Quantum phases of antiferromagnets and the underdoped cuprates

Talk online: sachdev.physics.harvard.edu

Outline

1. Coupled dimer antiferromagnets

   Landau-Ginzburg quantum criticality

2. Spin liquids and valence bond solids

   (a) Schwinger-boson mean-field theory - square lattice
   (b) Gauge theories of perturbative fluctuations
   (c) Non-perturbative effects: Berry phases
   (d) Schwinger-boson mean-field theory - triangular lattice
   (e) Visons and the Kitaev model

3. Cuprate superconductivity

   (a) Review of experiments, old and new
   (b) Fermi surfaces and the spin density wave theory
   (c) Fermi pockets and the underdoped cuprates

References

- Exotic phases and quantum phase transitions: model systems and experiments, Rapporteur talk at the 24th Solvay Conference on Physics, "Quantum Theory of Condensed Matter", arXiv:0901.4103

- Quantum magnetism and criticality, *Nature Physics* **4**, 173 (2008), arXiv:0711.3015

- Quantum phases and phase transitions of Mott insulators, arXiv:cond-mat/0401041

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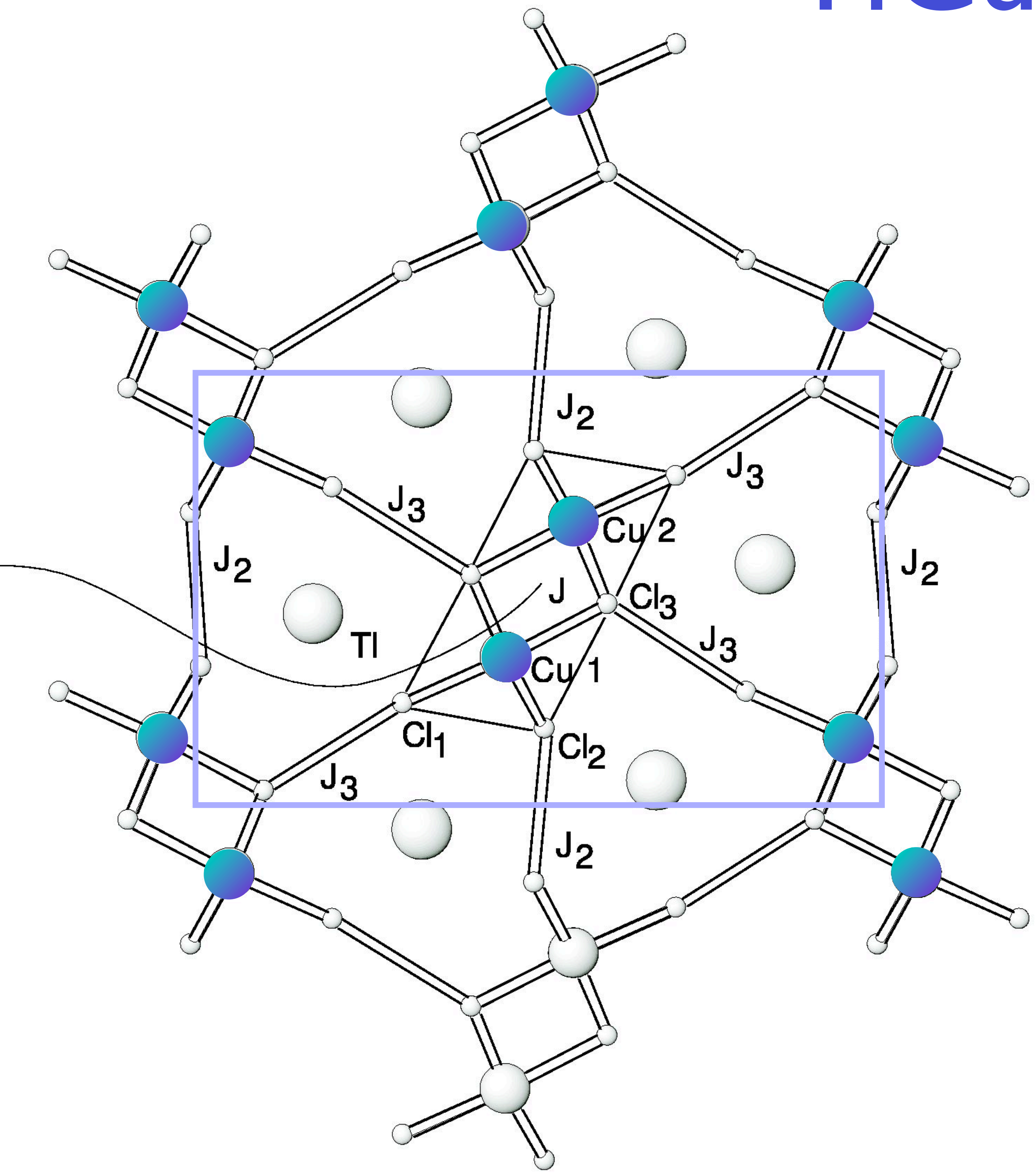
$TlCuCl_3$



