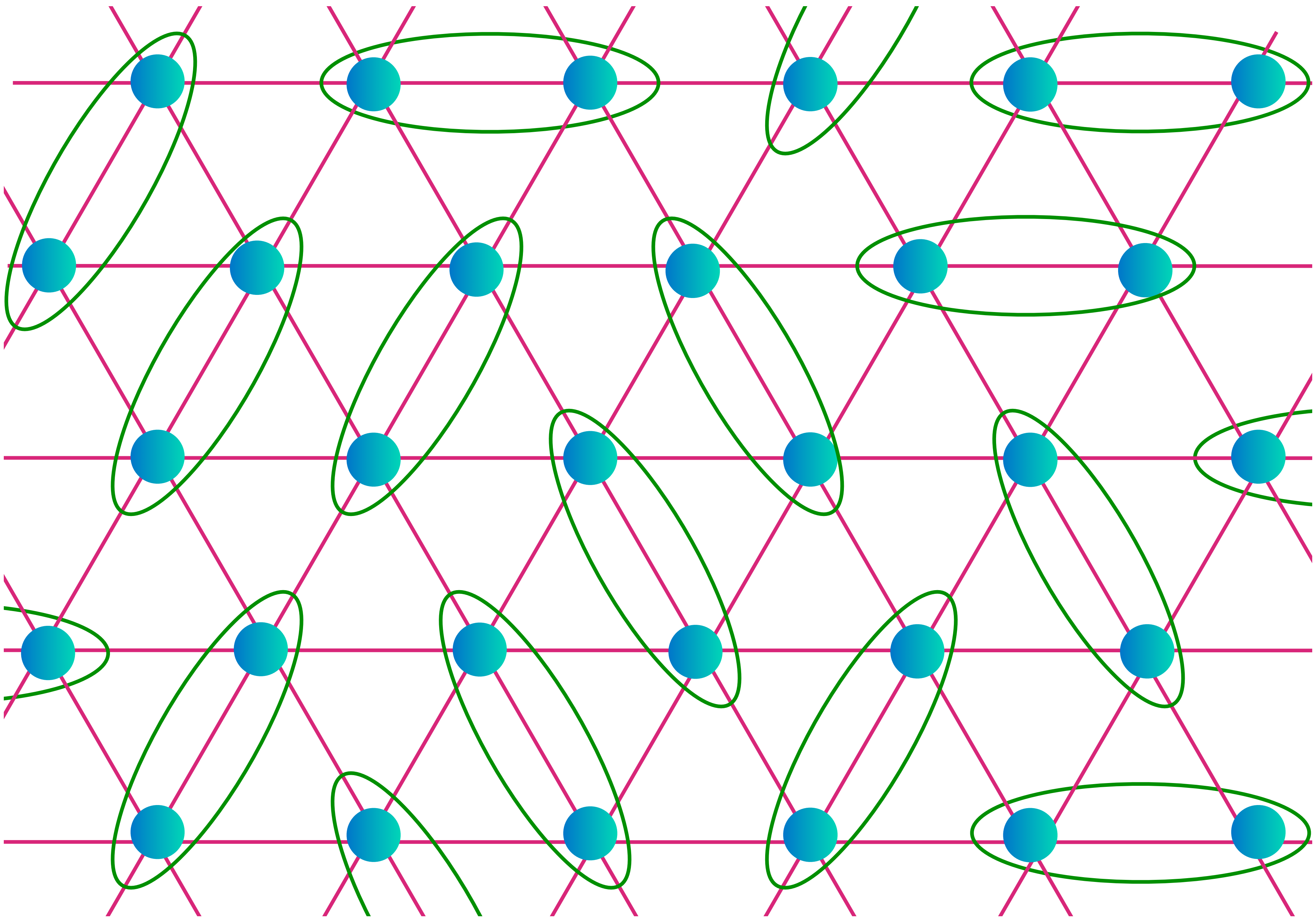

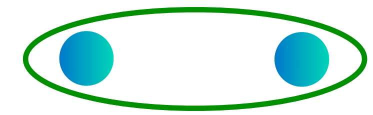

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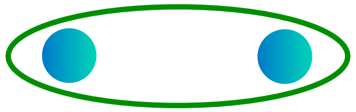
Excitations of the Z_2 Spin liquid

A spinon



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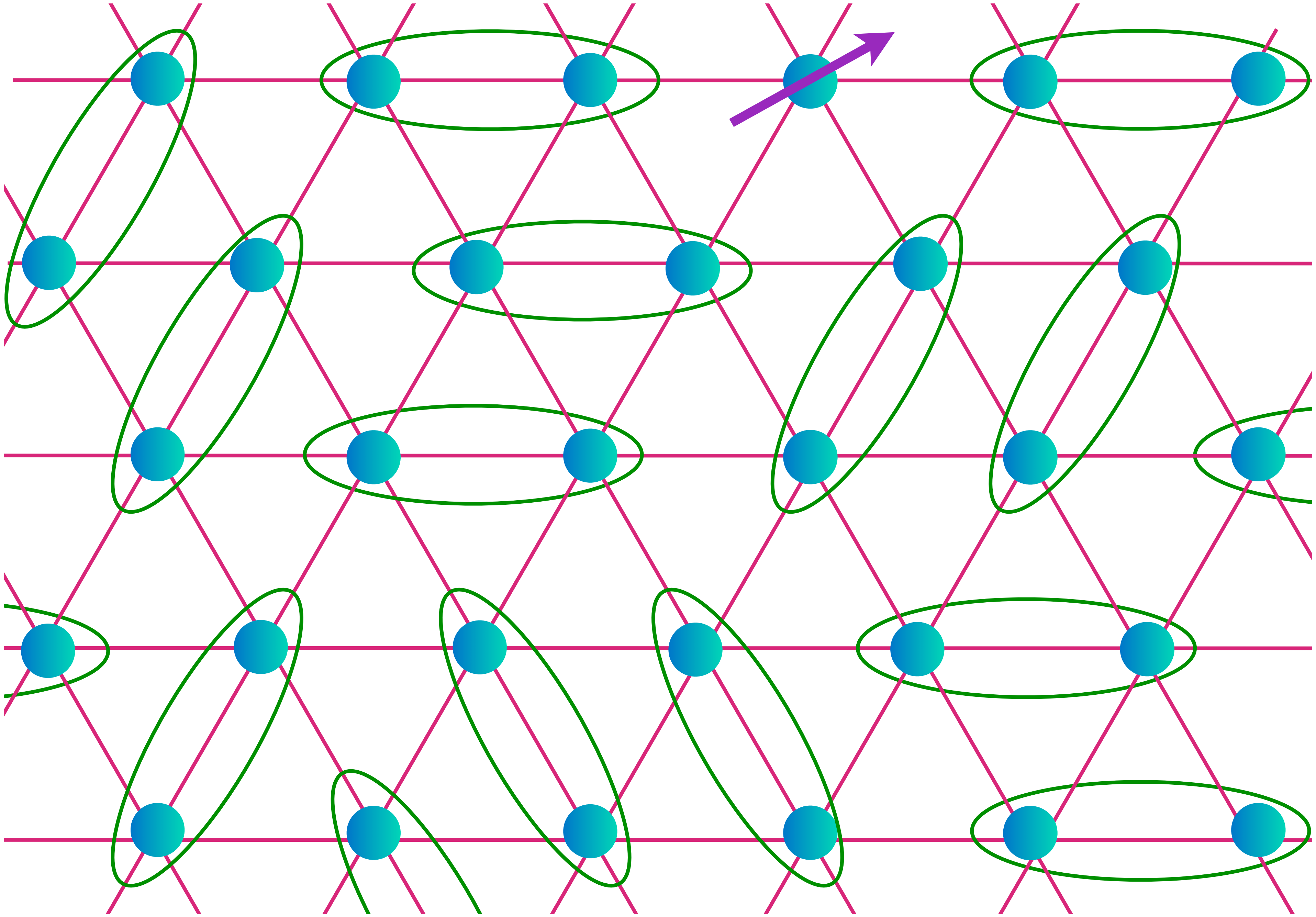
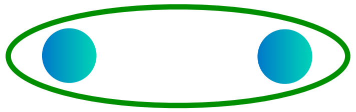


Excitations of the Z_2 Spin liquid



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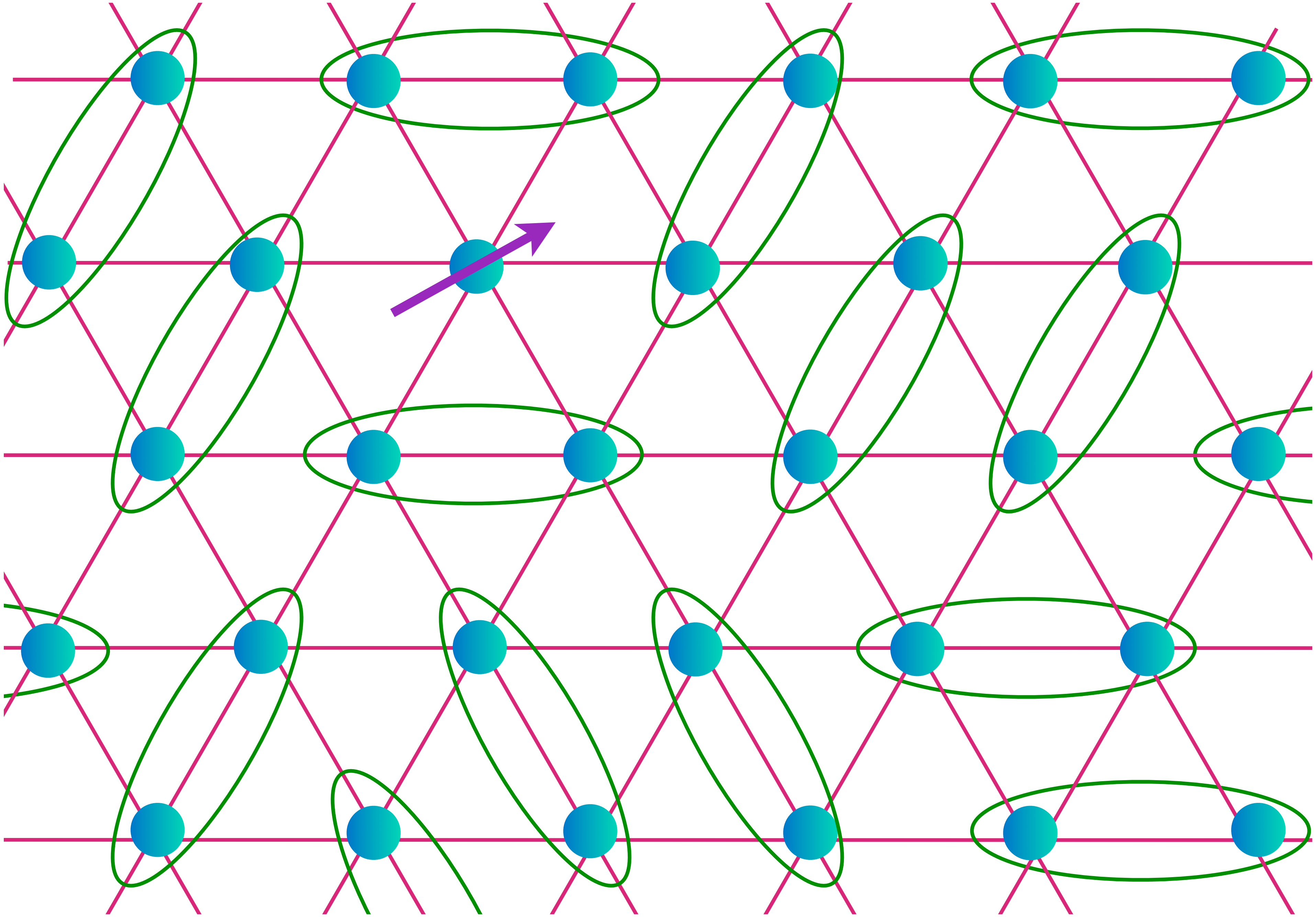

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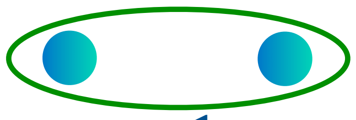

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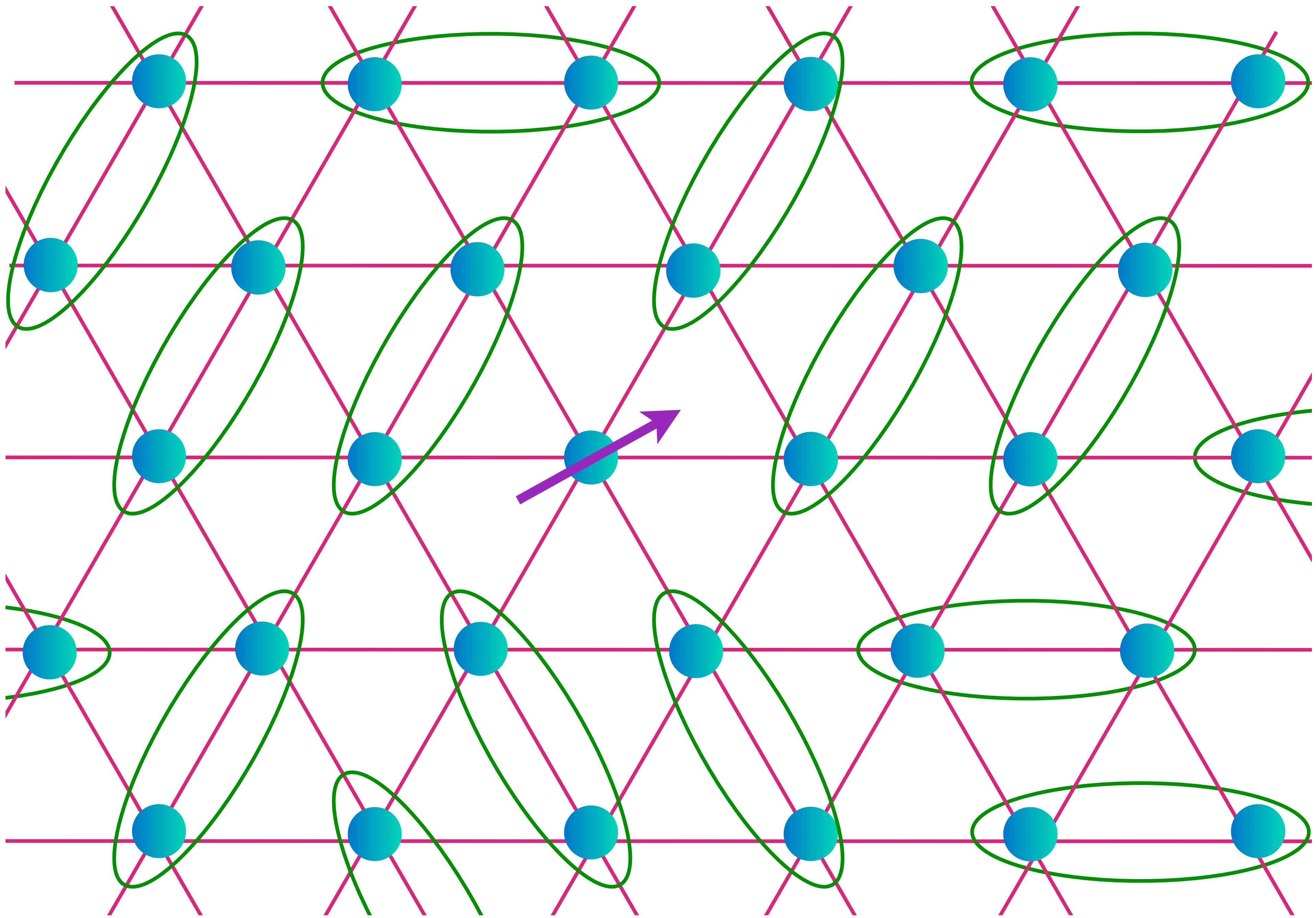

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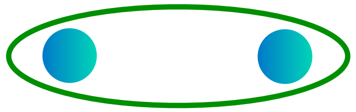

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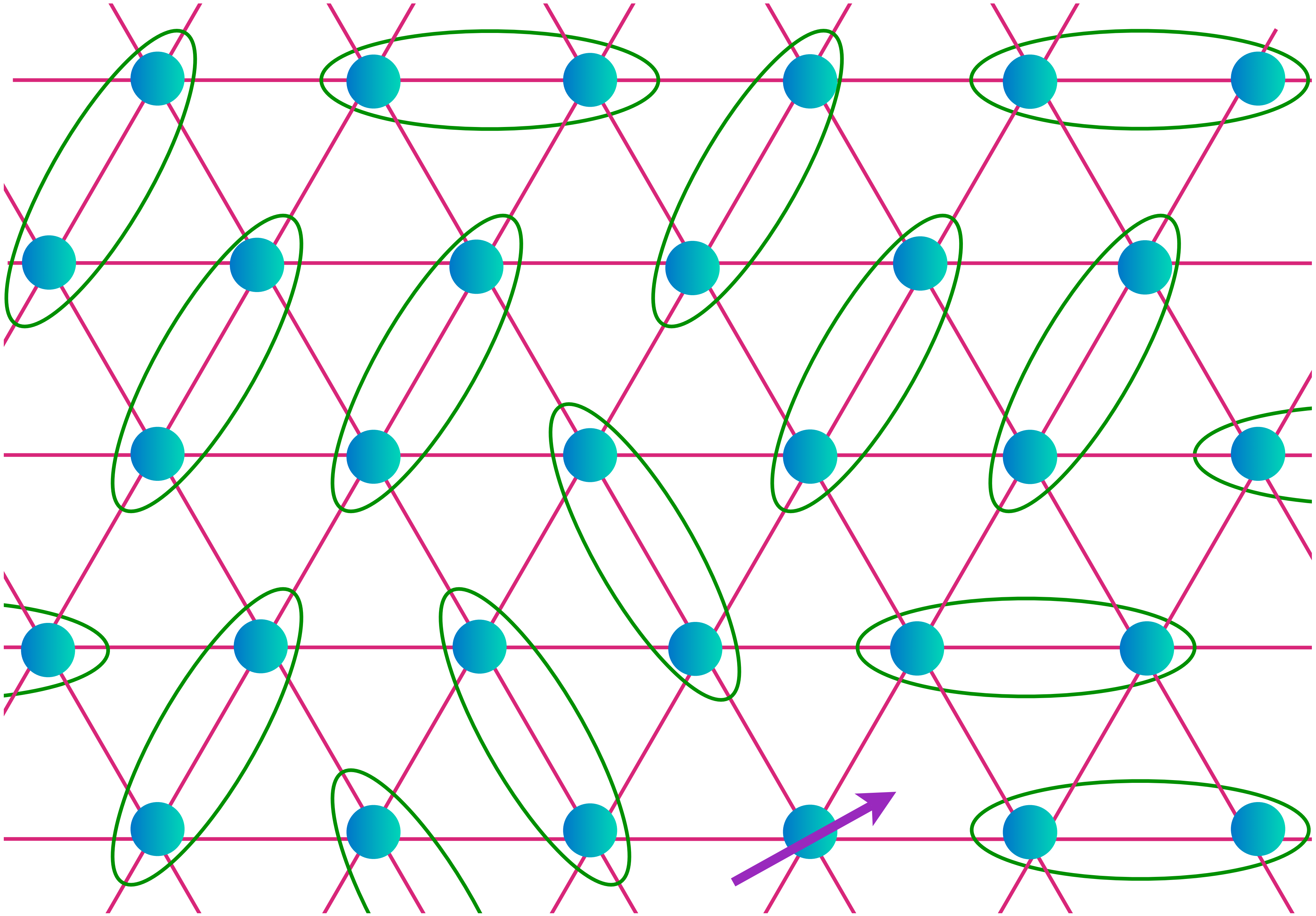
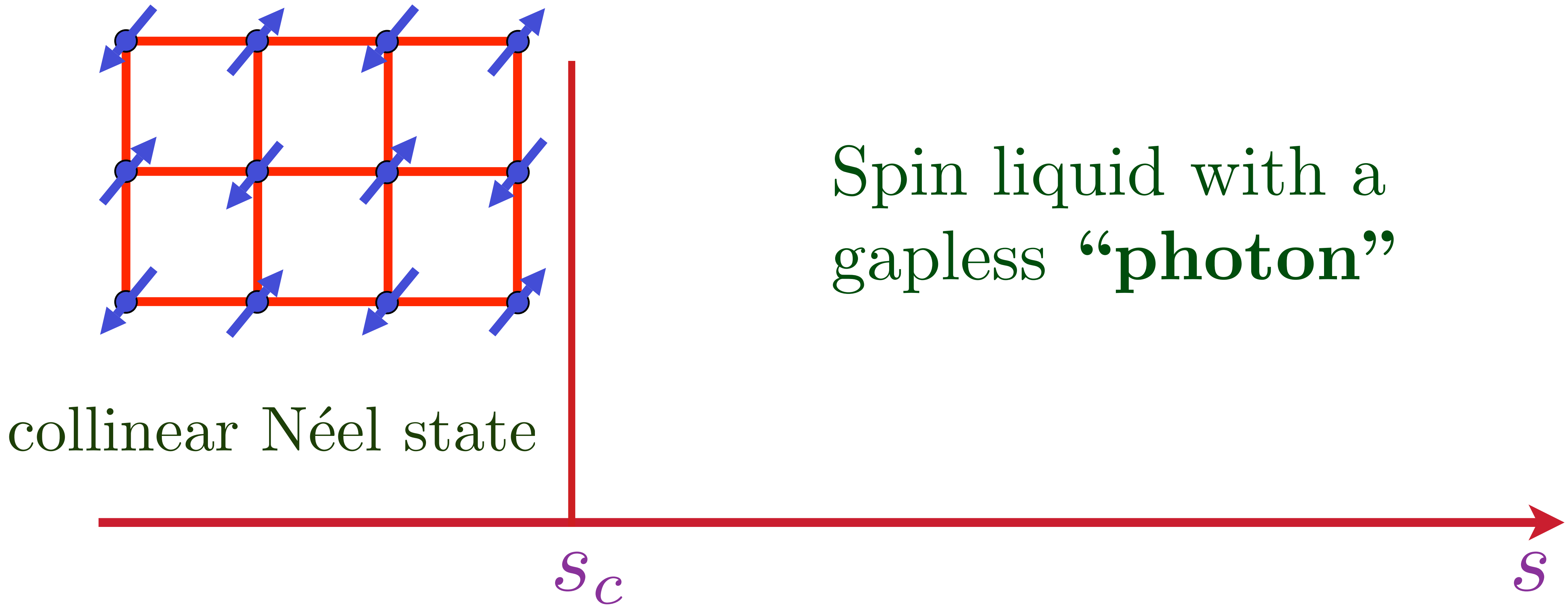