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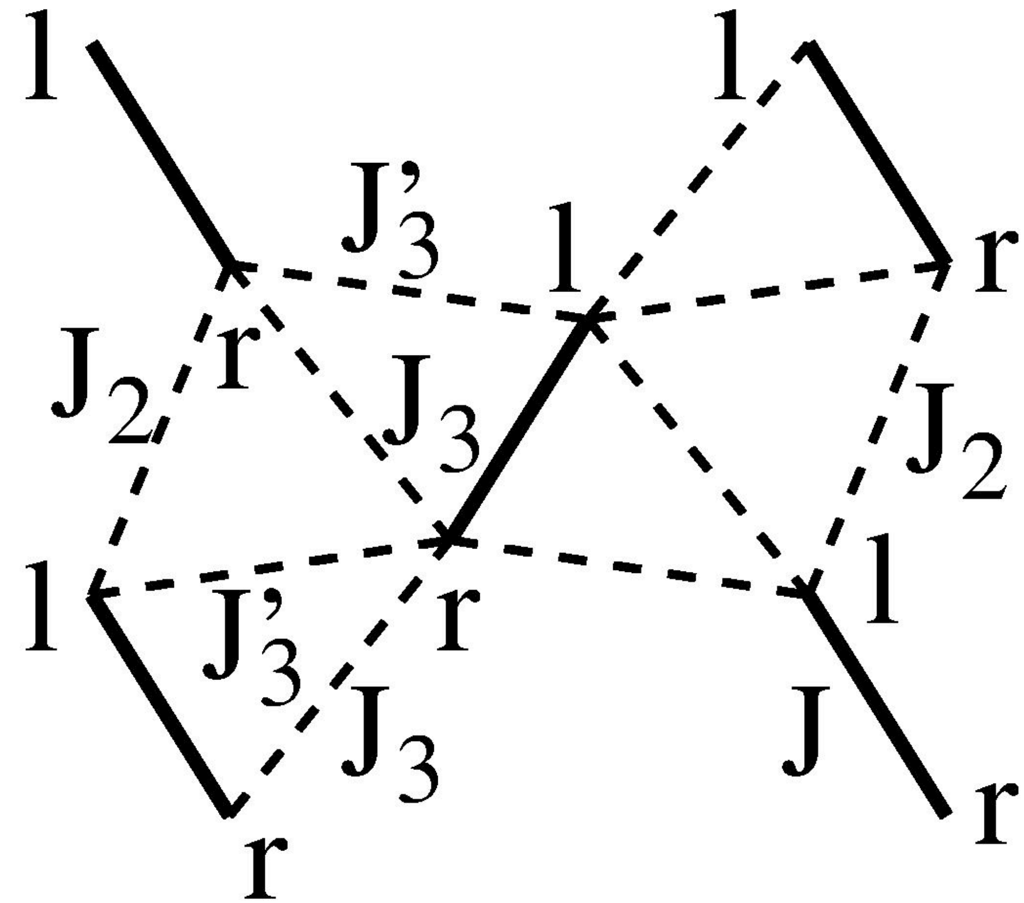

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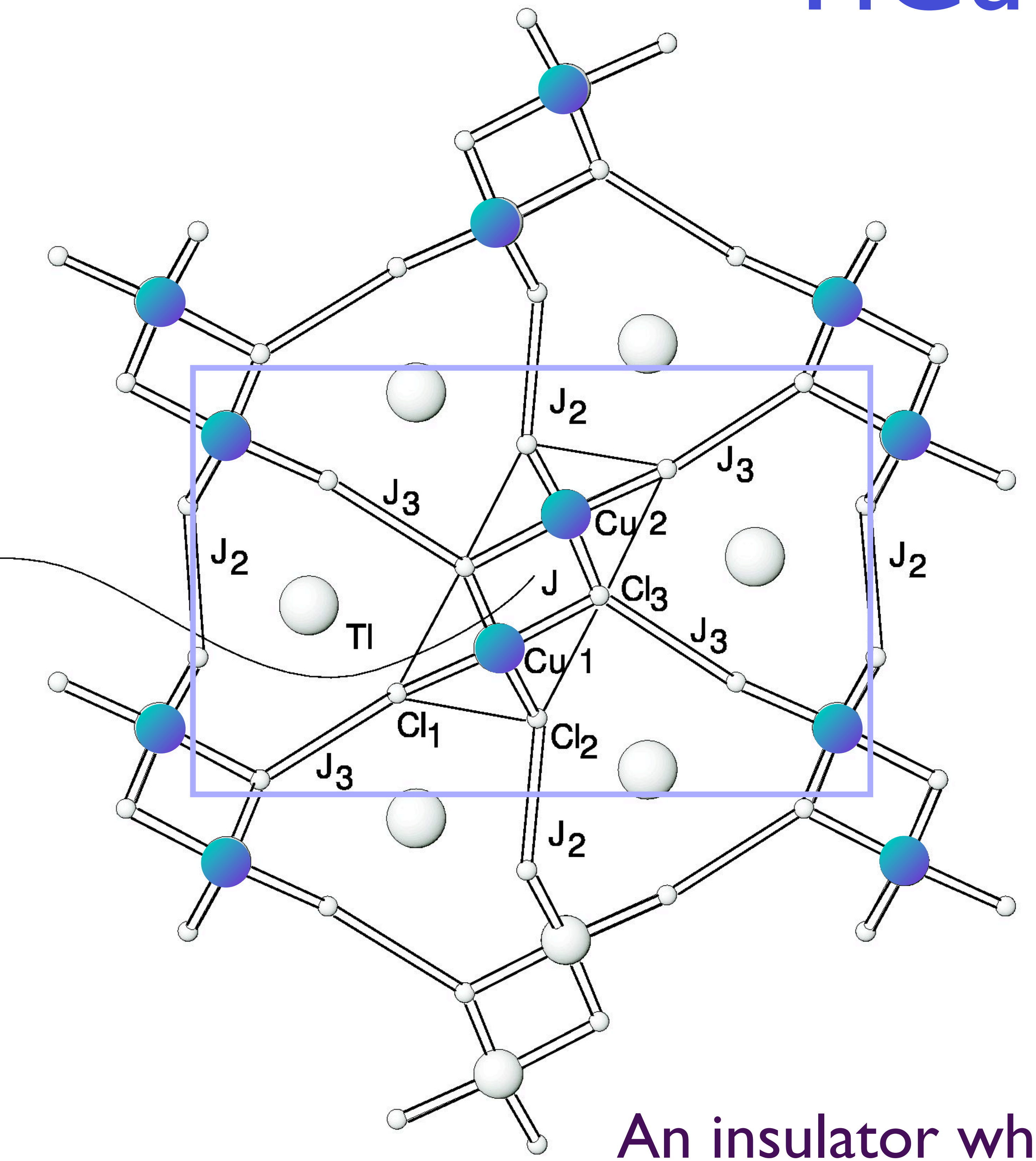