Discussion of Schwinger bosons on the square lattice and U(1) gauge theory

http://qpt.physics.harvard.edu/leshouches/schwinger_bosons.pdf

Quantum "disordering" magnetic order



Perturbative analysis of fluctuations about the Schwinger boson mean-field theory leads to the following CP^1 field theory with spacetime action

$$\mathcal{S} = \int d^2x d\tau \left[|(\partial_\mu - iA_\tau) z_\alpha|^2 + c^2 |(\vec{\nabla}_x - i\vec{A}) z_\alpha|^2 + s|z_\alpha|^2 + \frac{u}{2} \left(|z_\alpha|^2 \right)^2 \right]$$

where z_α is the $S = 1/2$ bosonic spinon, and A_μ is an emergent photon.

D. P. Arovas and A. Auerbach, *Phys. Rev. B* **38**, 316 (1988).
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

Non-perturbative effects in the U(1) spin liquid phase of the CP^1 field theory

- Discretize spacetime into a cubic lattice of sites labeled a.
- The U(1) gauge field $A_{a\mu}$ ($\mu = \tau, x, y$) lives on the link connecting site a to $a + \hat{\mu}$.
- Proposed effective action which is invariant under $A_{a\mu} \rightarrow A_{a\mu} + 2\pi$:

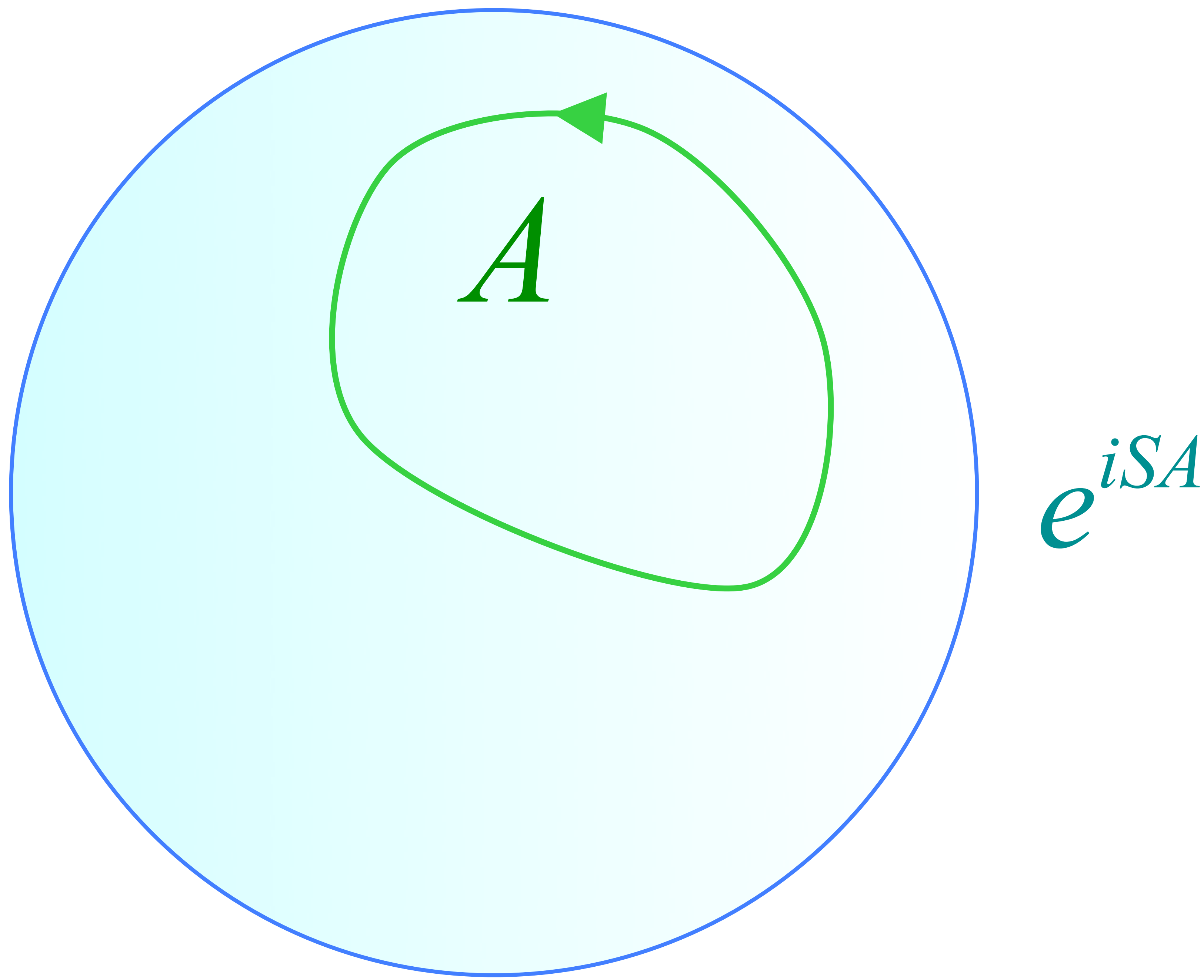
$$\mathcal{Z} = \prod_{a\mu} \int dA_{a\mu} \exp\left(\frac{1}{e^2} \sum_{\square} \cos(\epsilon_{\mu\nu\lambda} \Delta_\nu A_{a\lambda}) \right)$$

  where Δ_μ is the discrete lattice derivative: $\Delta_\mu f_a \equiv f_{a+\hat{\mu}} - f_a$. This is *compact* U(1) electrodynamics.

- This theory is missing the spin Berry phases.

Non-perturbative effects in the U(1) spin liquid phase of the CP^1 field theory

Missing ingredient: Spin Berry Phases



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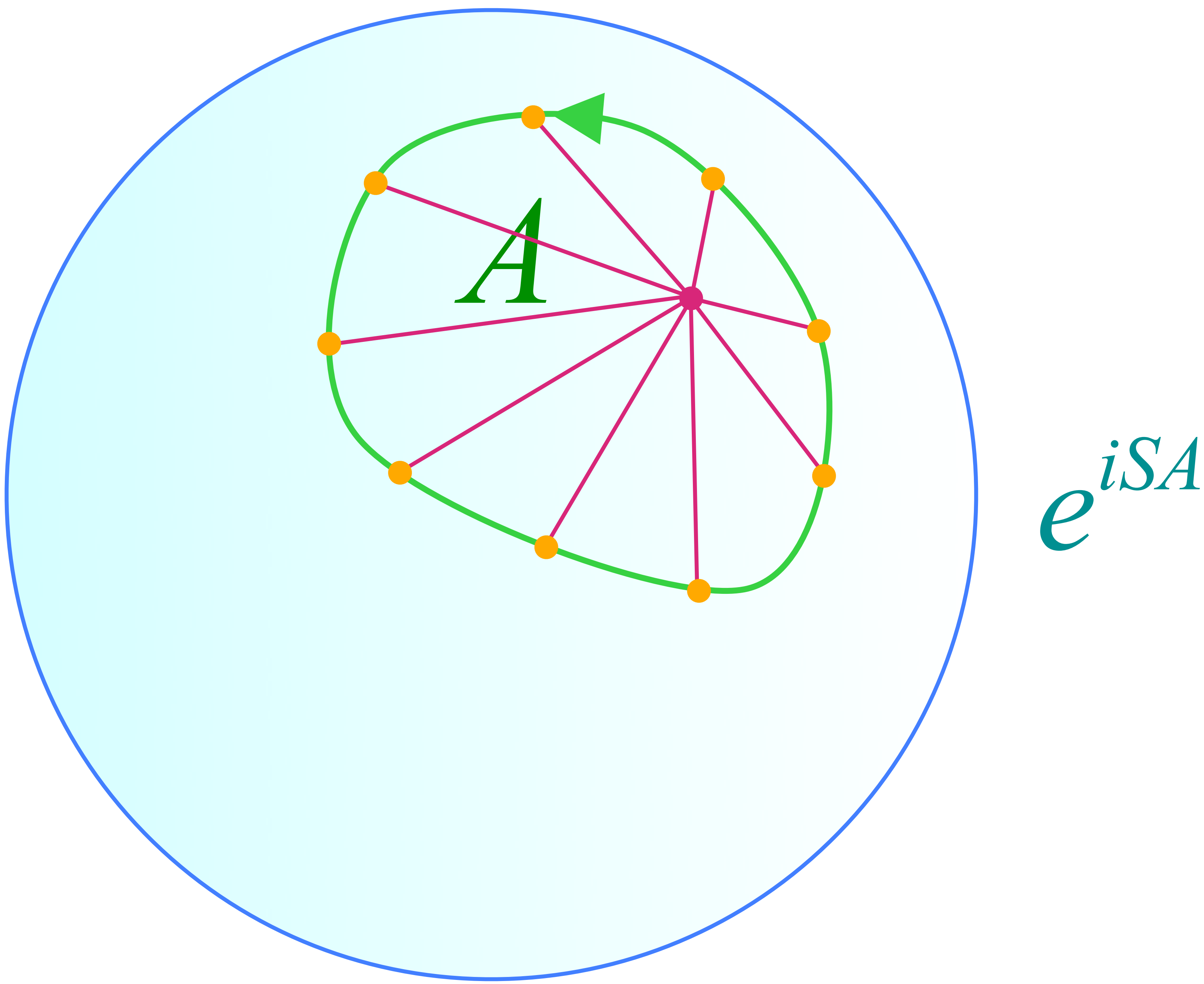
Missing ingredient: Spin Berry Phases



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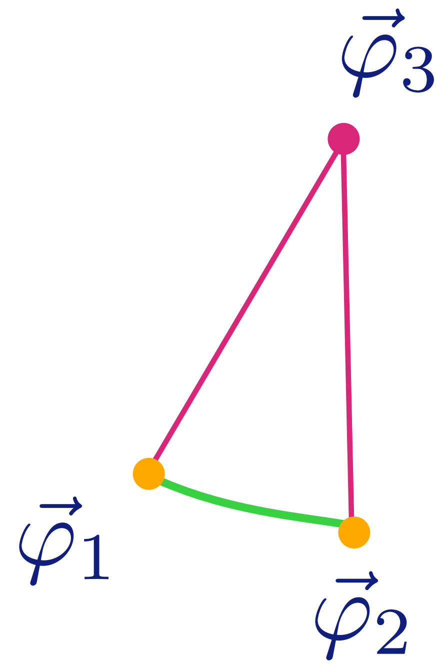

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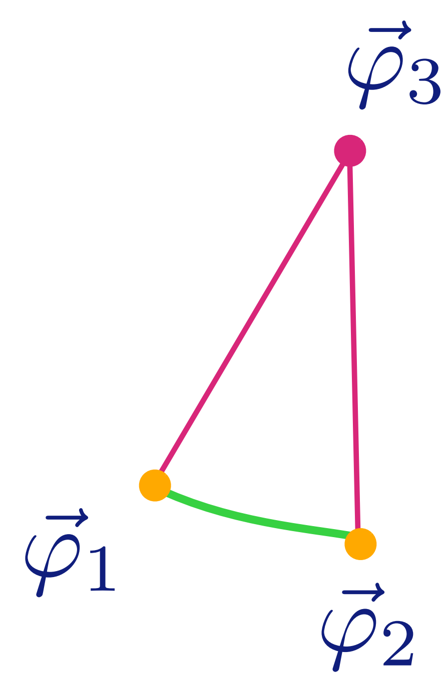
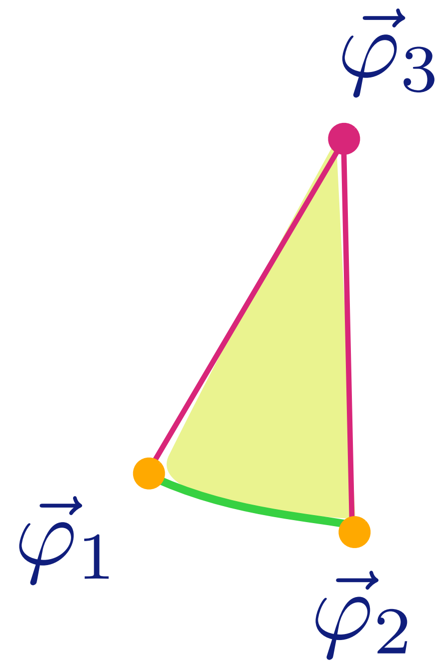


Define $\vec{\varphi}_i \equiv z_{i\alpha}^* \vec{\sigma}_{\alpha\beta} z_{i\beta}$
and $A_{ij} = \arg\left[z_{i\alpha}^* z_{j\alpha}\right]$.

Non-perturbative effects in the U(1) spin liquid phase of the CP^1 field theory

Missing ingredient: Spin Berry Phases



$$\text{Area of triangle} = 2(A_{12} + A_{23} + A_{31})$$

Non-perturbative effects in the U(1) spin liquid phase of the CP^1 field theory

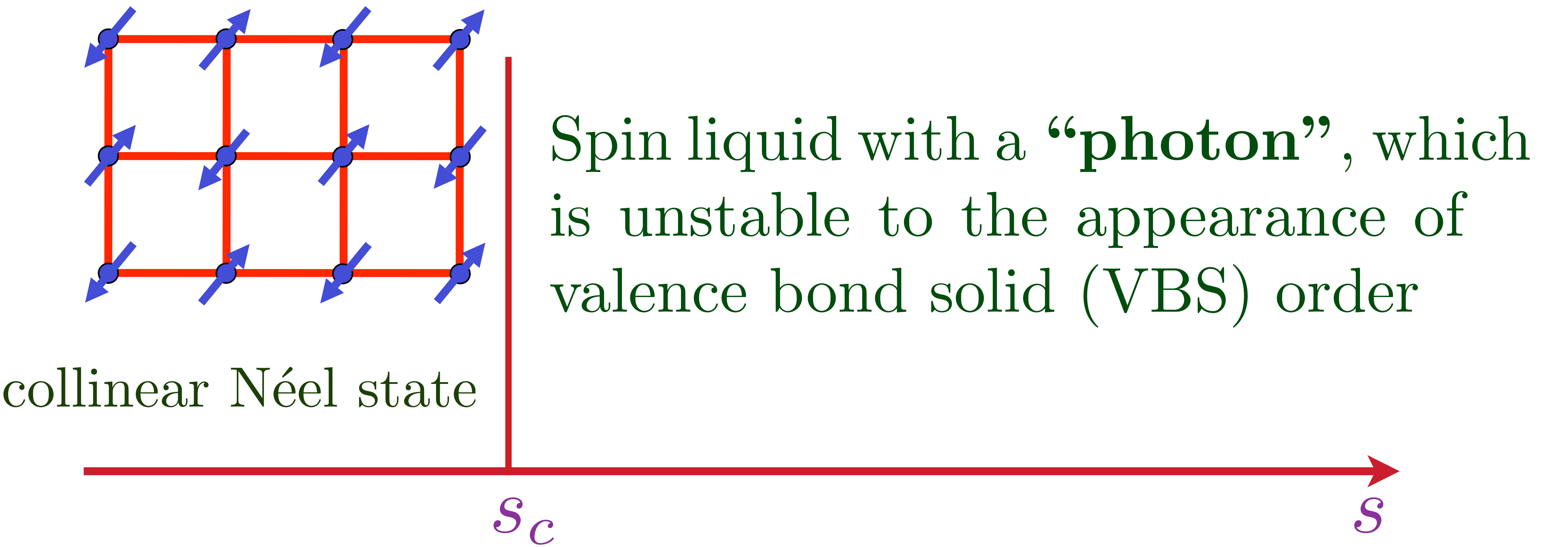
- Proposed effective action which is invariant under $A_{a\mu} \to A_{a\mu} + 2\pi$, and includes spin Berry phases:

$$\mathcal{Z} = \prod_{a\mu} \int dA_{a\mu} \exp\left(\frac{1}{e^2} \sum_{\square} \cos(\epsilon_{\mu\nu\lambda}\Delta_\nu A_{a\lambda}) + i2S \sum_a \eta_a A_{a\tau} \right)$$

  where $\eta_a = \pm 1$ on the two square sublattices. The is compact quantum electrodynamics with fixed background charges $= 2S\eta_a$ on each lattice site.

- This theory can be solved by duality mappings: the spin liquid is unstable to valence bond solid order, and the photon acquires an energy gap.

Quantum "disordering" magnetic order



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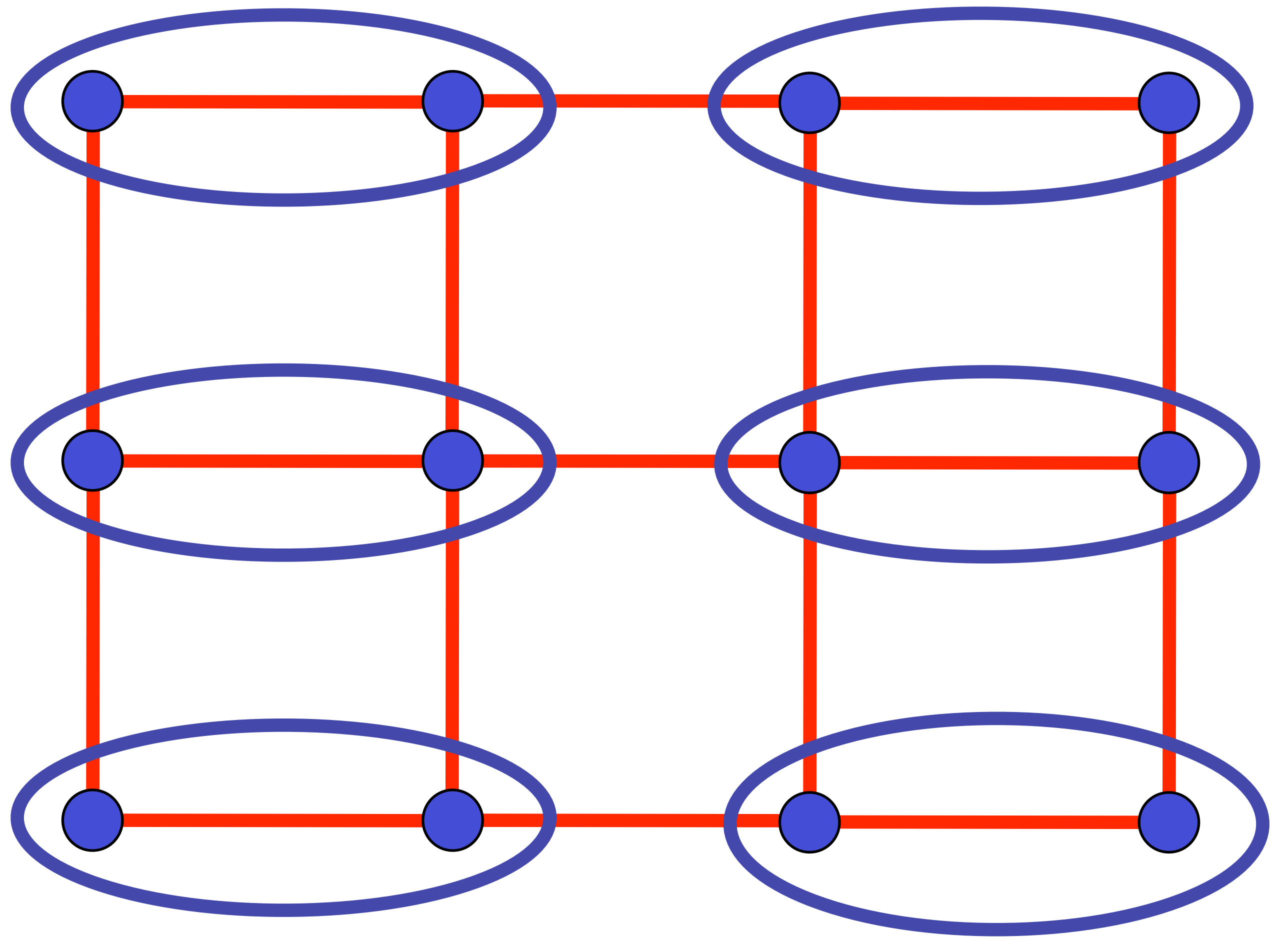
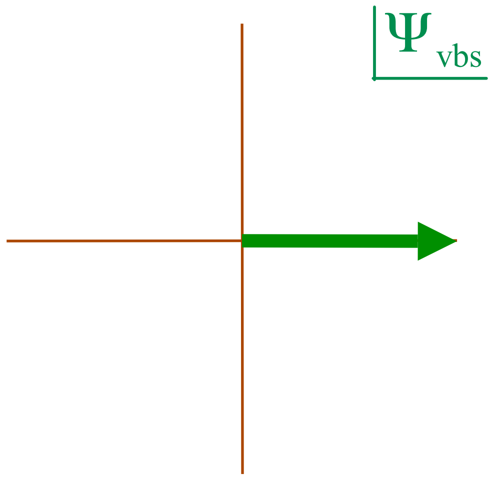
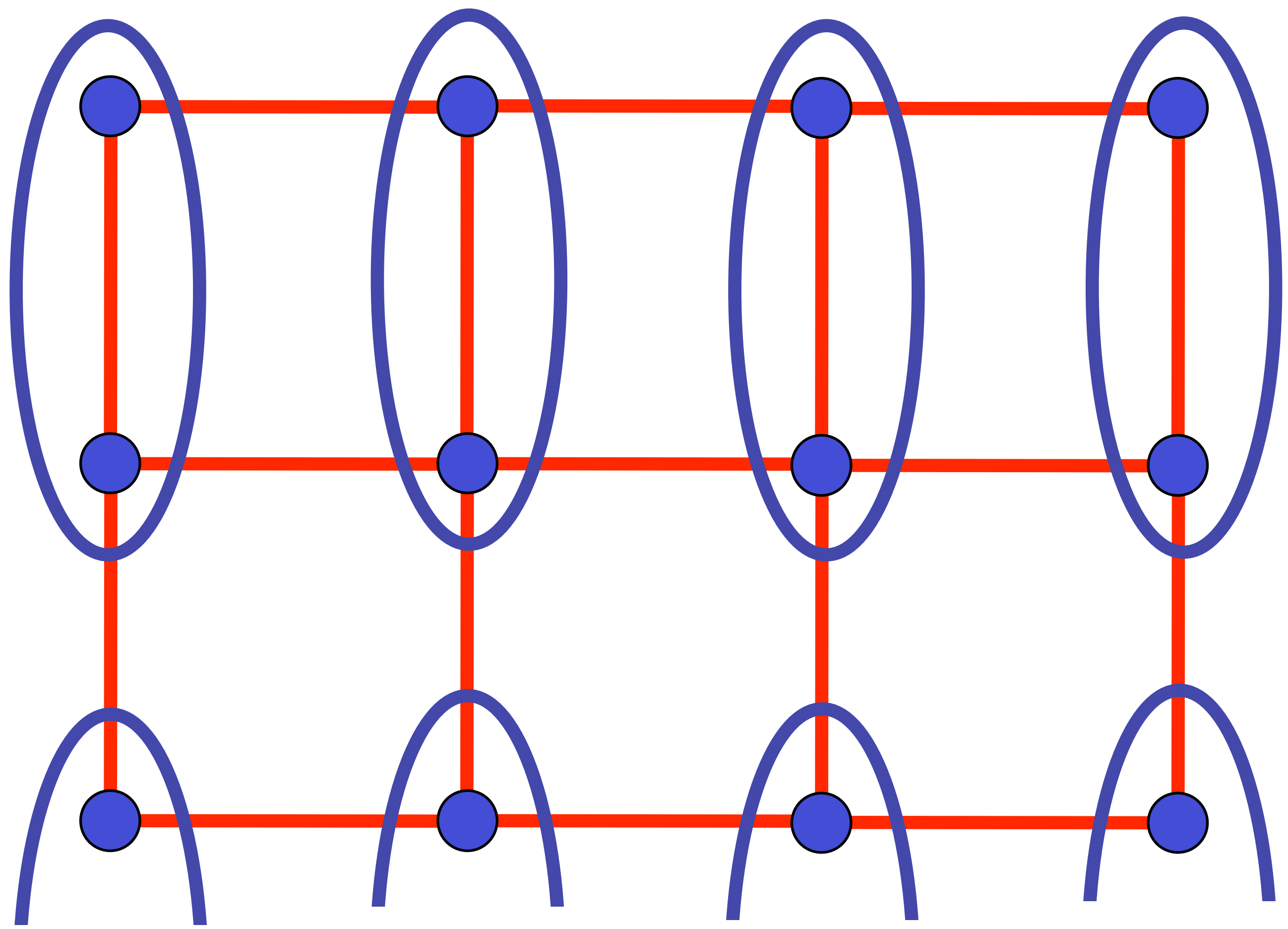
Order parameter of VBS state

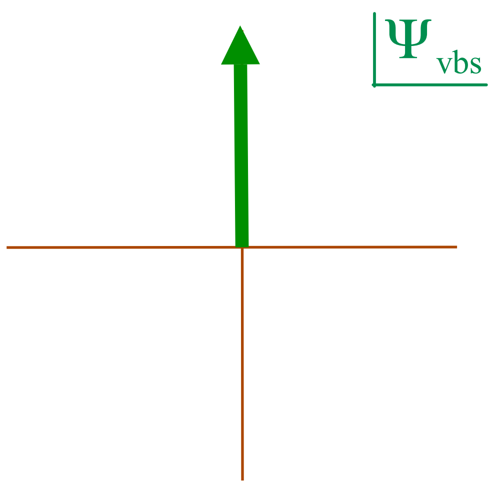




$$\Psi_{\mathrm{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Order parameter of VBS state





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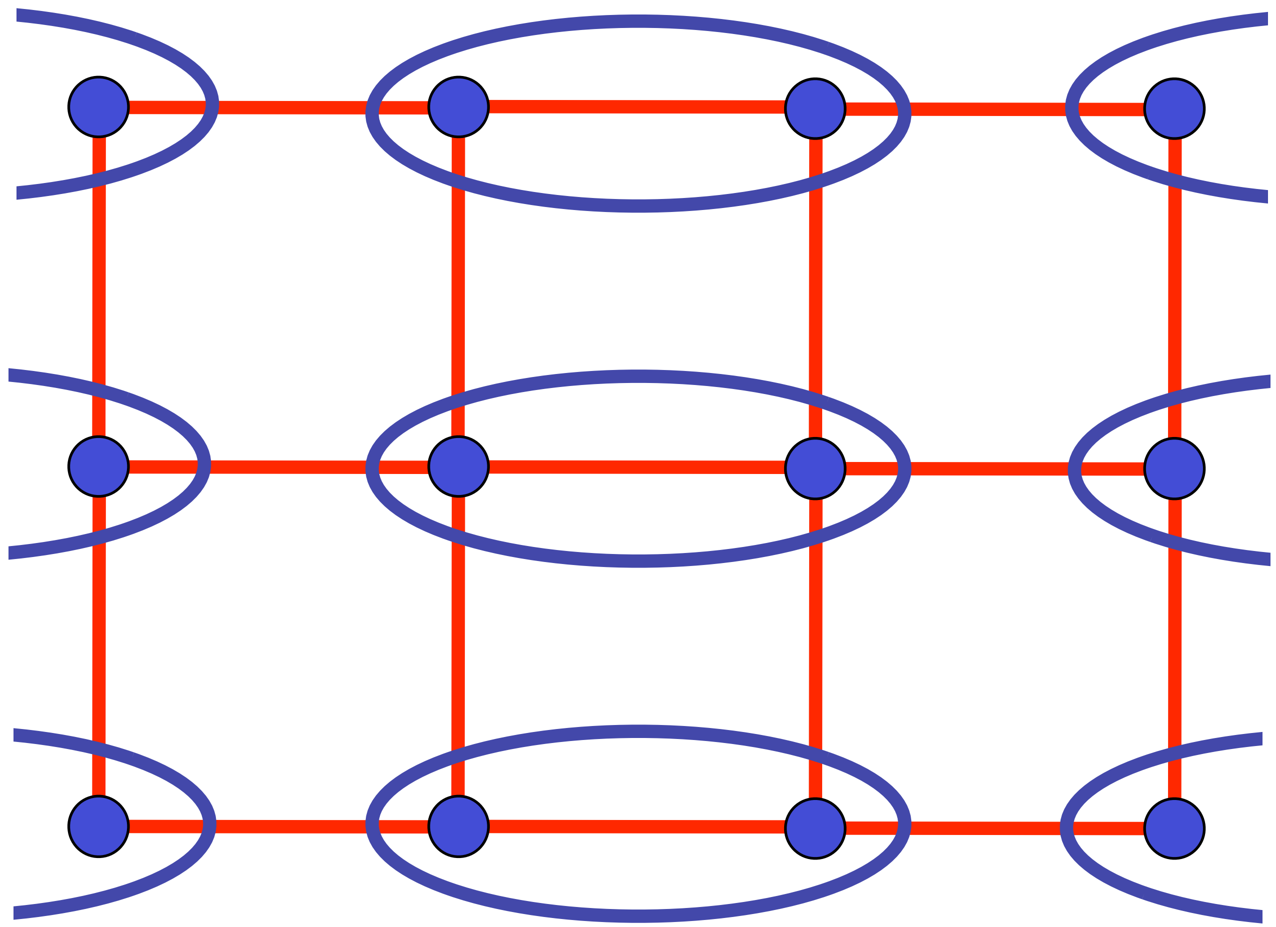

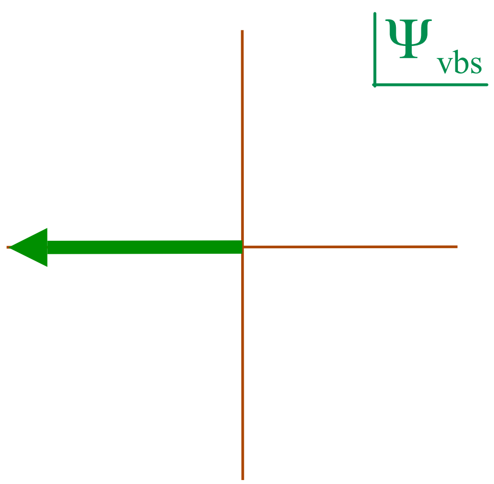

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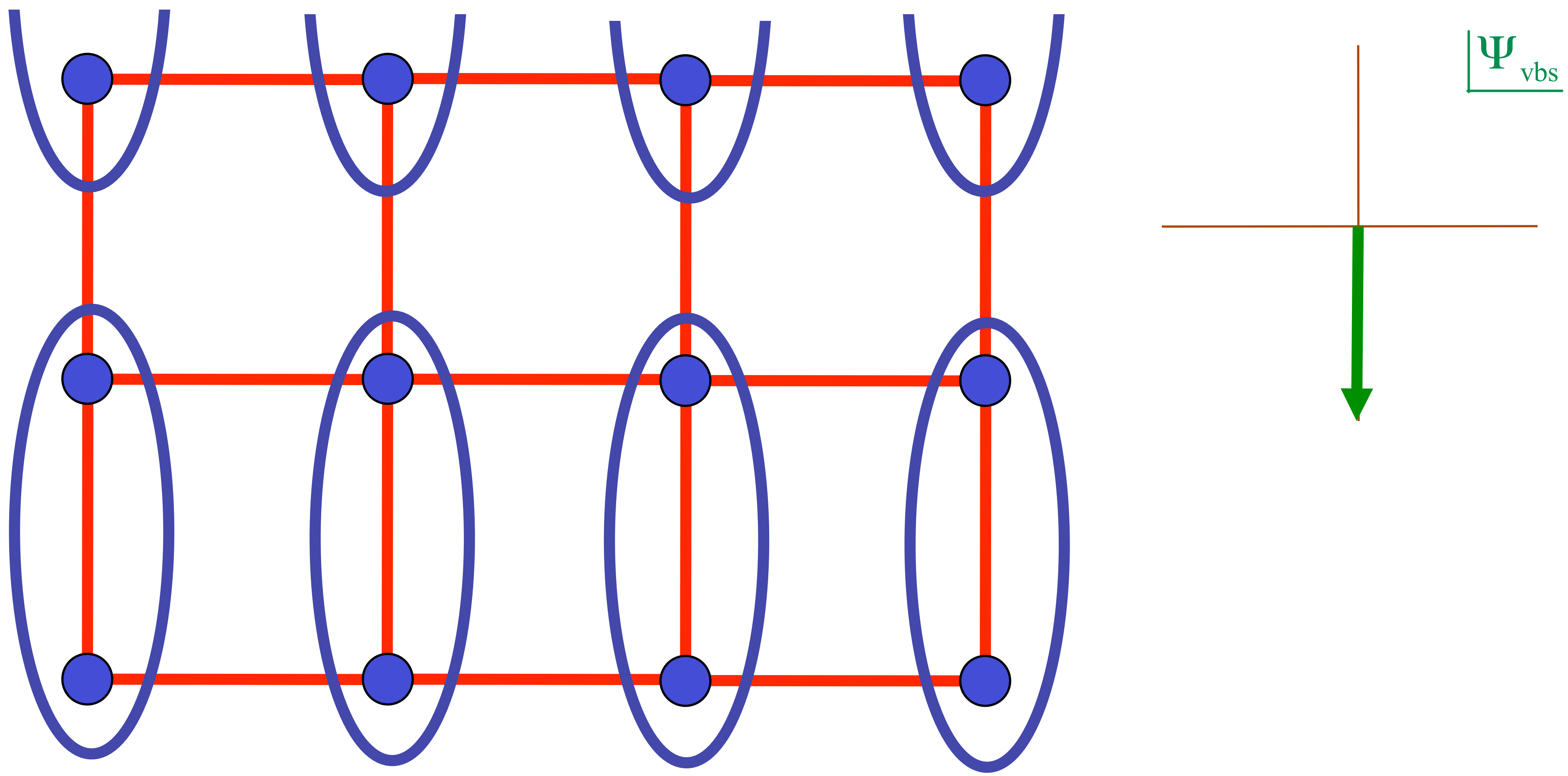

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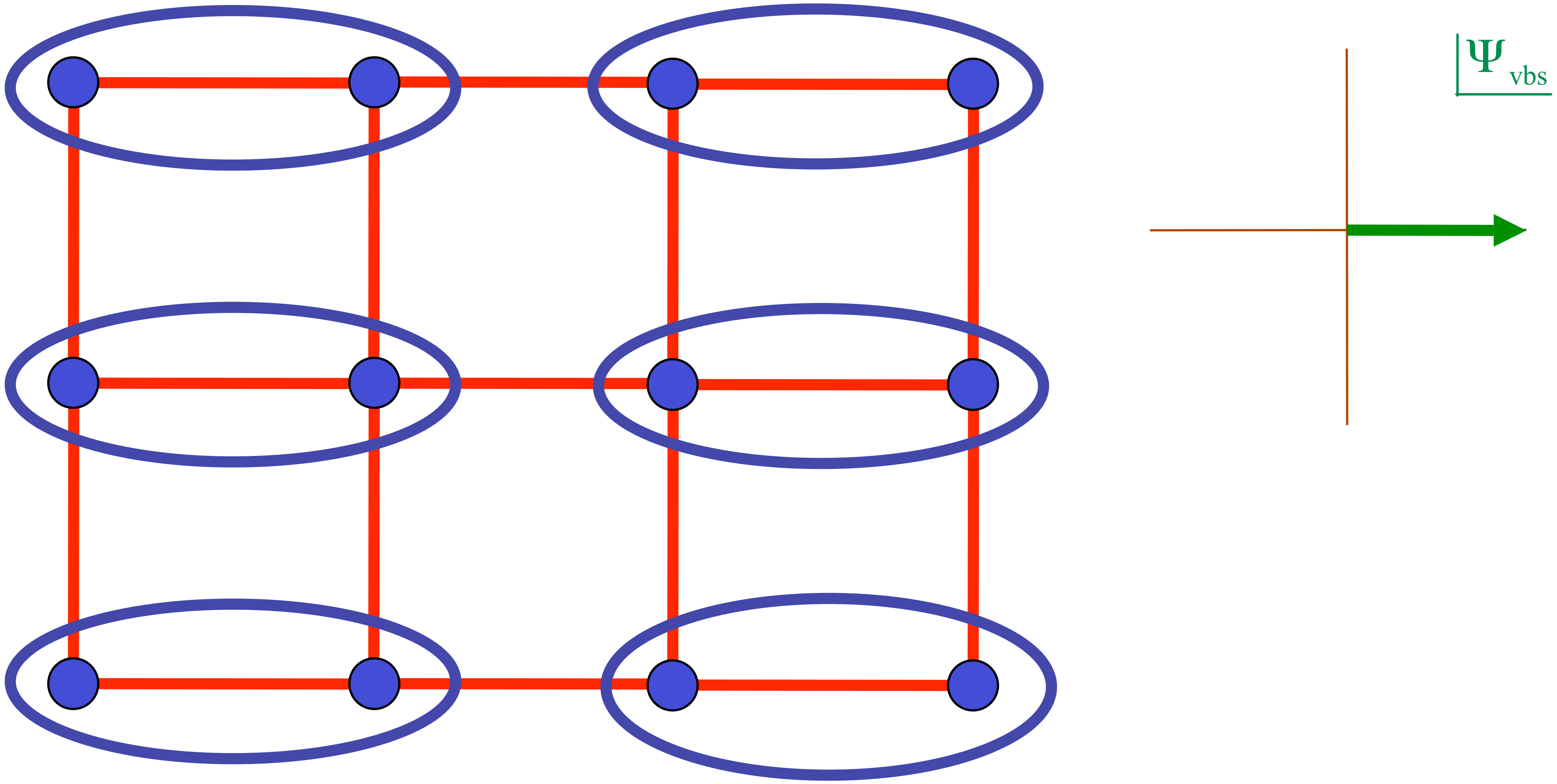