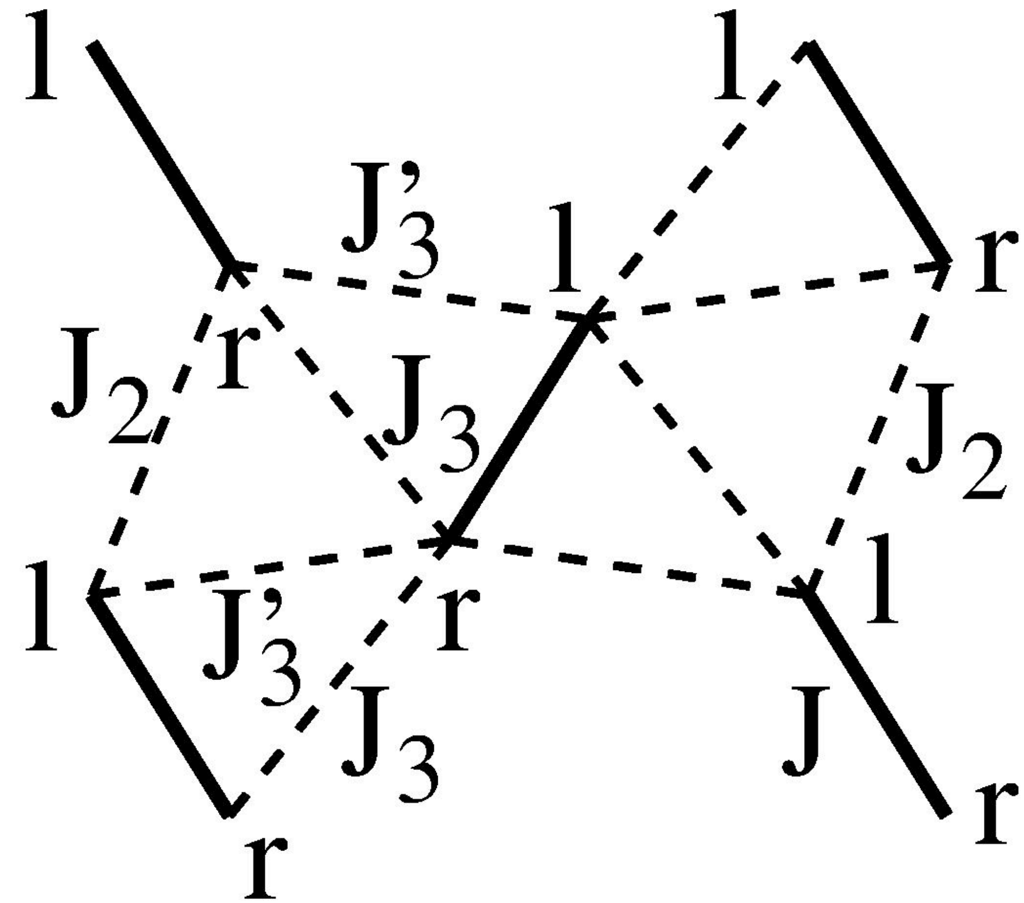
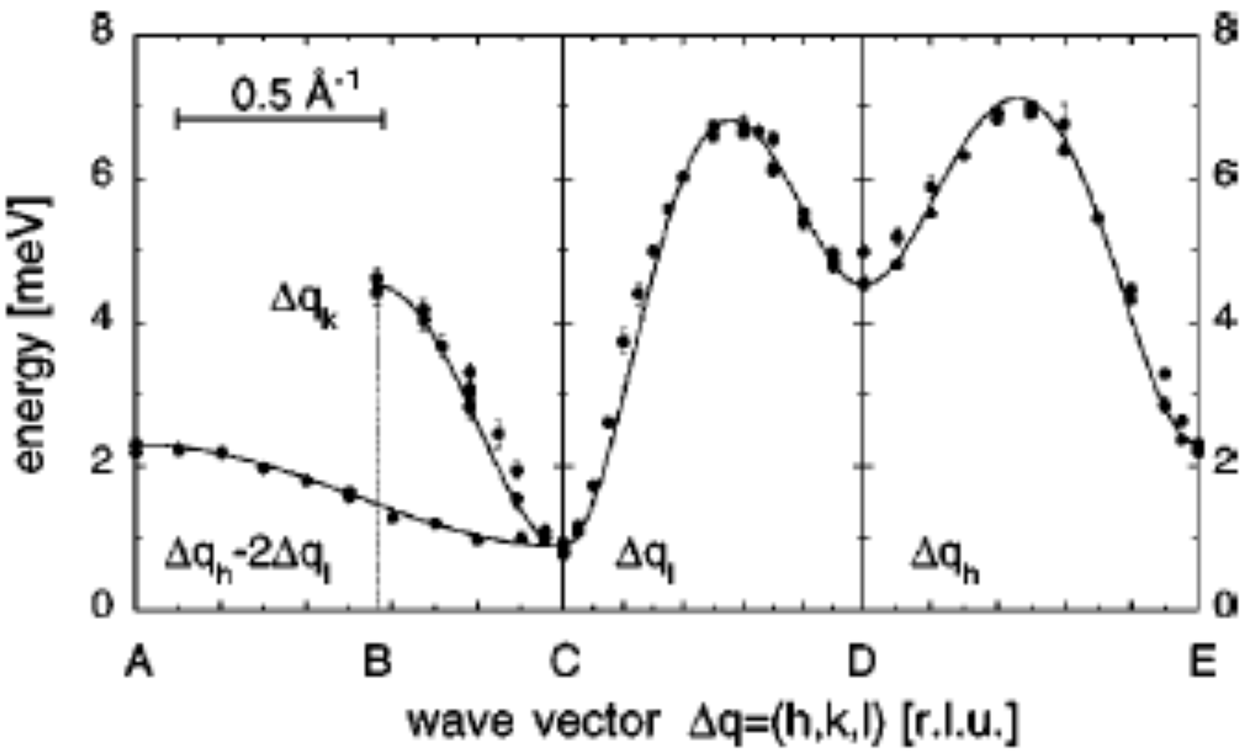
TlCuCl$_3$