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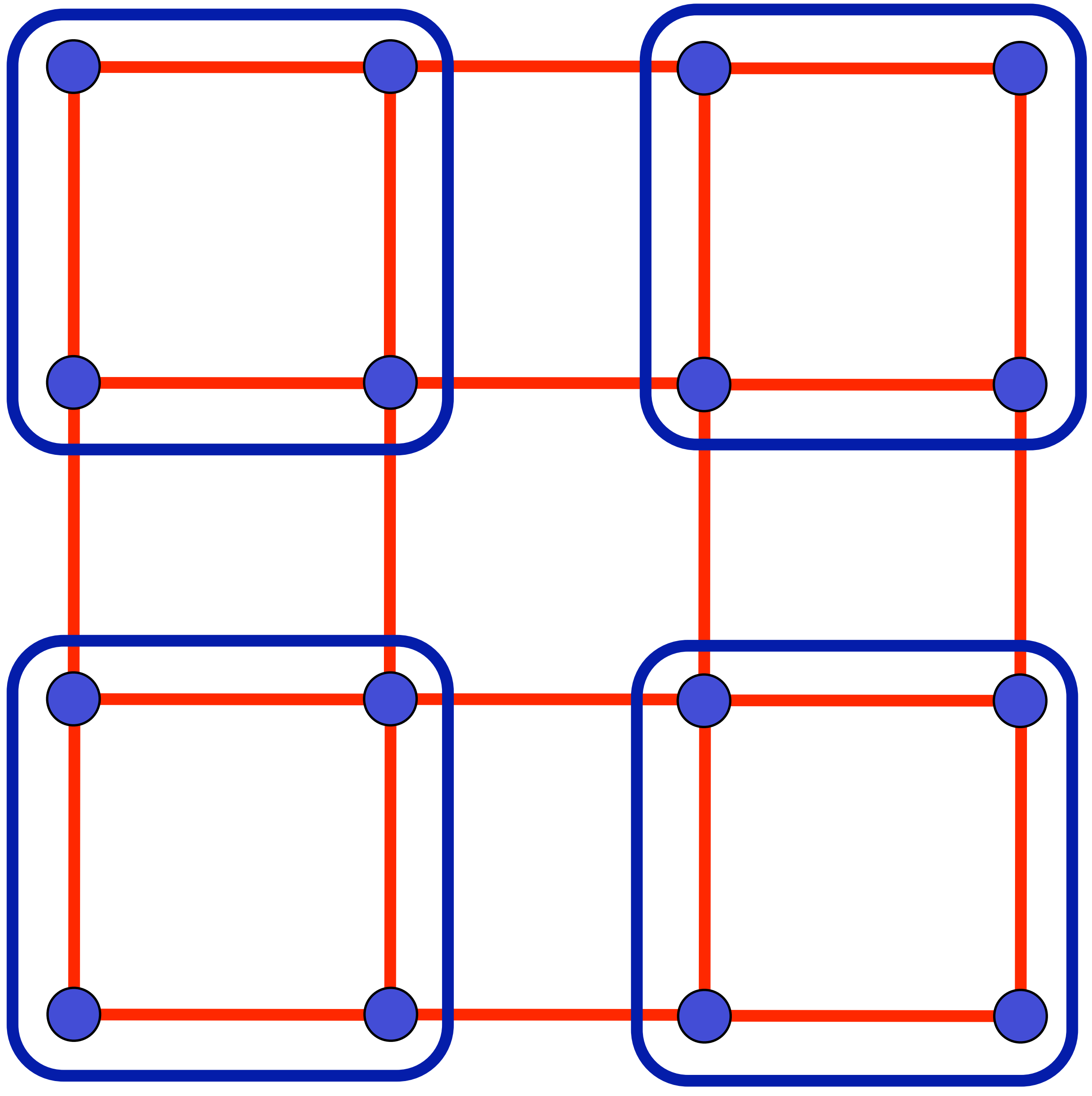

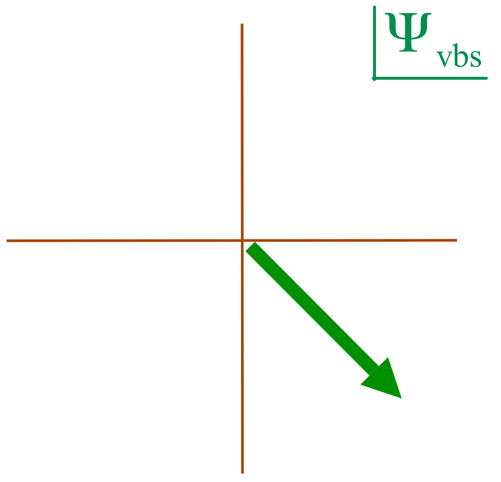

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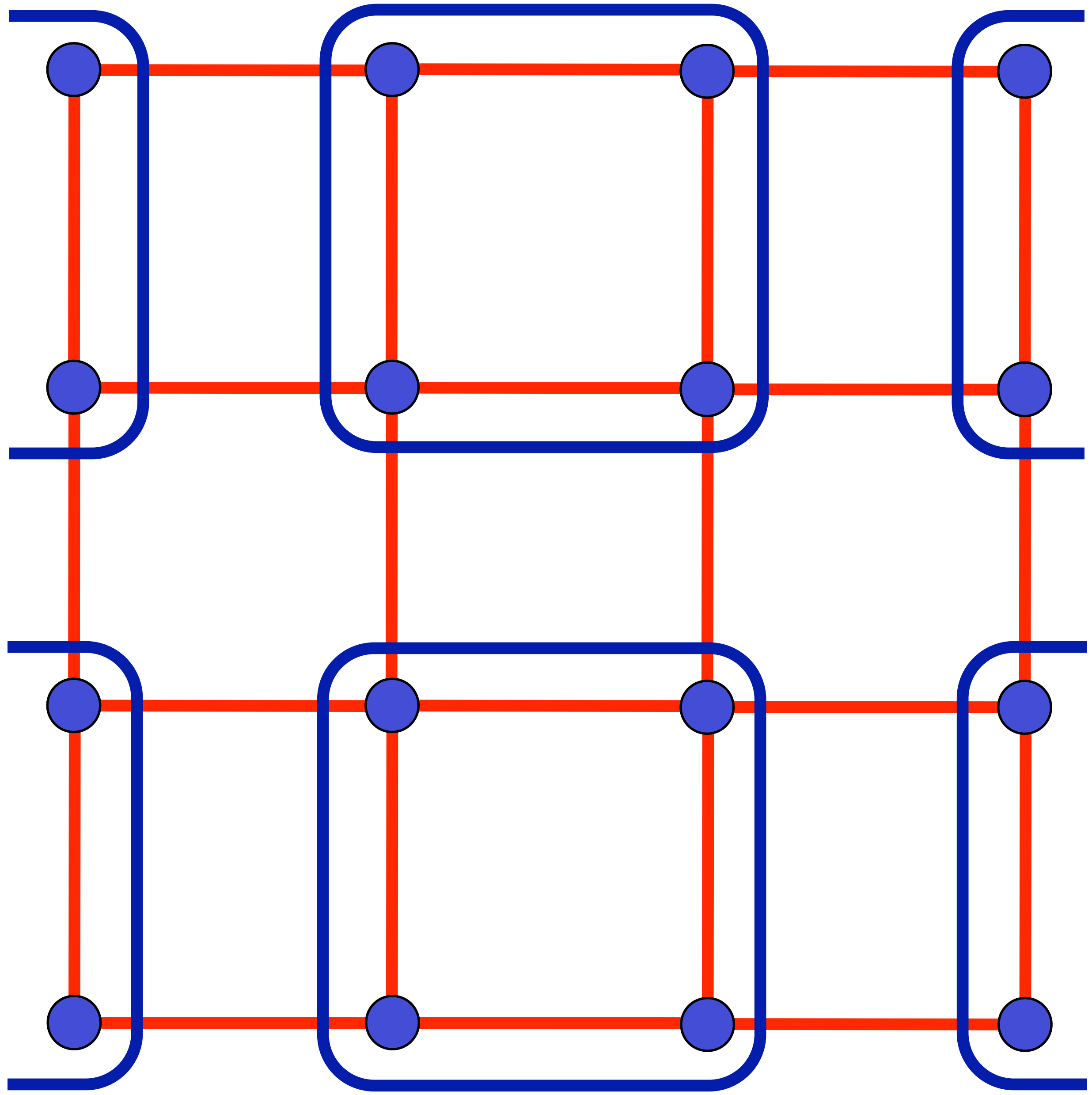

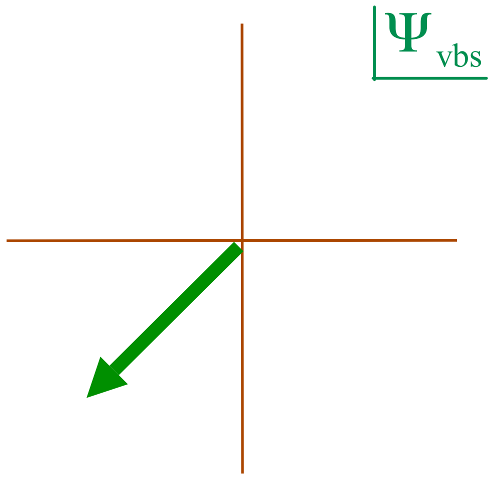

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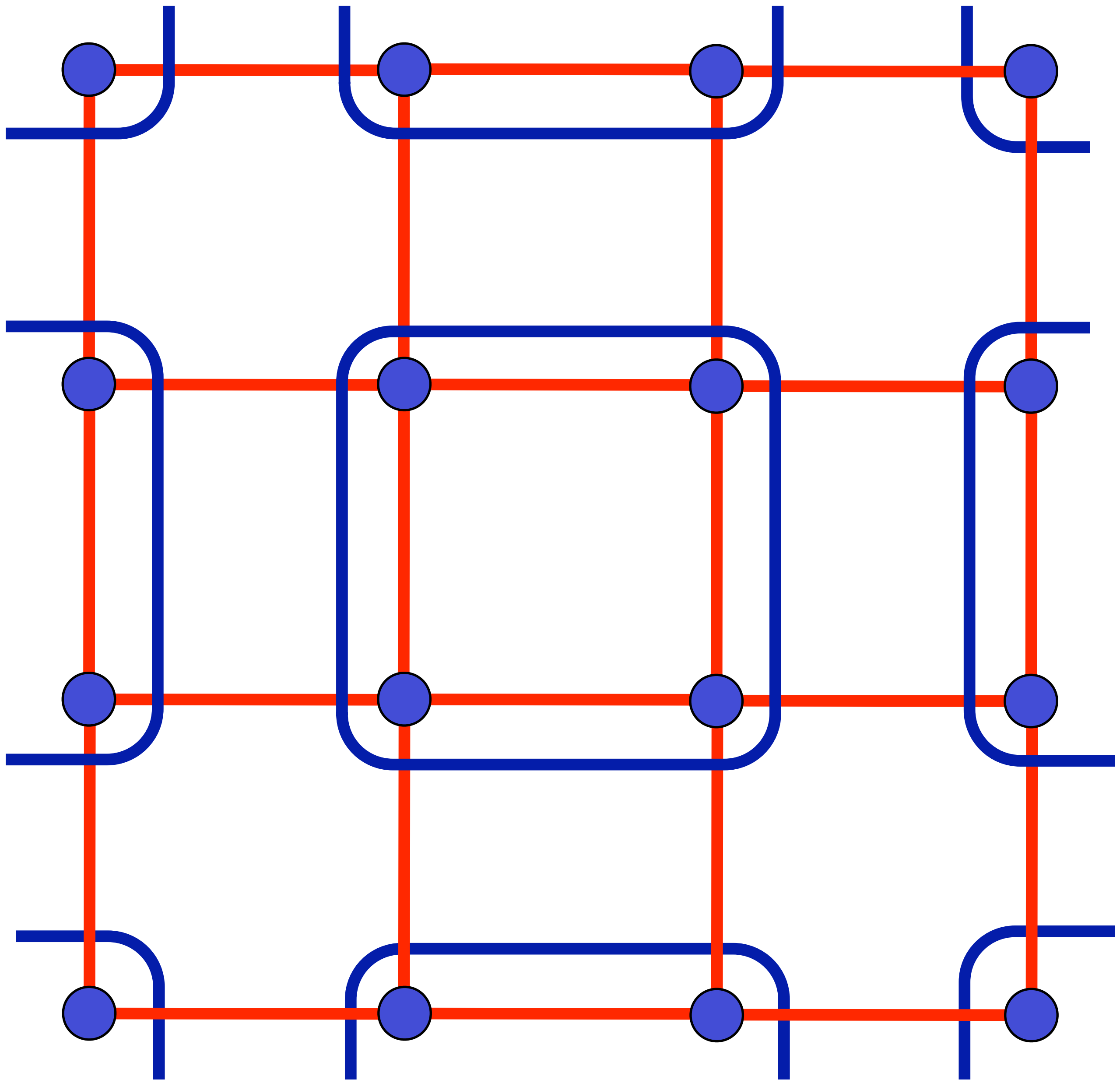

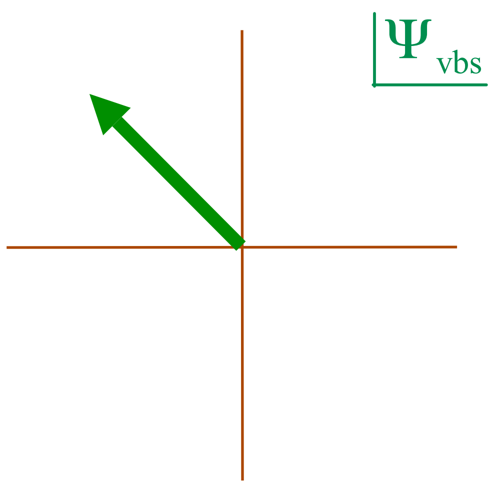

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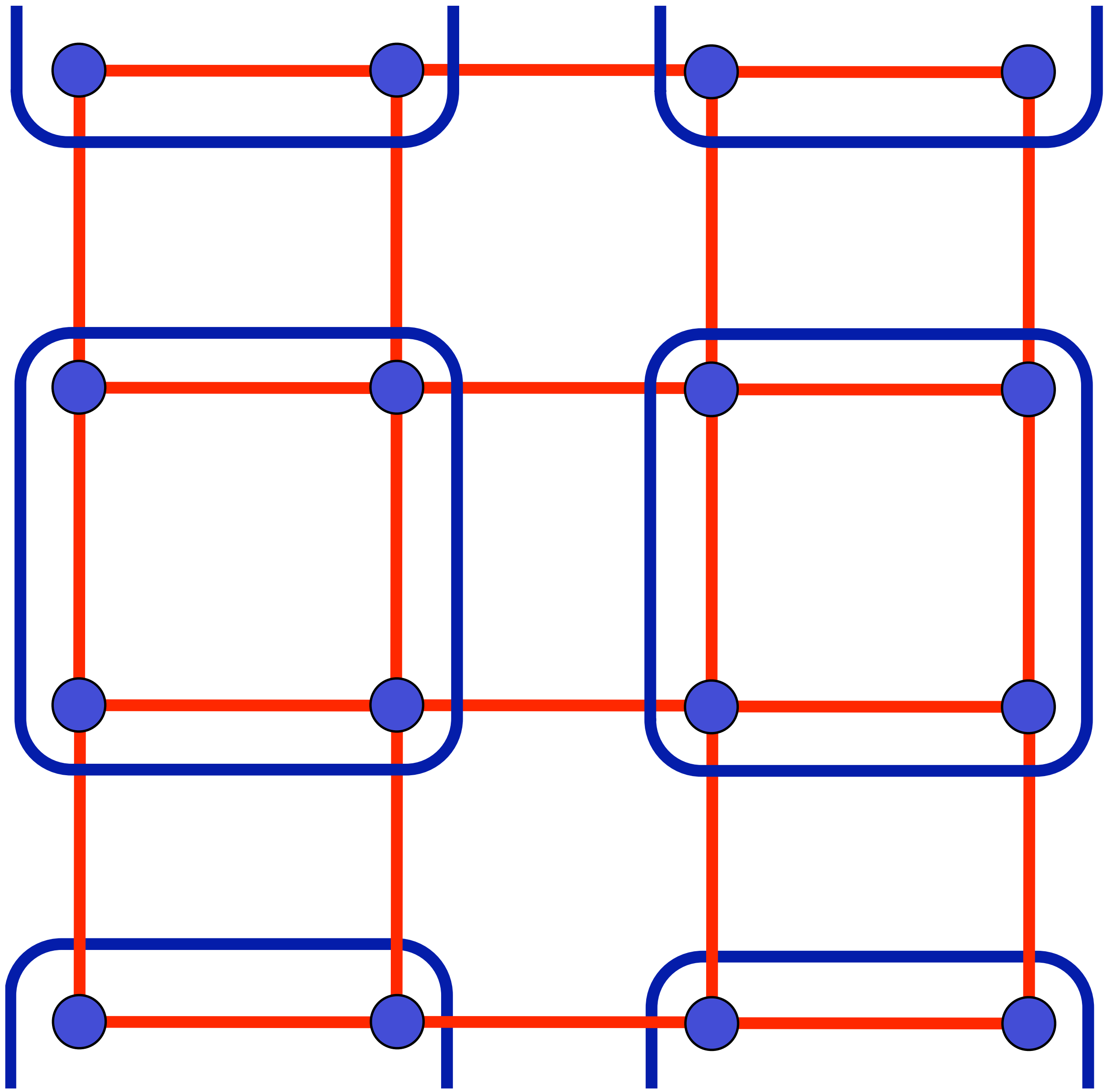

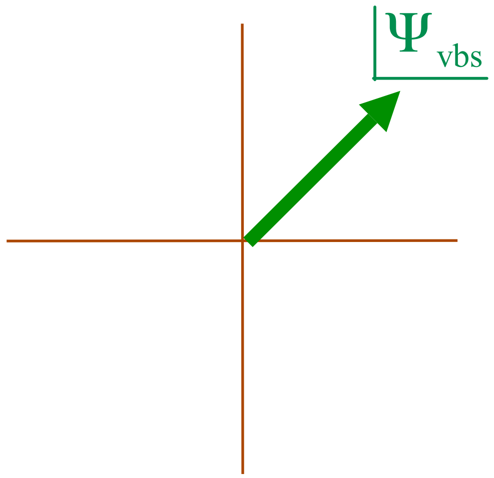

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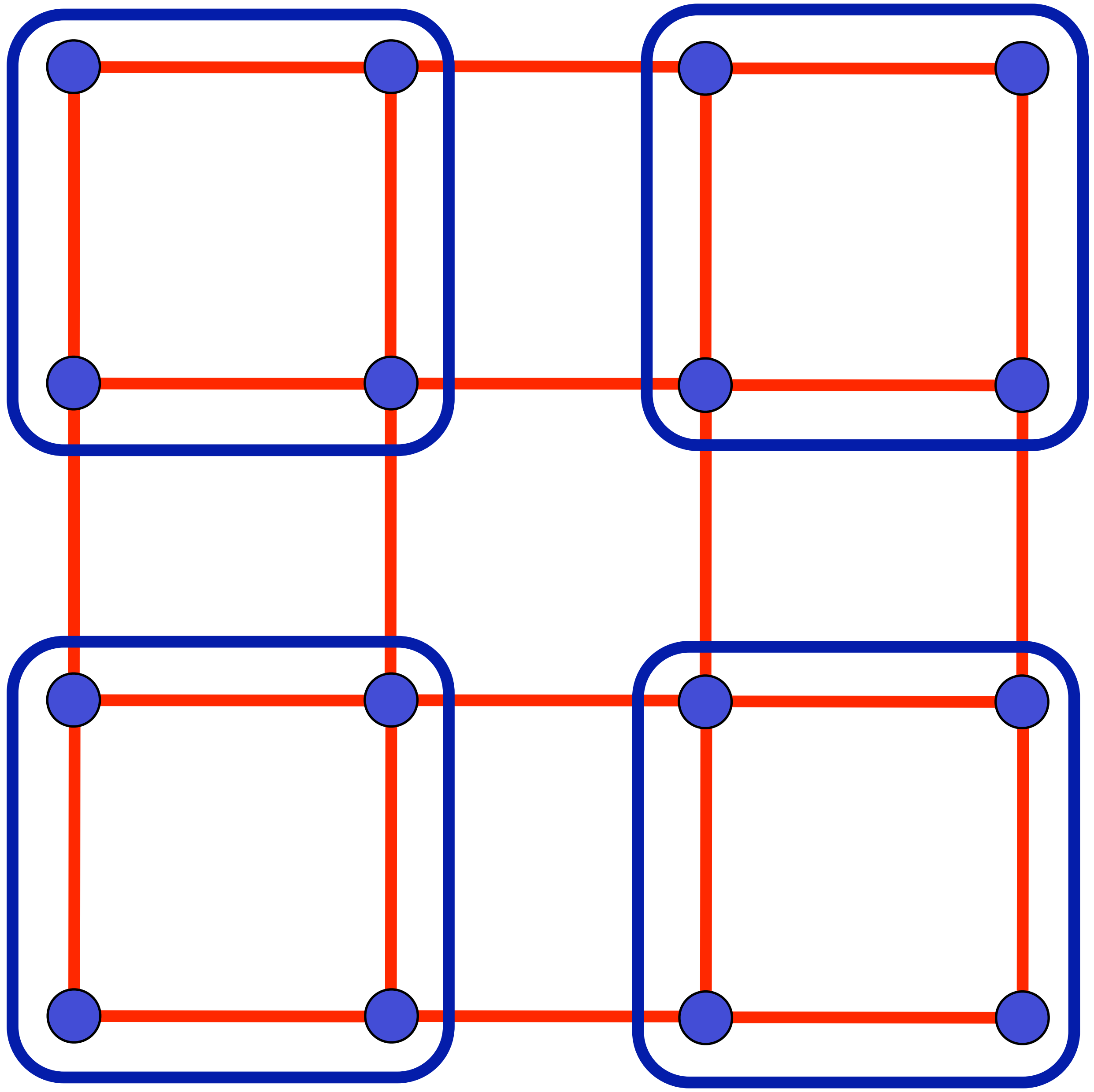

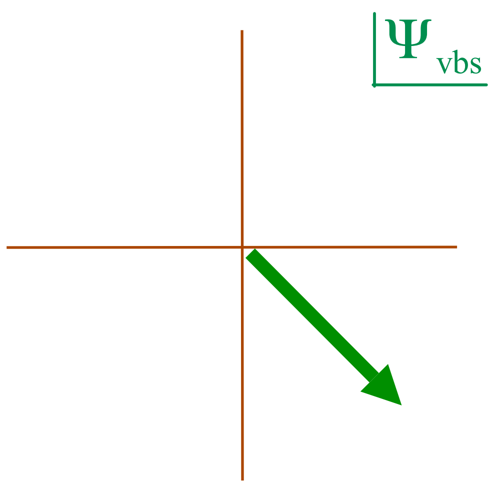

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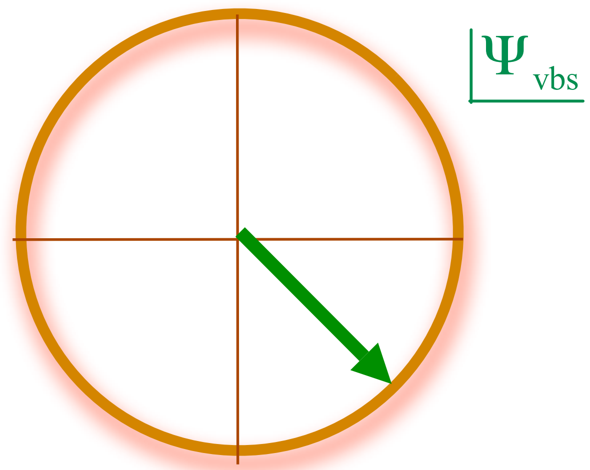
- Near the Néel-VBS transition, the (nearly) gapless photon can be identified with the Goldstone mode associated with an emergent circular symmetry



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N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)
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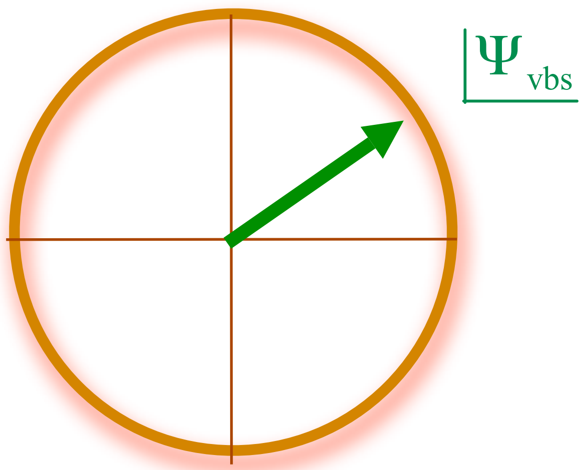

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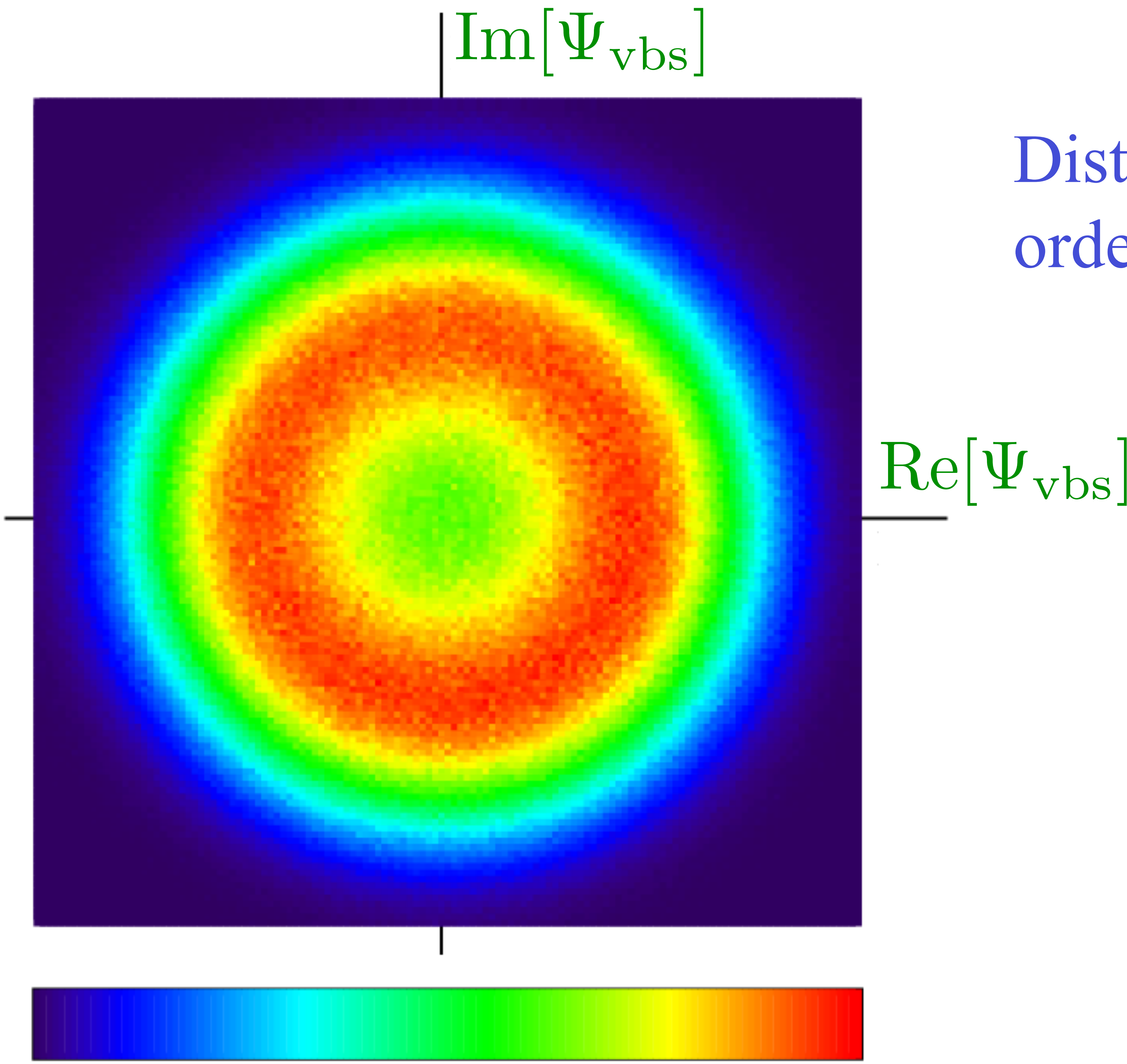
$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$

Quantum Monte Carlo simulations display convincing evidence for a transition from a

Neel state at small Q
to a
VBS state at large Q

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).
R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).
F.-J. Jiang, M. Nyfeler, S. Chandrasekharan, and U.-J. Wiese, arXiv:0710.3926

$$\mathcal{H}_{\mathrm{SU(2)}} = J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j - Q\sum_{\langle ijkl\rangle}\left(\mathbf{S}_i\cdot\mathbf{S}_j-\tfrac{1}{4}\right)\left(\mathbf{S}_k\cdot\mathbf{S}_l-\tfrac{1}{4}\right)$$



Distribution of VBS order Ψ_{vbs} at large Q

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

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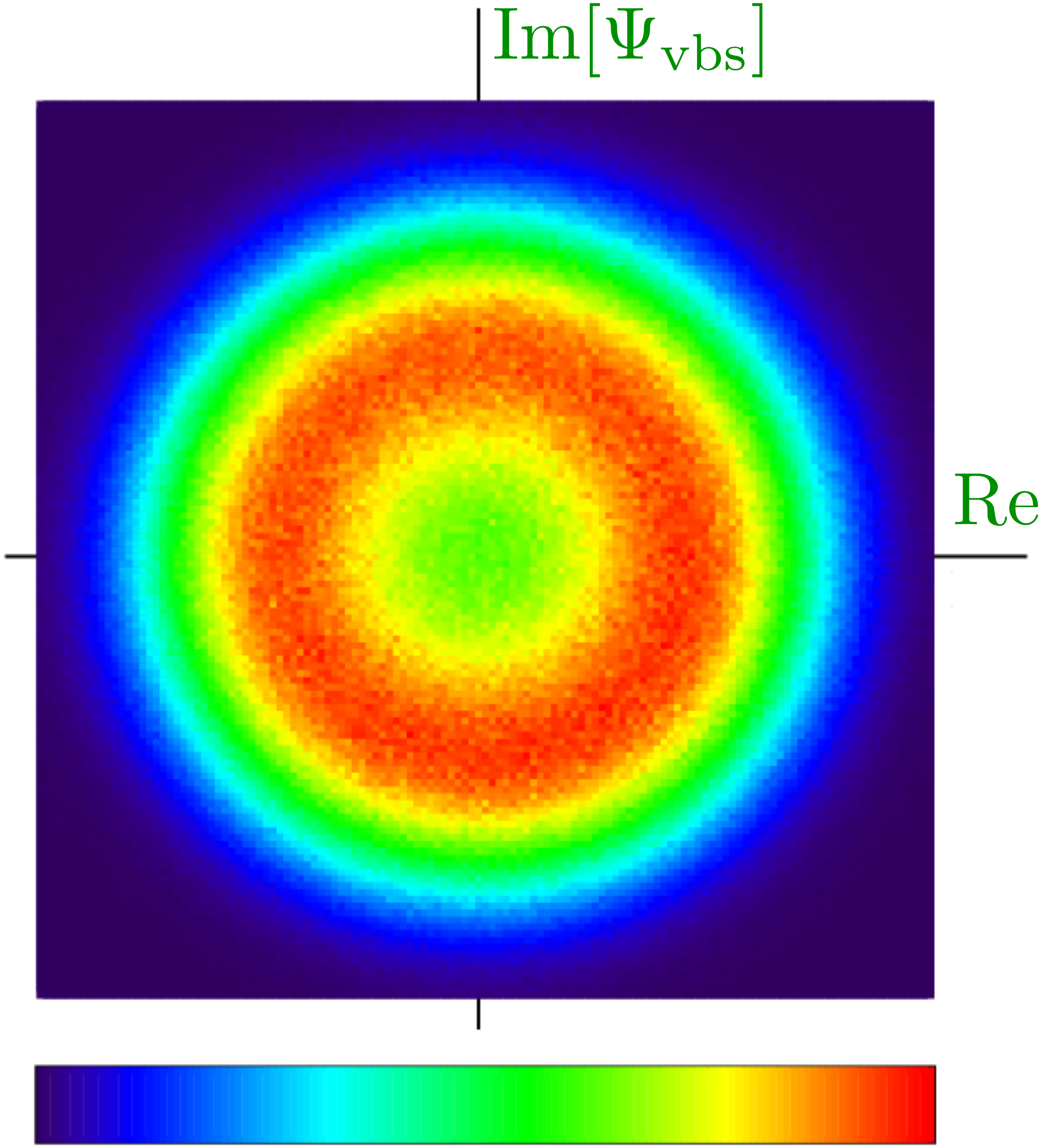
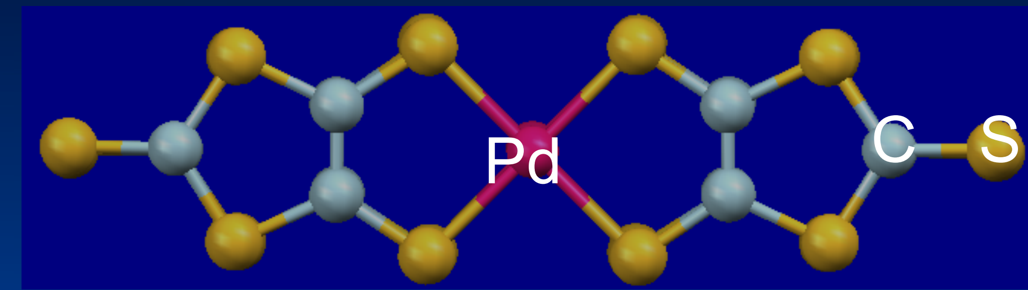


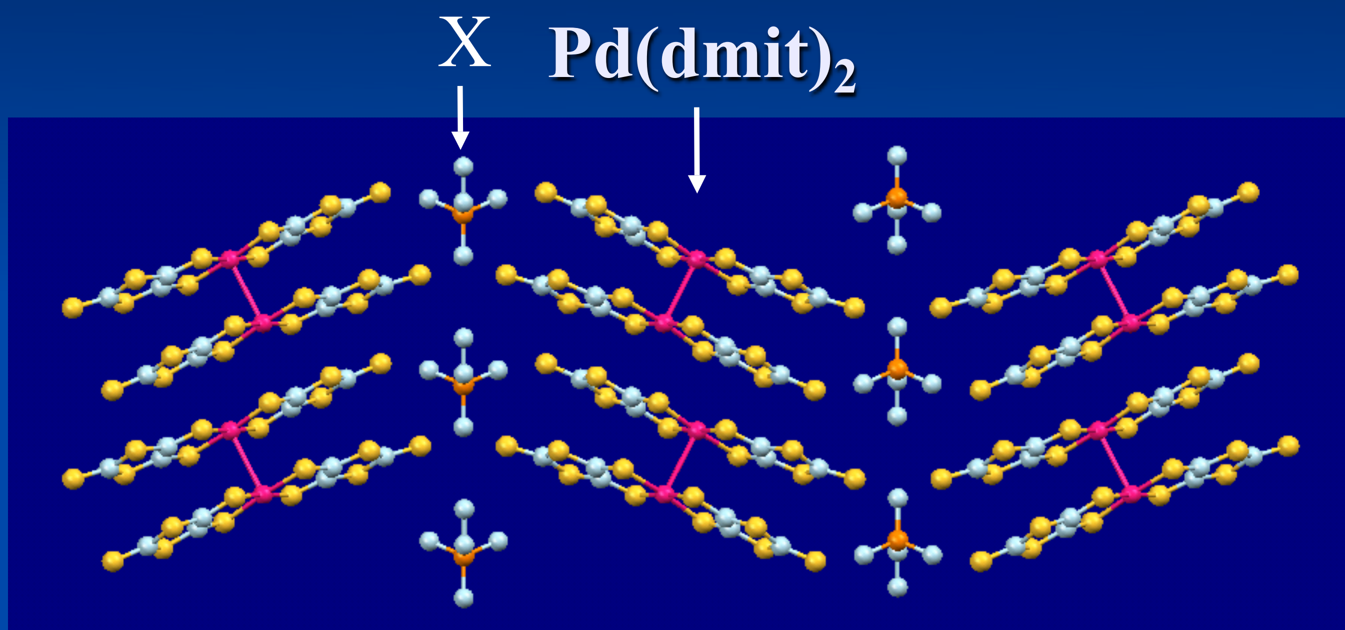
Distribution of VBS order Ψ_{vbs} at large Q

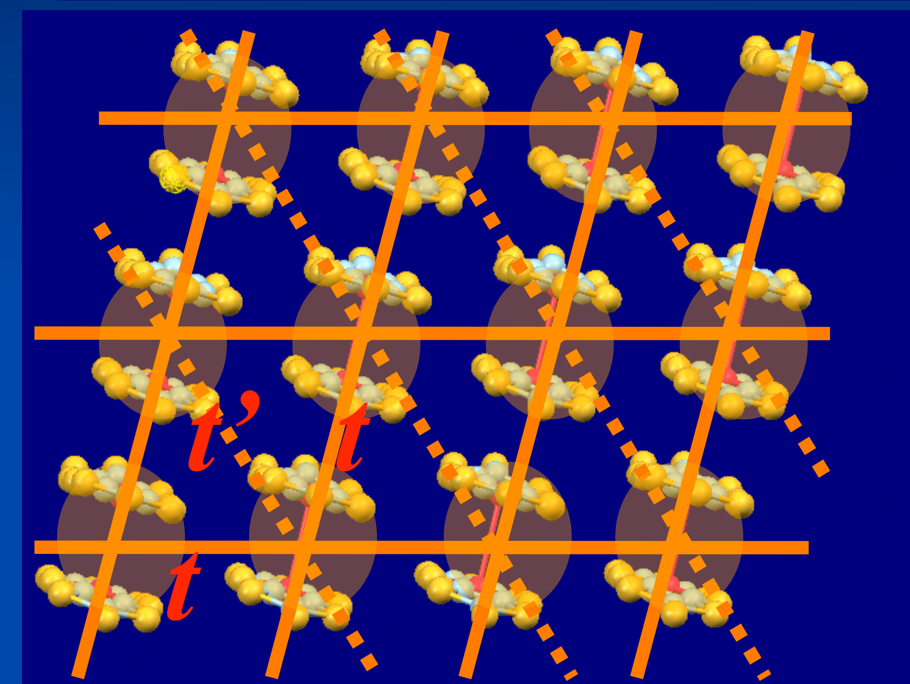
Emergent circular symmetry is evidence for U(1) photon and topological order