An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

$TlCuCl_3$ at ambient pressure



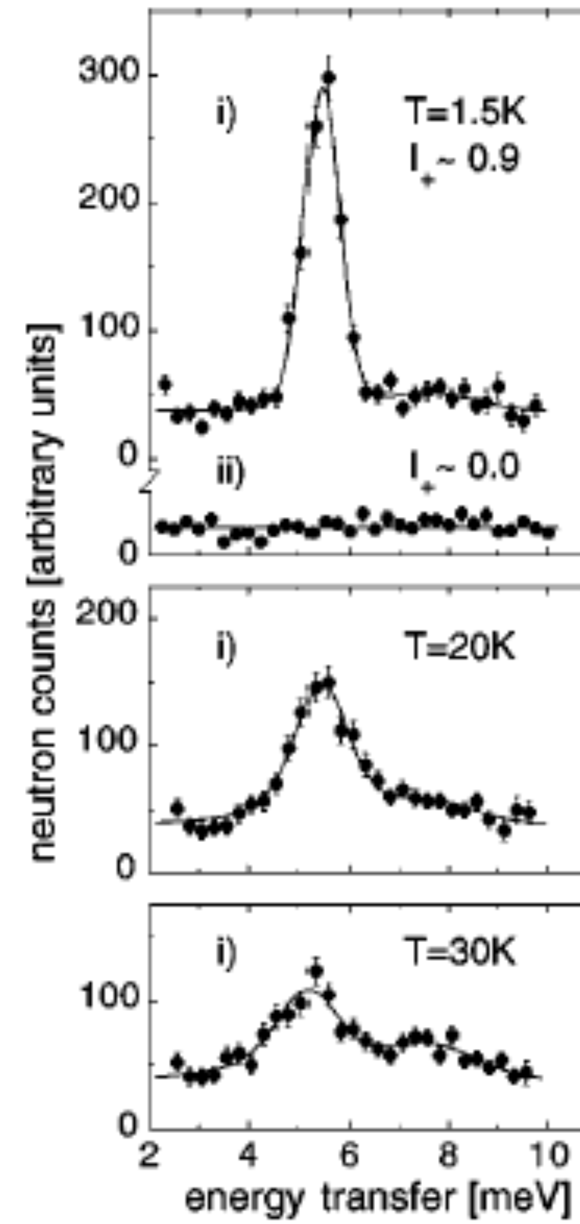
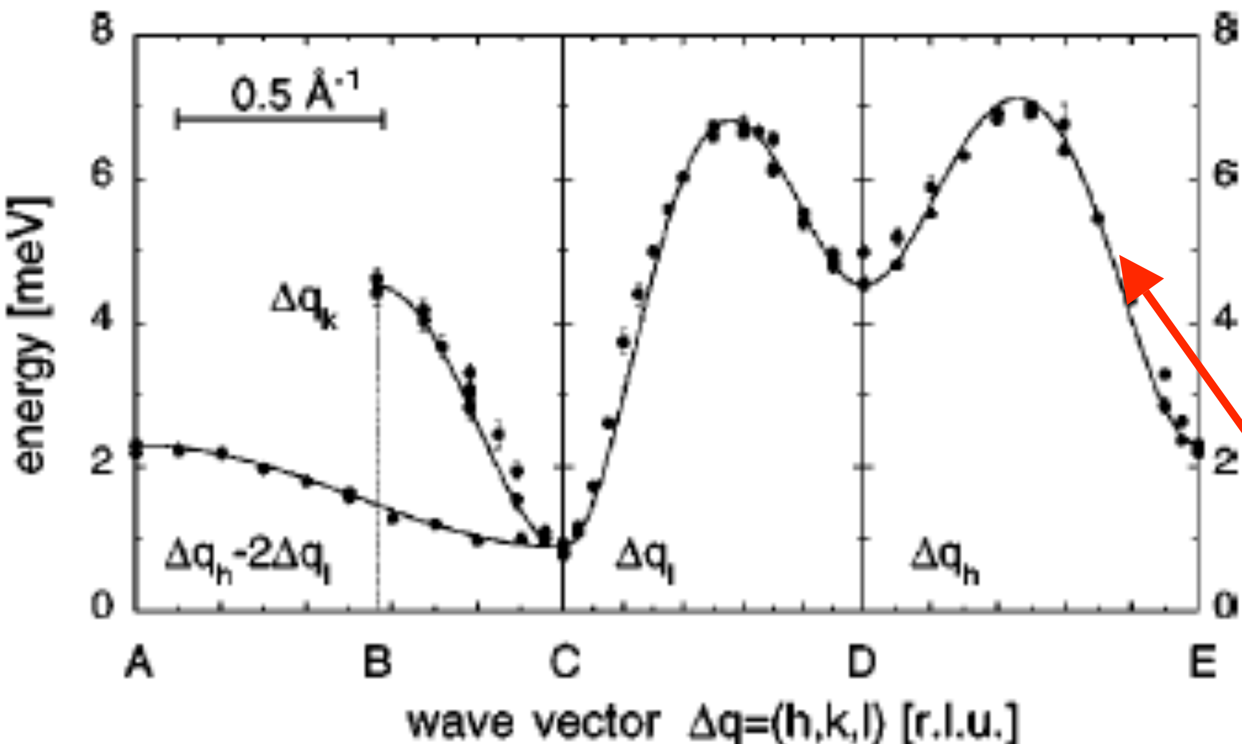
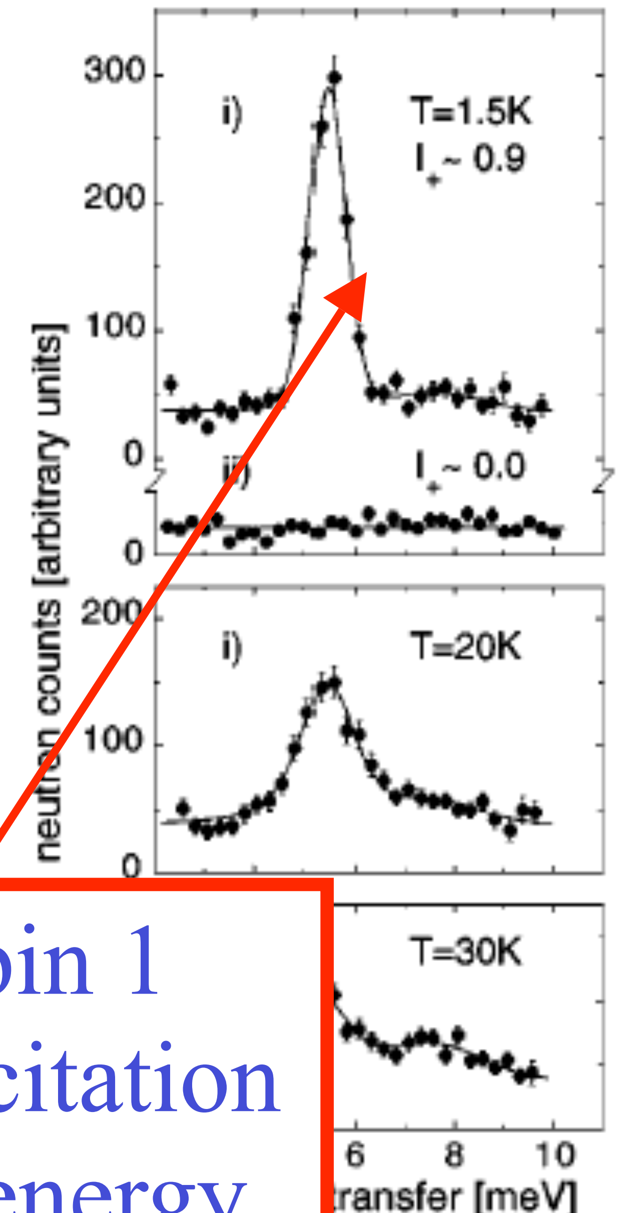


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

$TlCuCl_3$ at ambient pressure





Sharp spin 1 particle excitation above an energy gap (spin gap)

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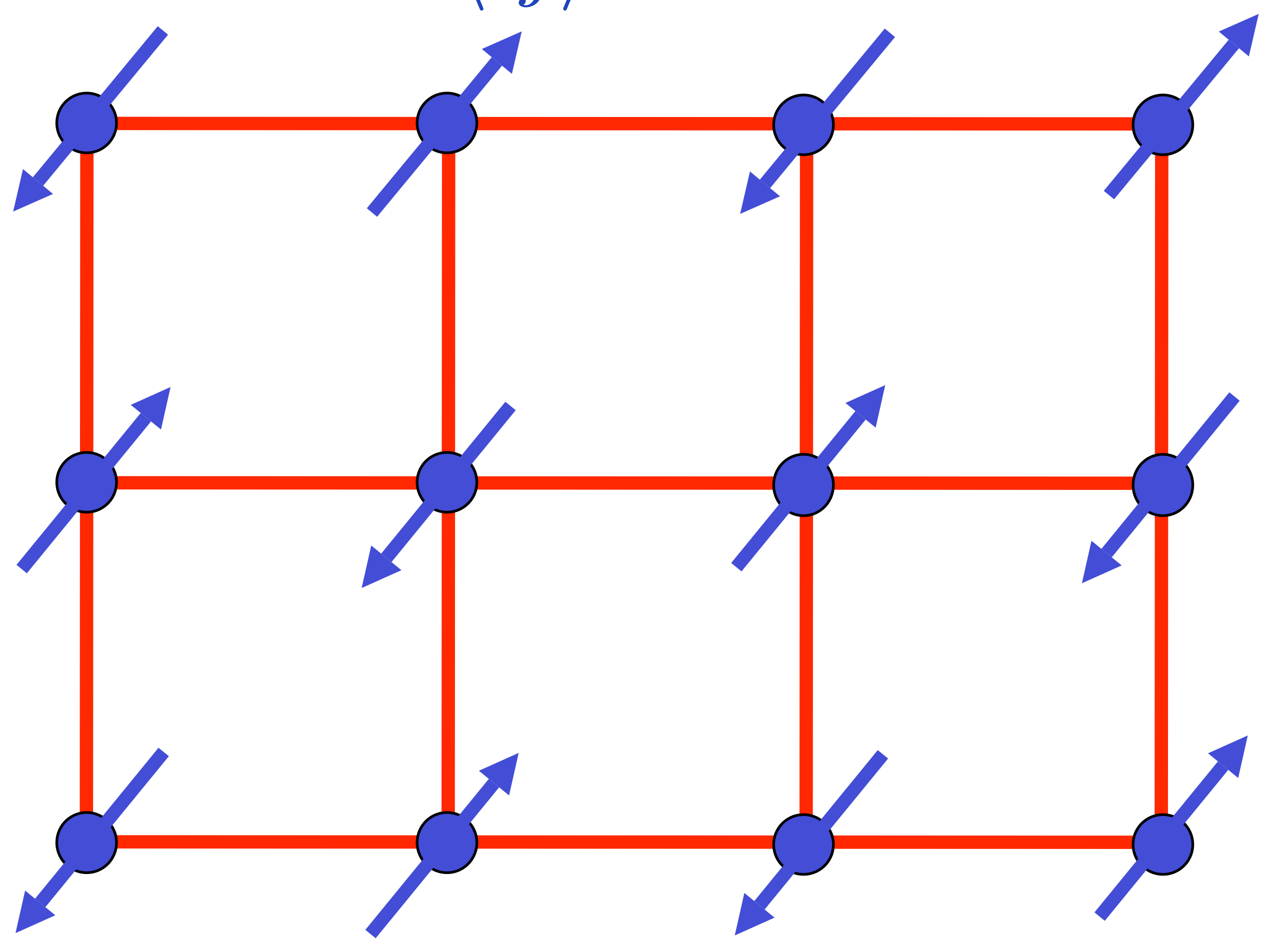