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

$X[Pd(dmit)_2]_2$







Half-filled band → Mott insulator with spin S = 1/2

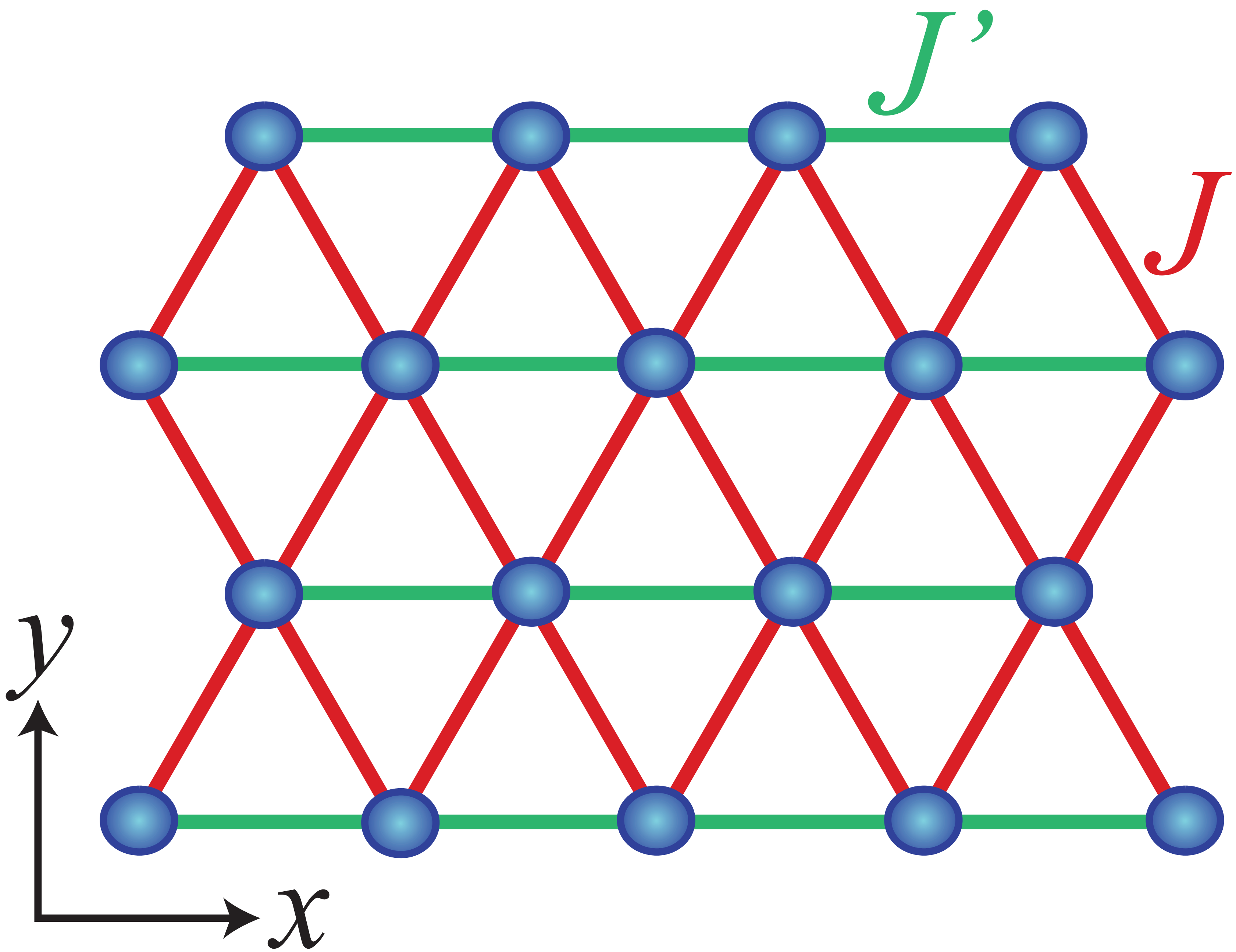
Triangular lattice of $[Pd(dmit)_2]_2$

→ frustrated quantum spin system

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

$\vec{S}_i \;\Rightarrow\;$ spin operator with $S = 1/2$



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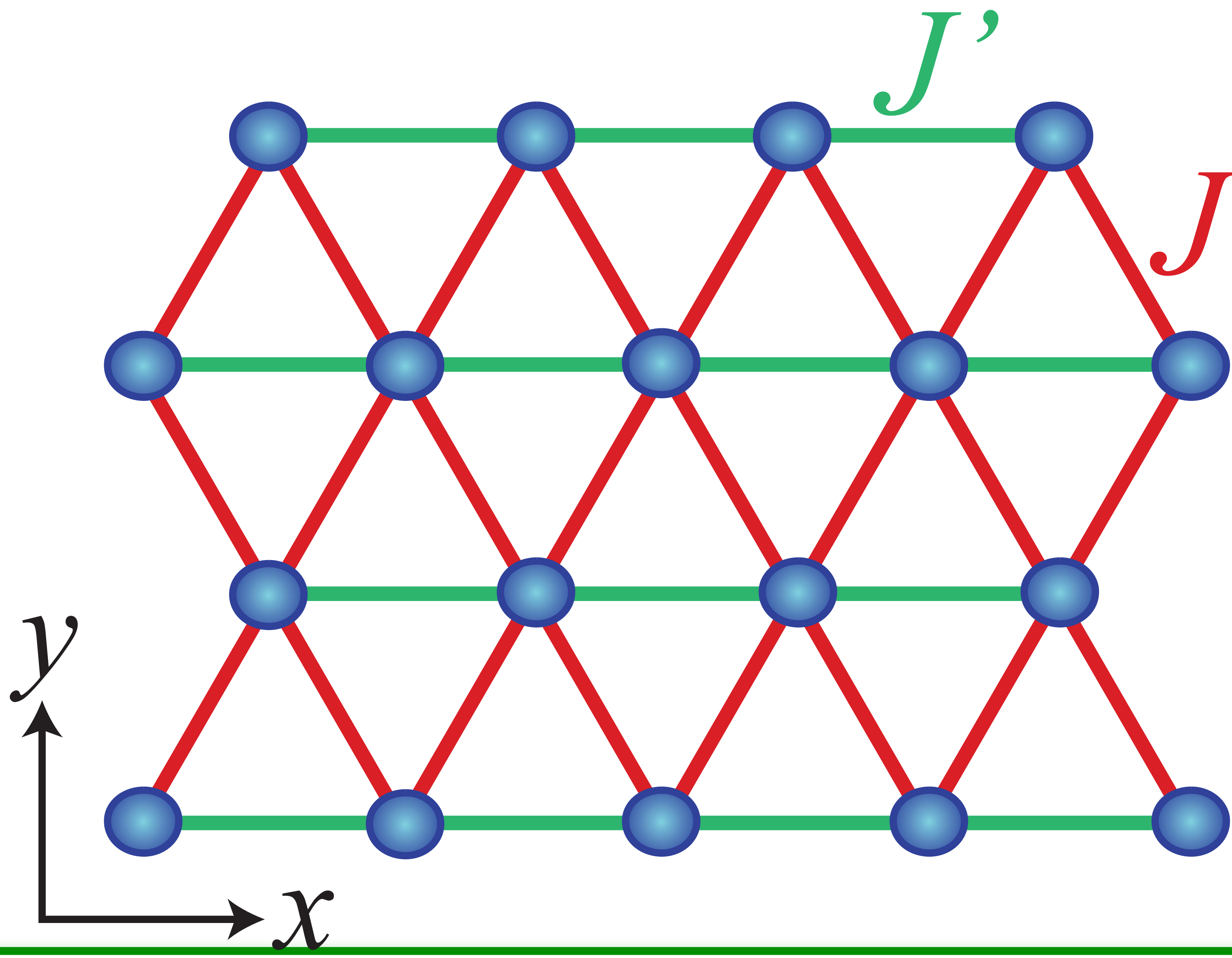
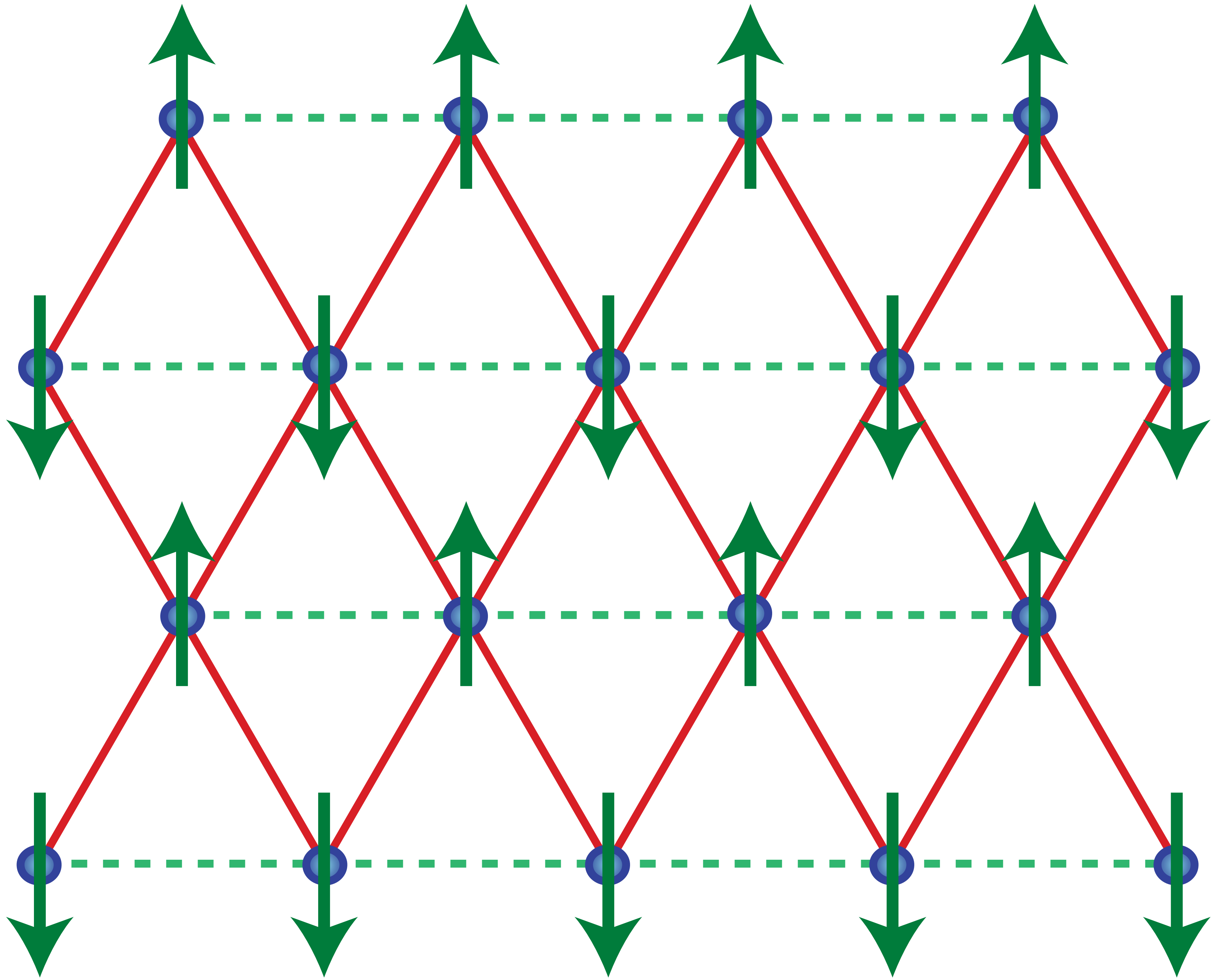


What is the ground state as a function of J'/J ?

Anisotropic triangular lattice antiferromagnet

Broken spin rotation symmetry



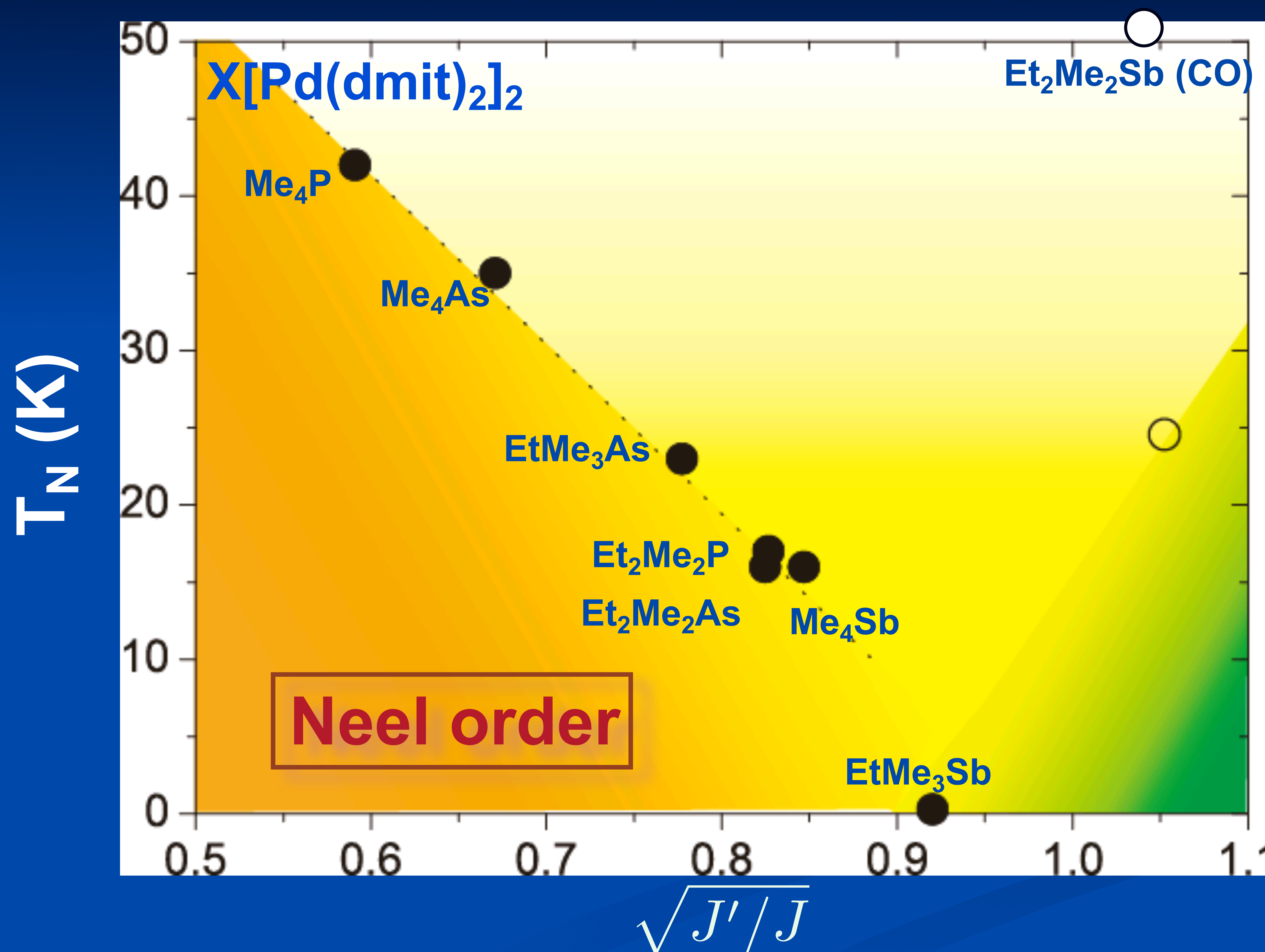
Neel ground state for small J'/J

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

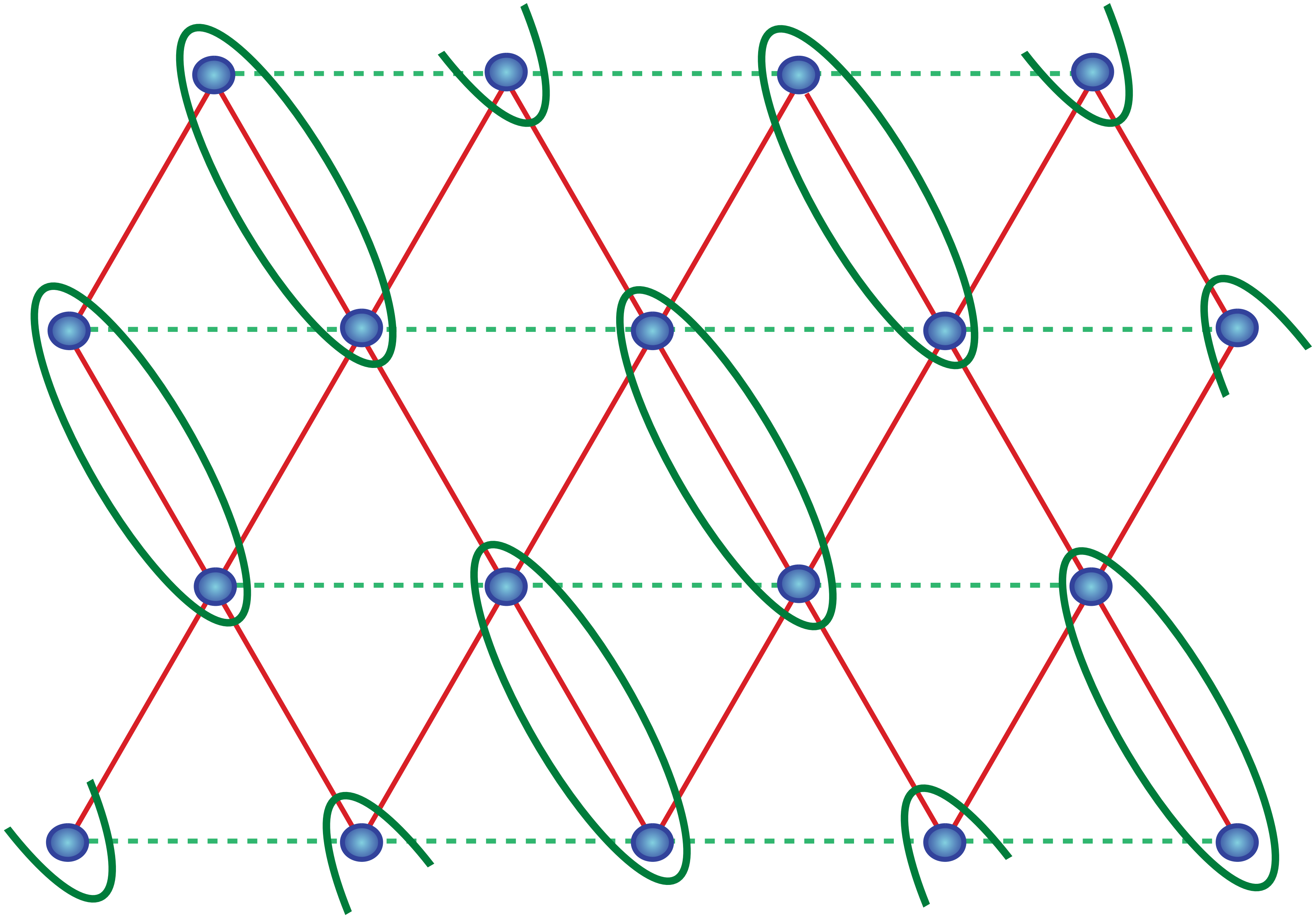
- Néel antiferromagnetic LRO

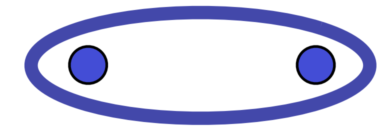
Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Anisotropic triangular lattice antiferromagnet





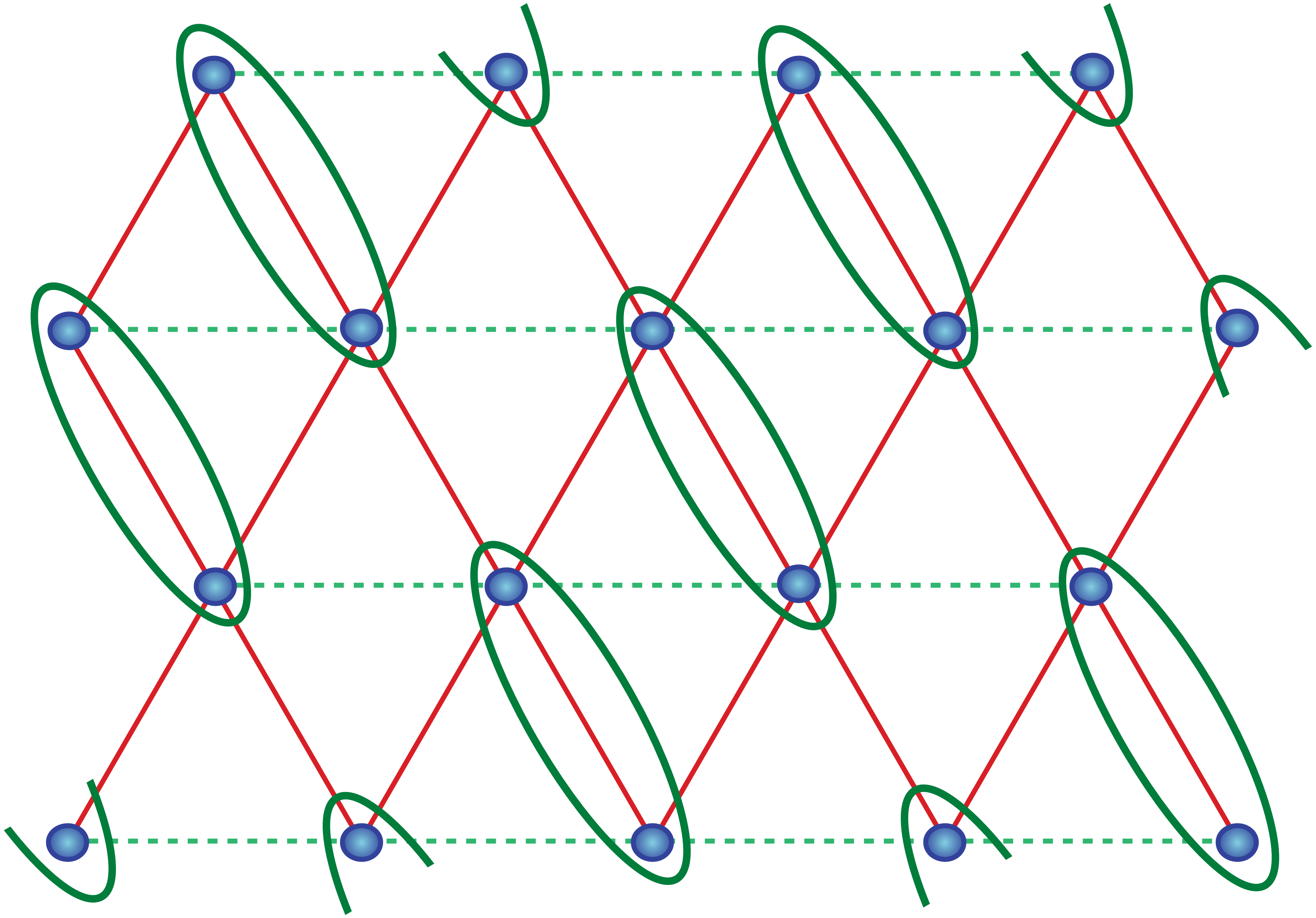
$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

Possible ground state for intermediate J'/J

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry



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Valence bond solid (VBS)

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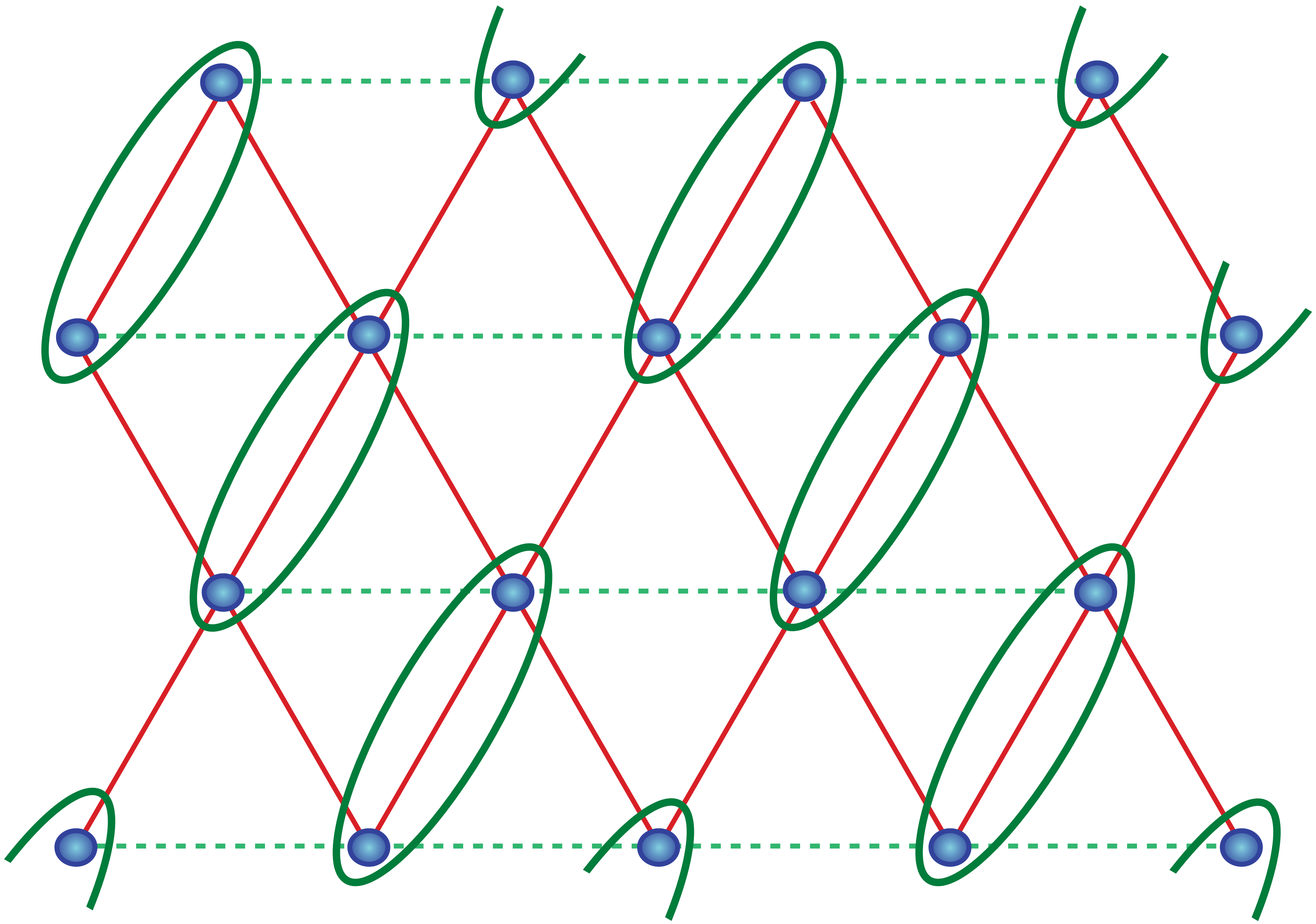

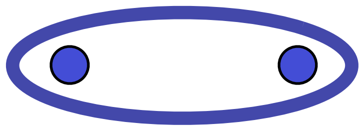

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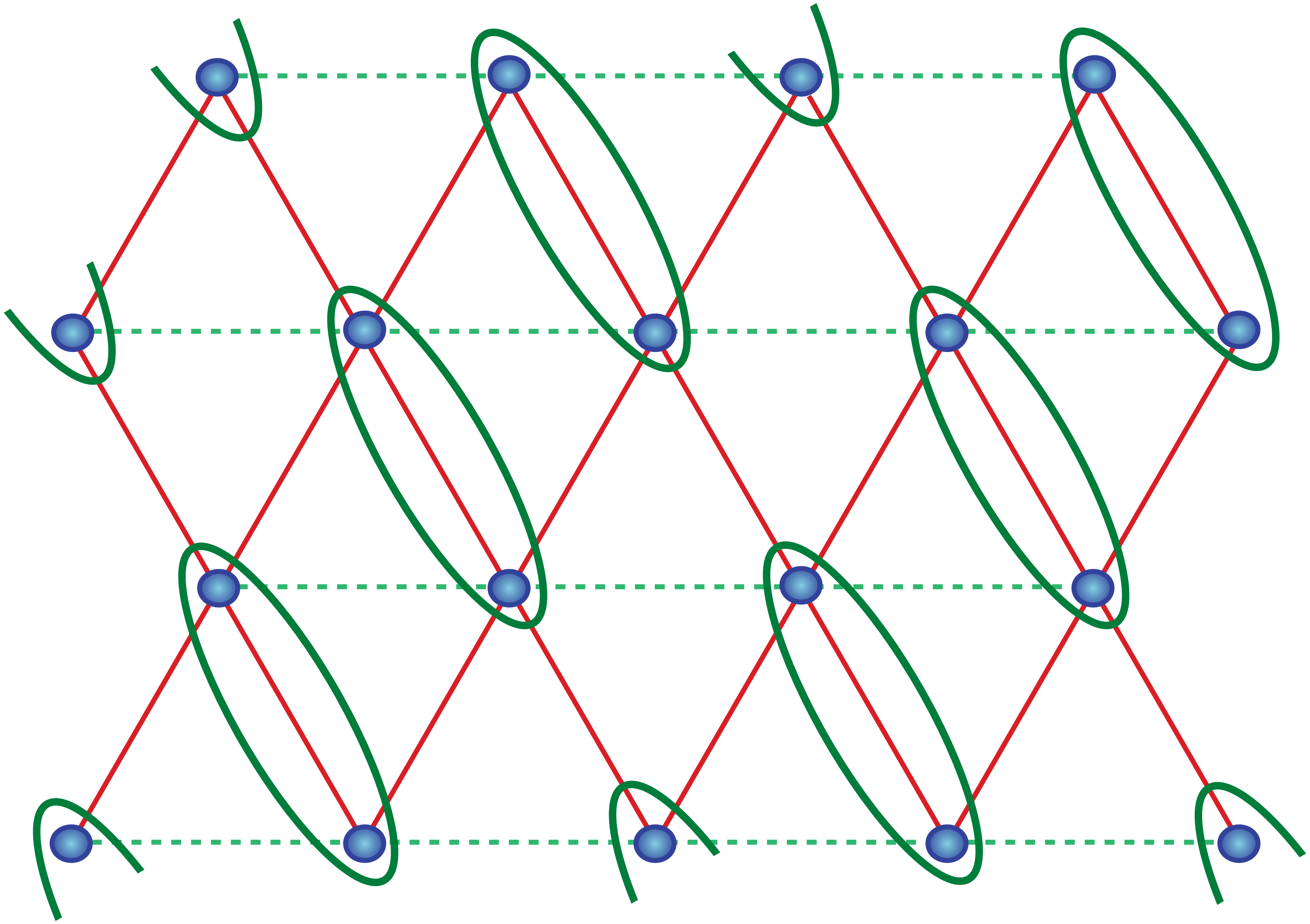

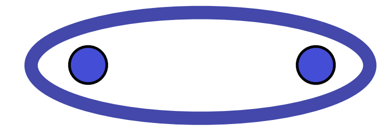

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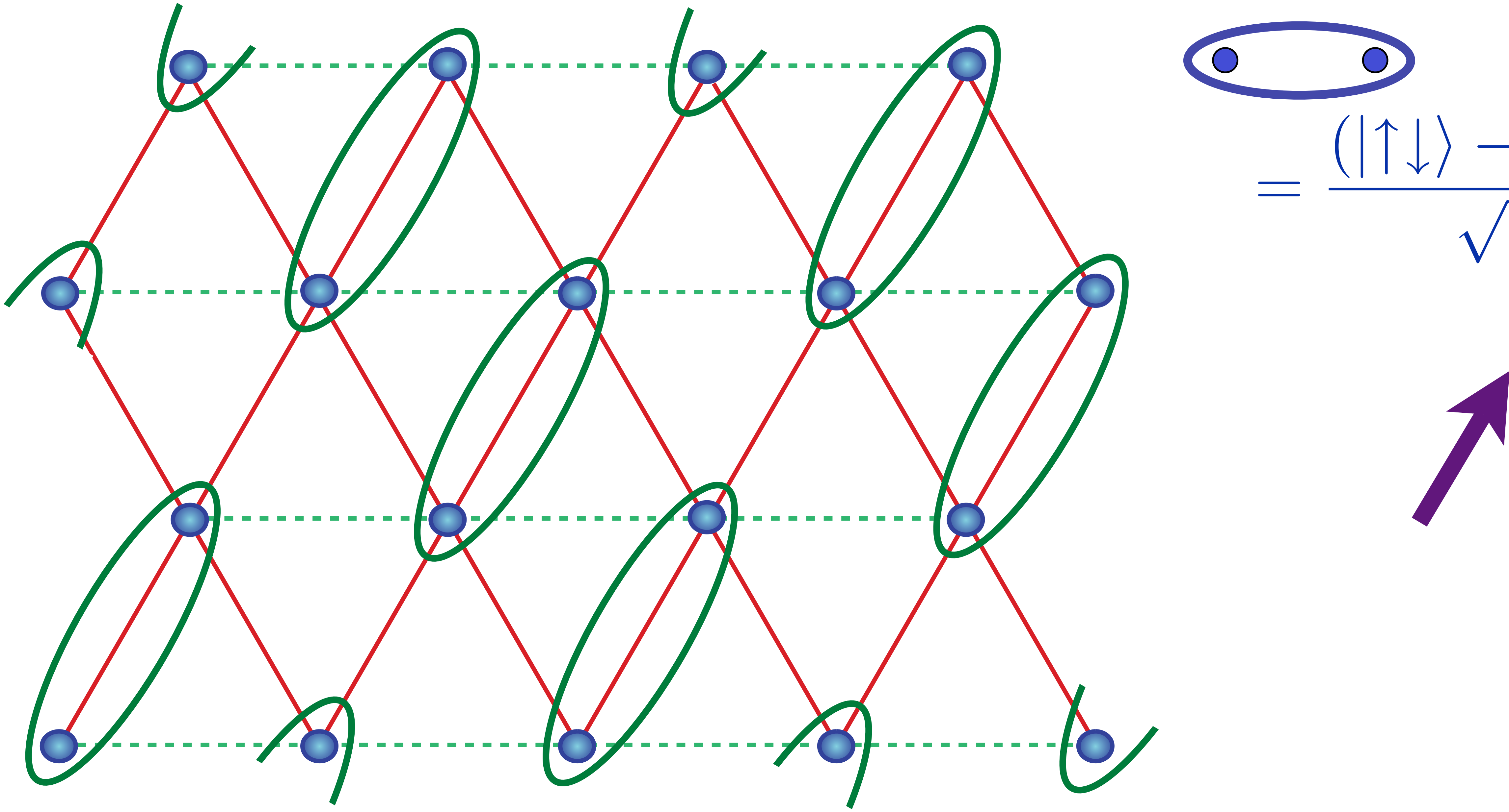

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Valence bond solid (VBS)

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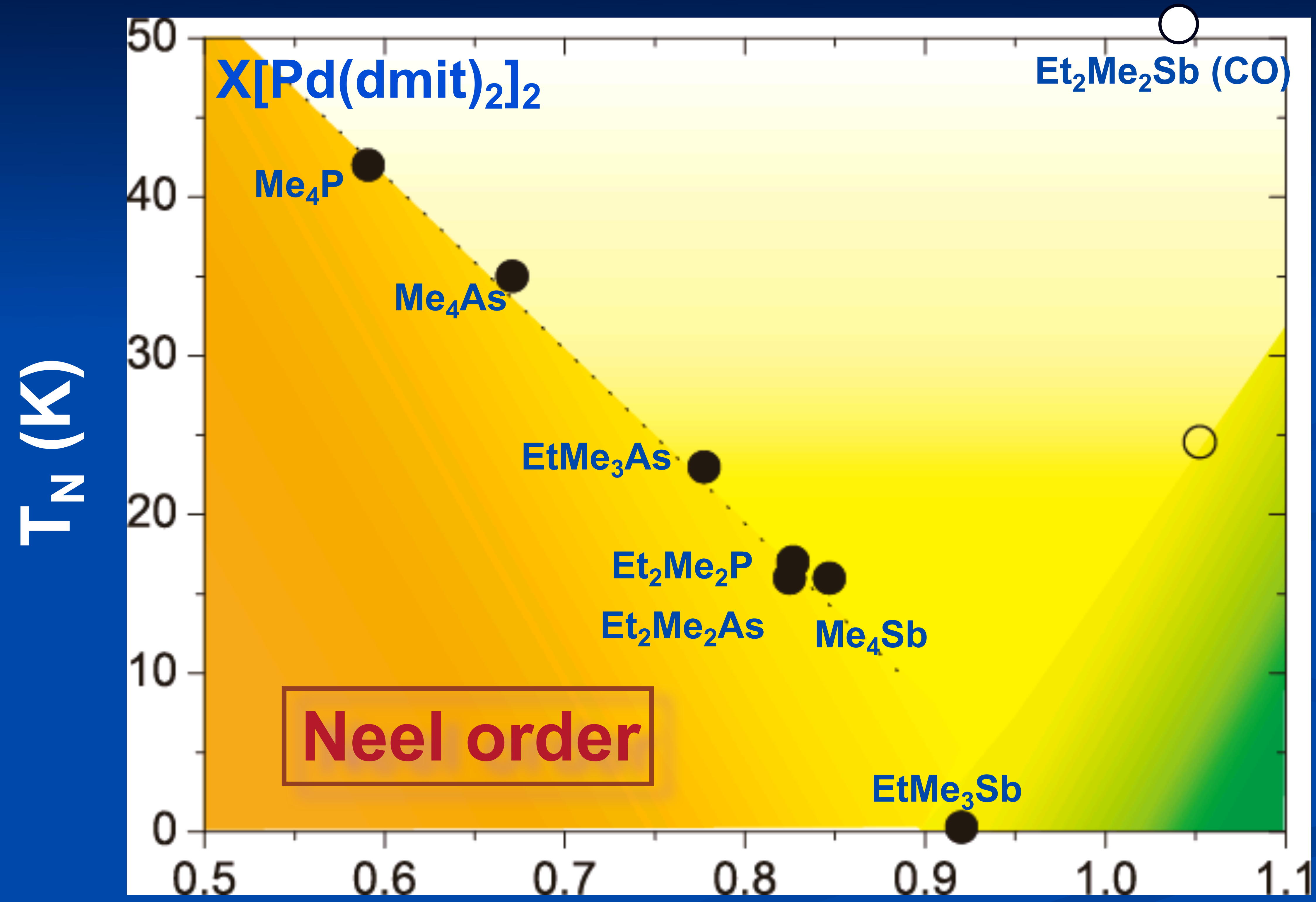
Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid