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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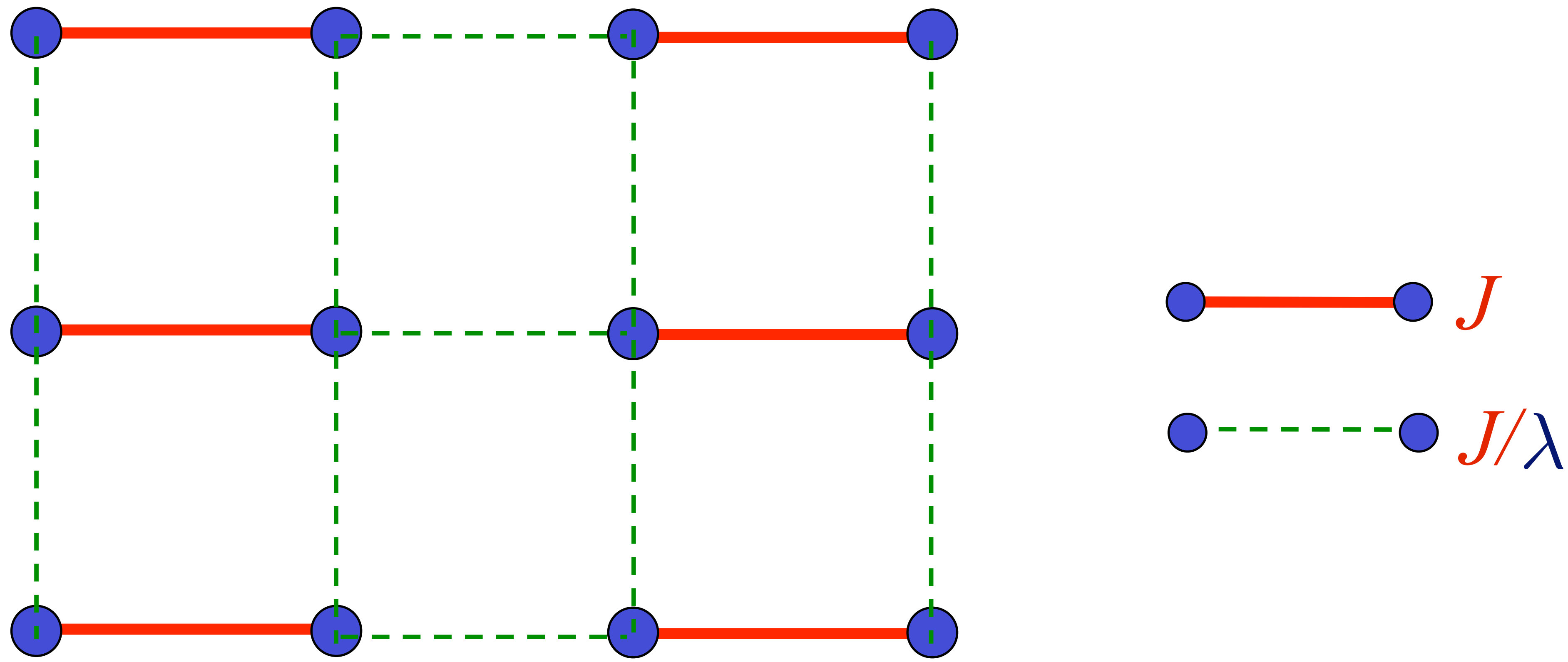
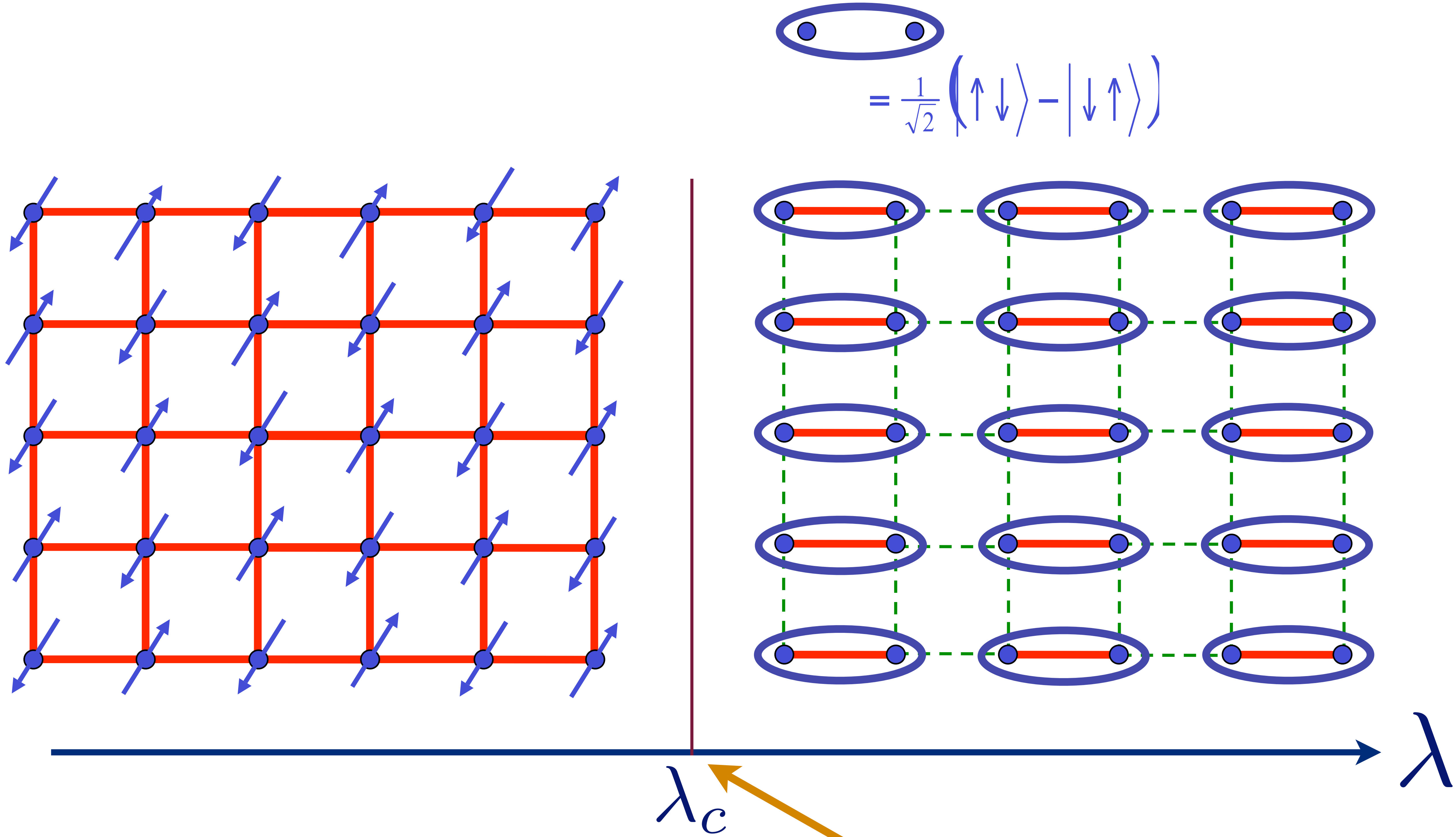
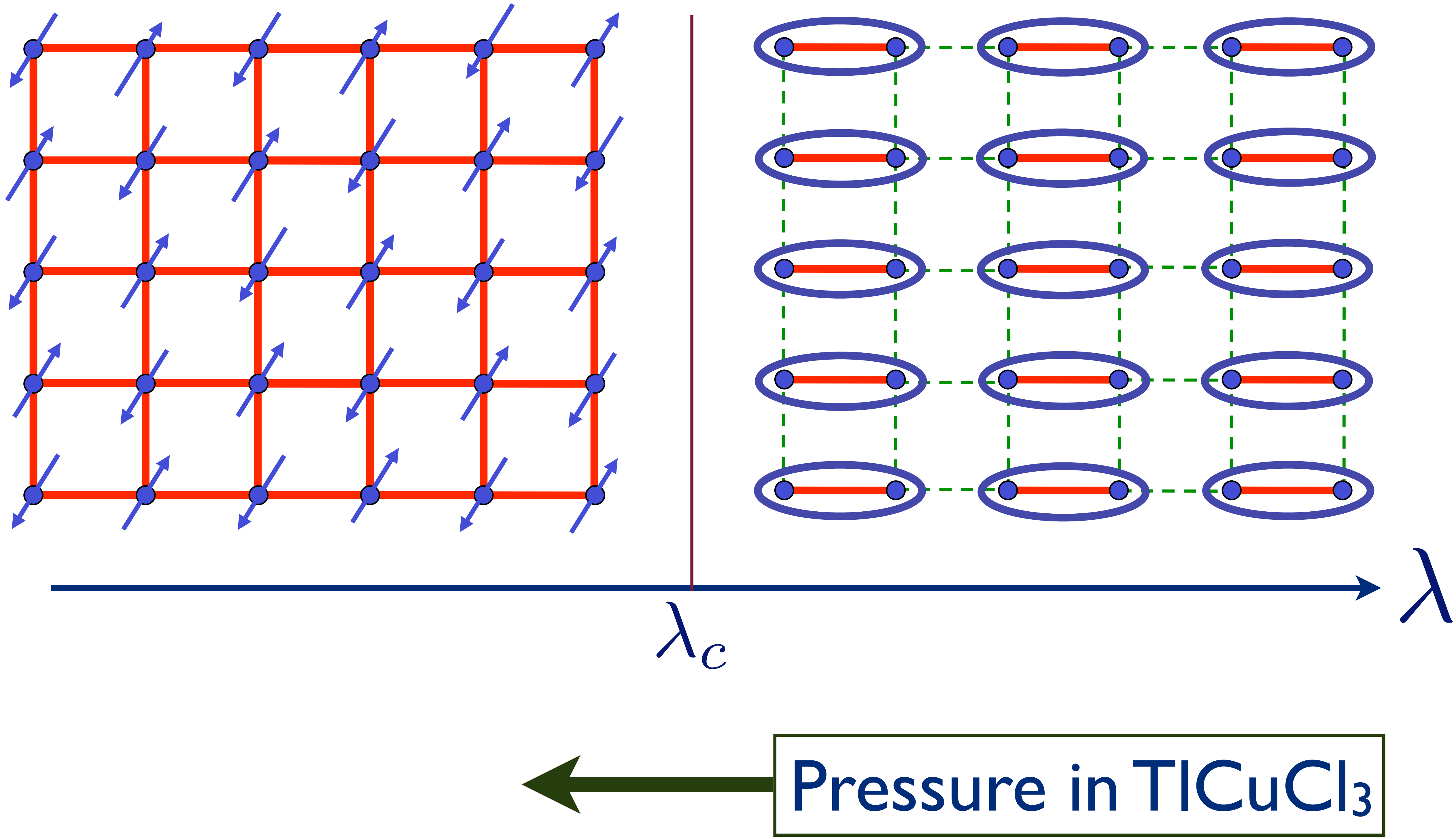