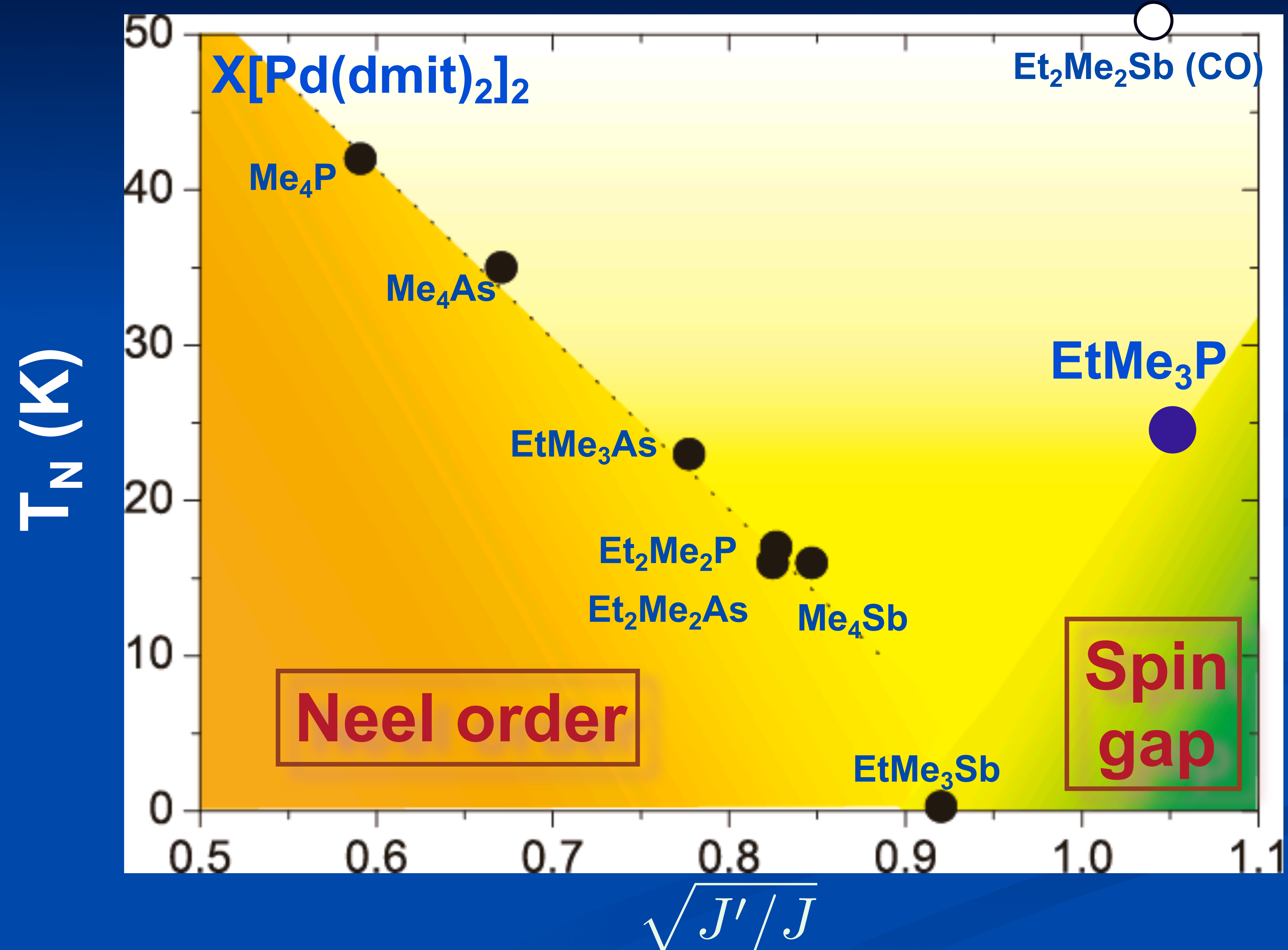
Magnetic Criticality



$\sqrt{J'/J}$

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Magnetic Criticality



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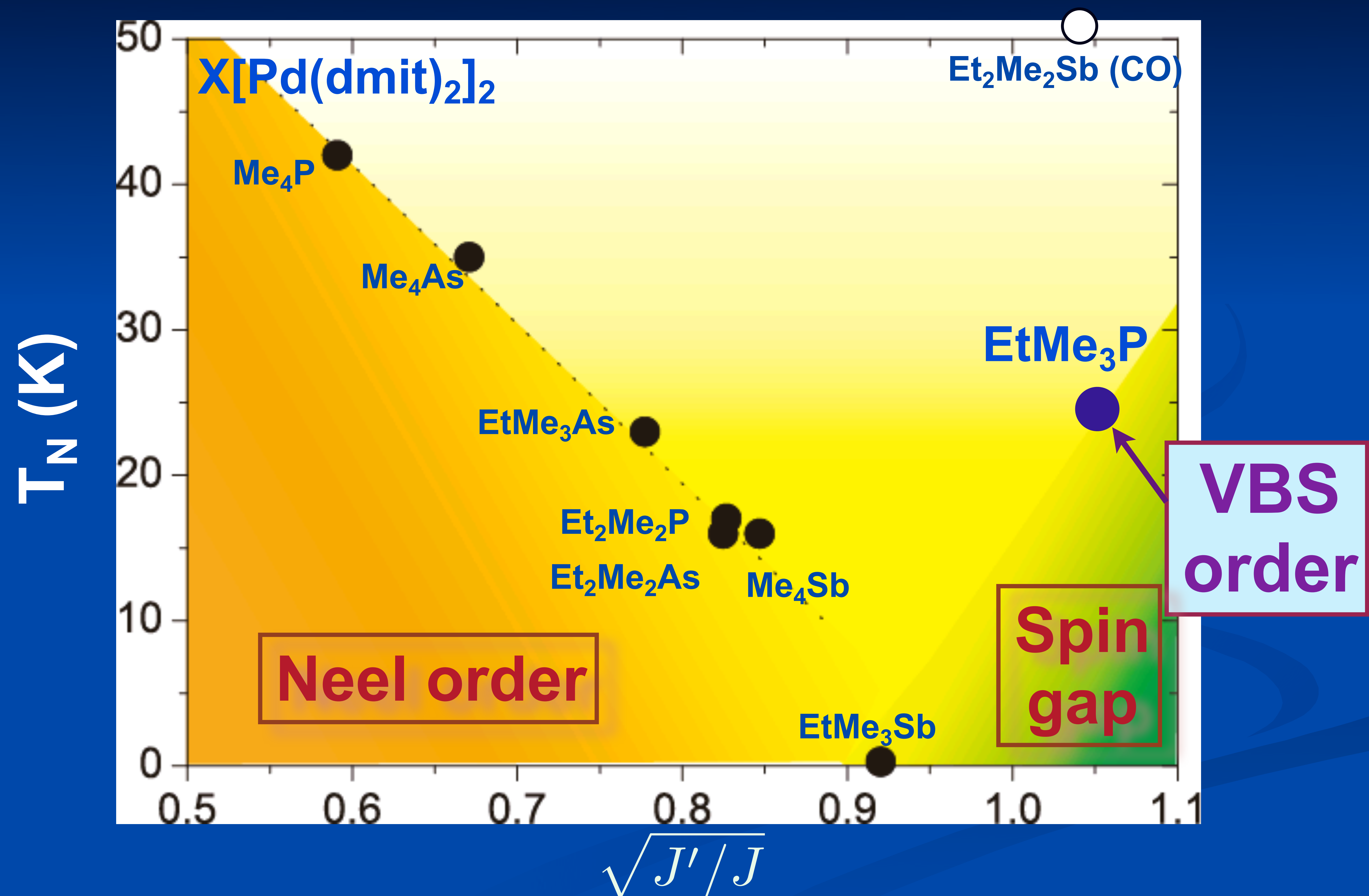
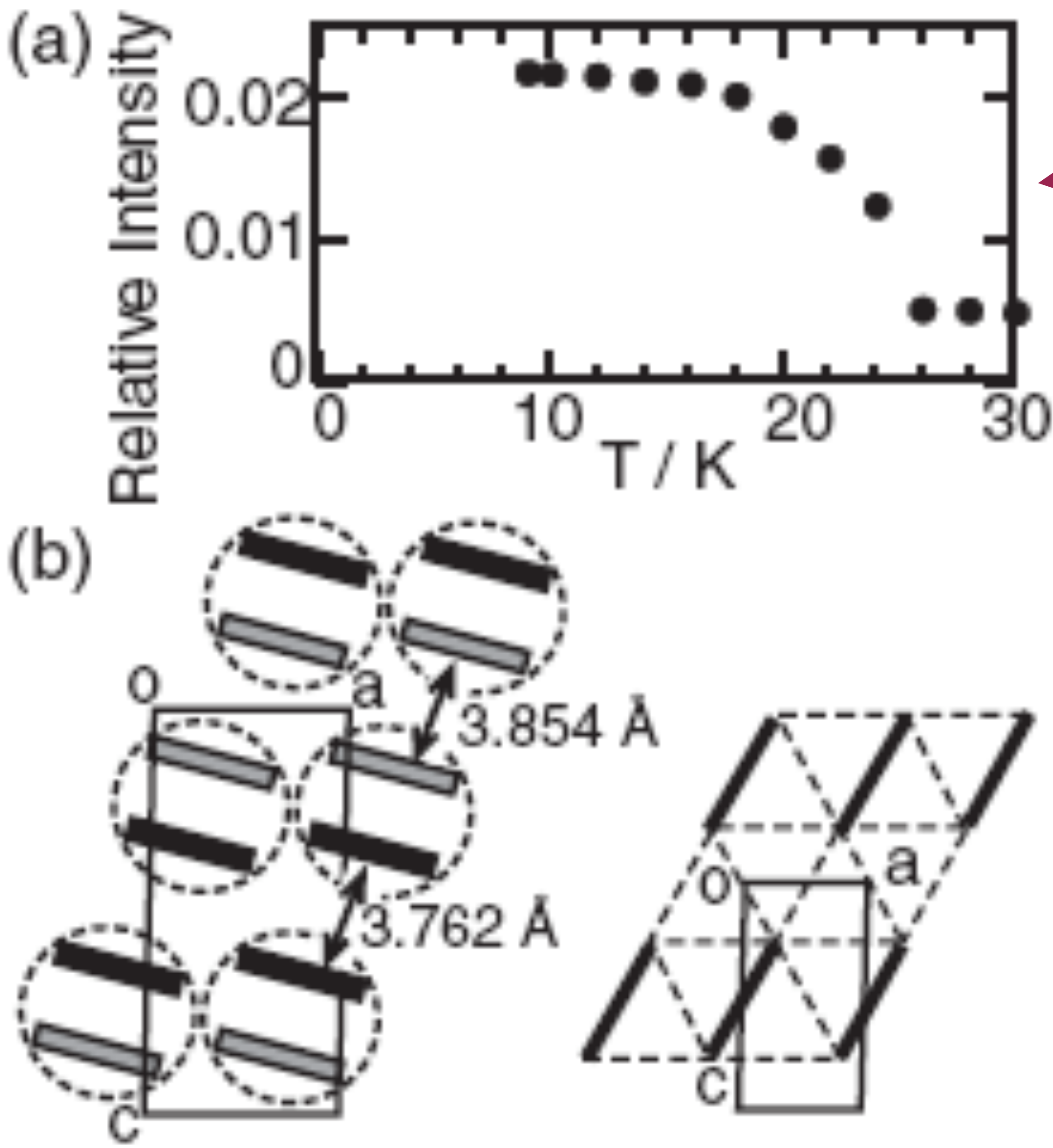


Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Observation of a valence bond solid (VBS) in $ETMe_3P[Pd(dmit)_2]_2$

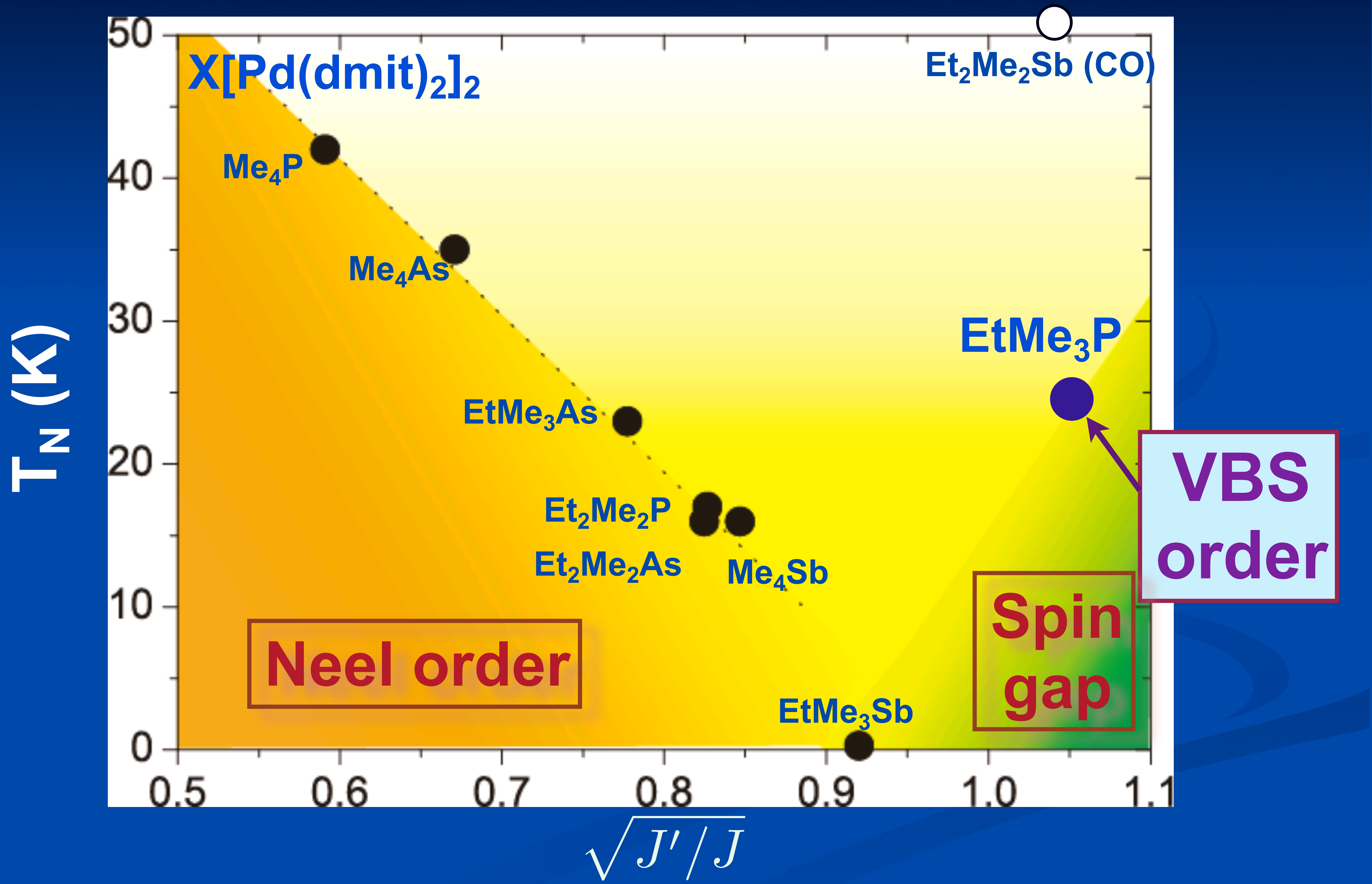


X-ray scattering

Spin gap ~ 40 K
J ~ 250 K

M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

Magnetic Criticality



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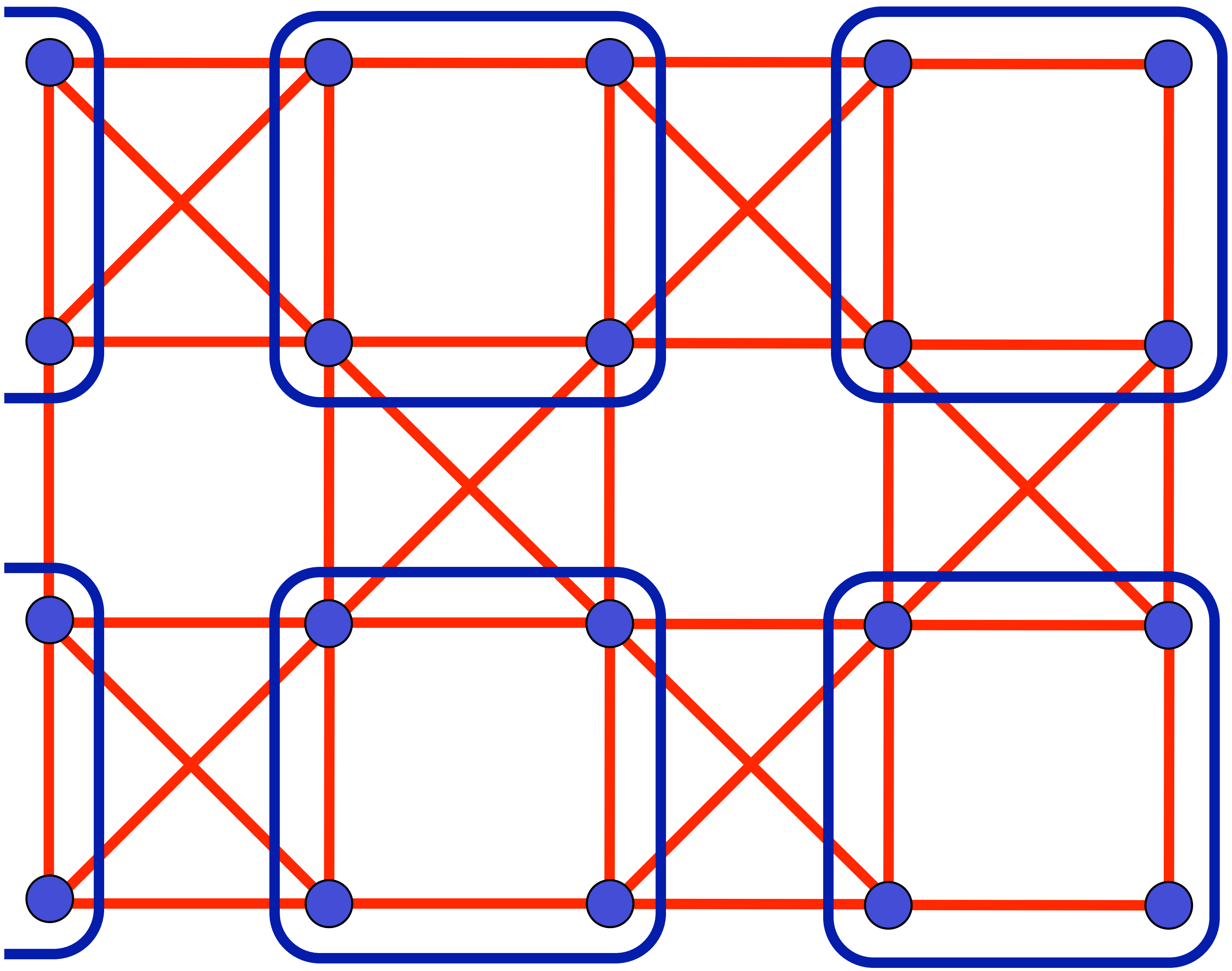
Many other models display VBS order: e.g. the planar pyrochlore lattice



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R. Moessner, O. Tchernyshyov, and S. L. Sondhi, *J. Stat. Phys.* **116**, 755 (2004).
O.A. Starykh, A. Furusaki, and L. Balents, *Phys. Rev. B* **72**, 094416 (2005)
M. Raczkowski and D. Poilblanc, arXiv:0810.0738

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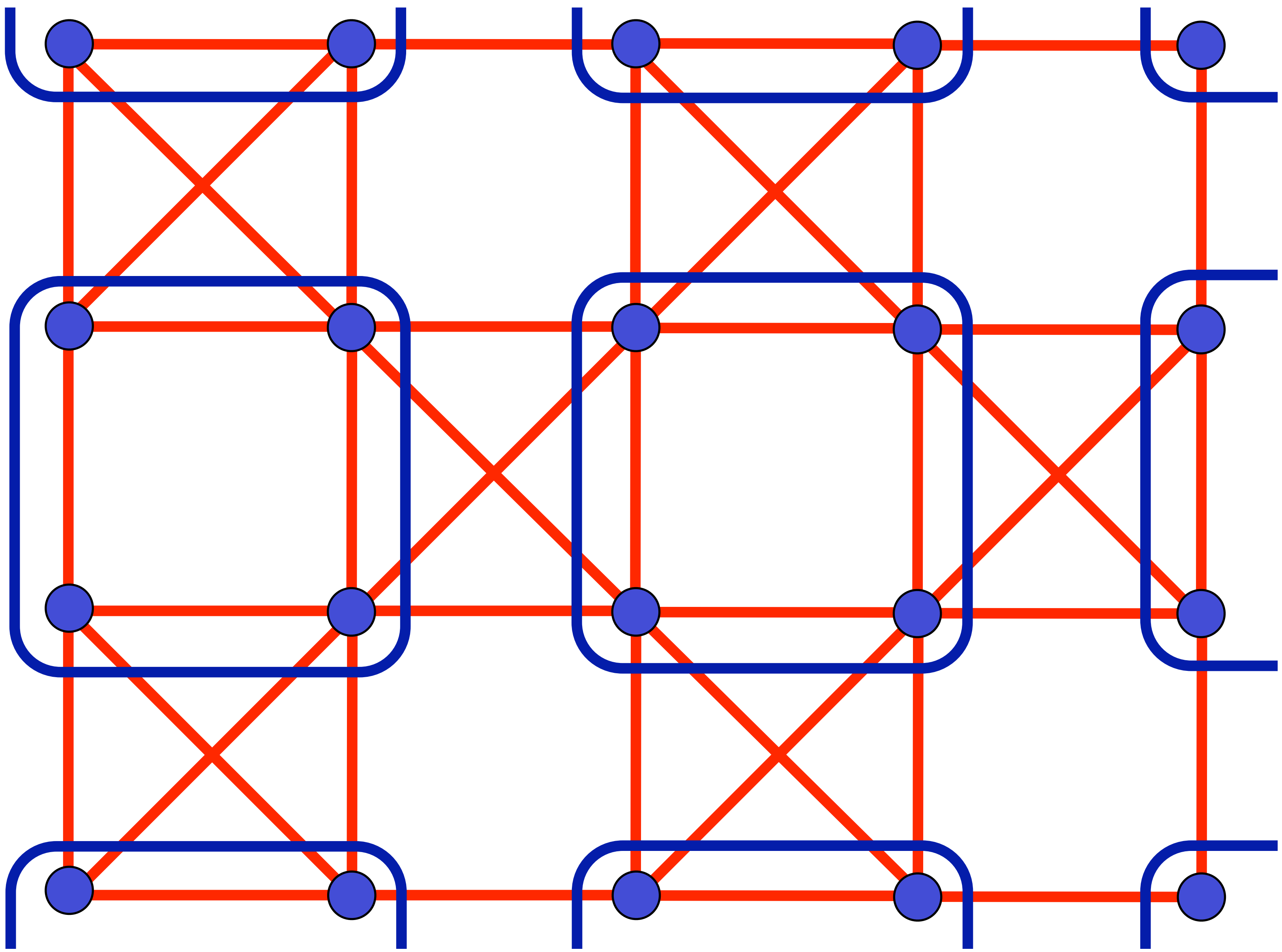

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