Weaken some bonds to induce spin entanglement in a new quantum phase

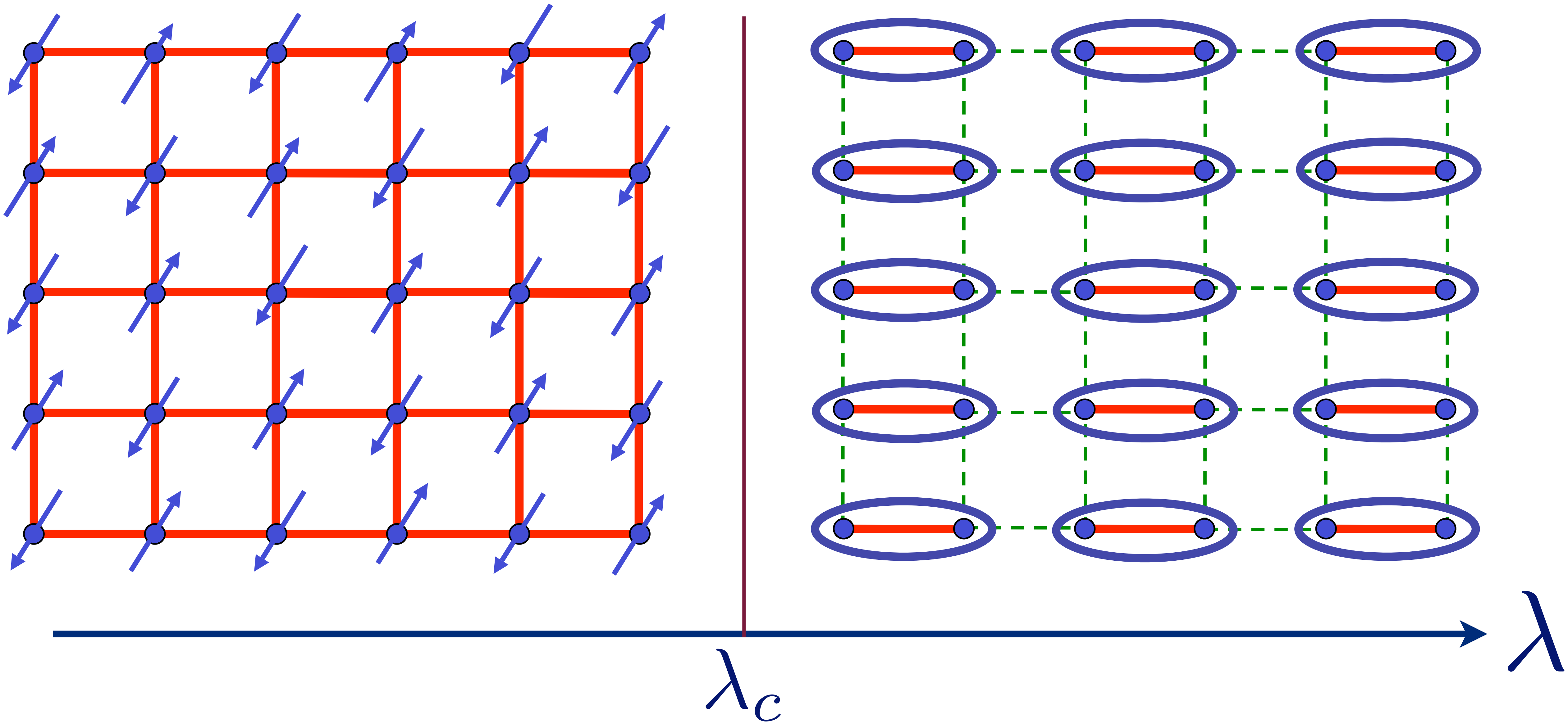


Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).


λ
λ_c
Pressure in $TlCuCl_3$

Excitation spectrum in the Néel phase



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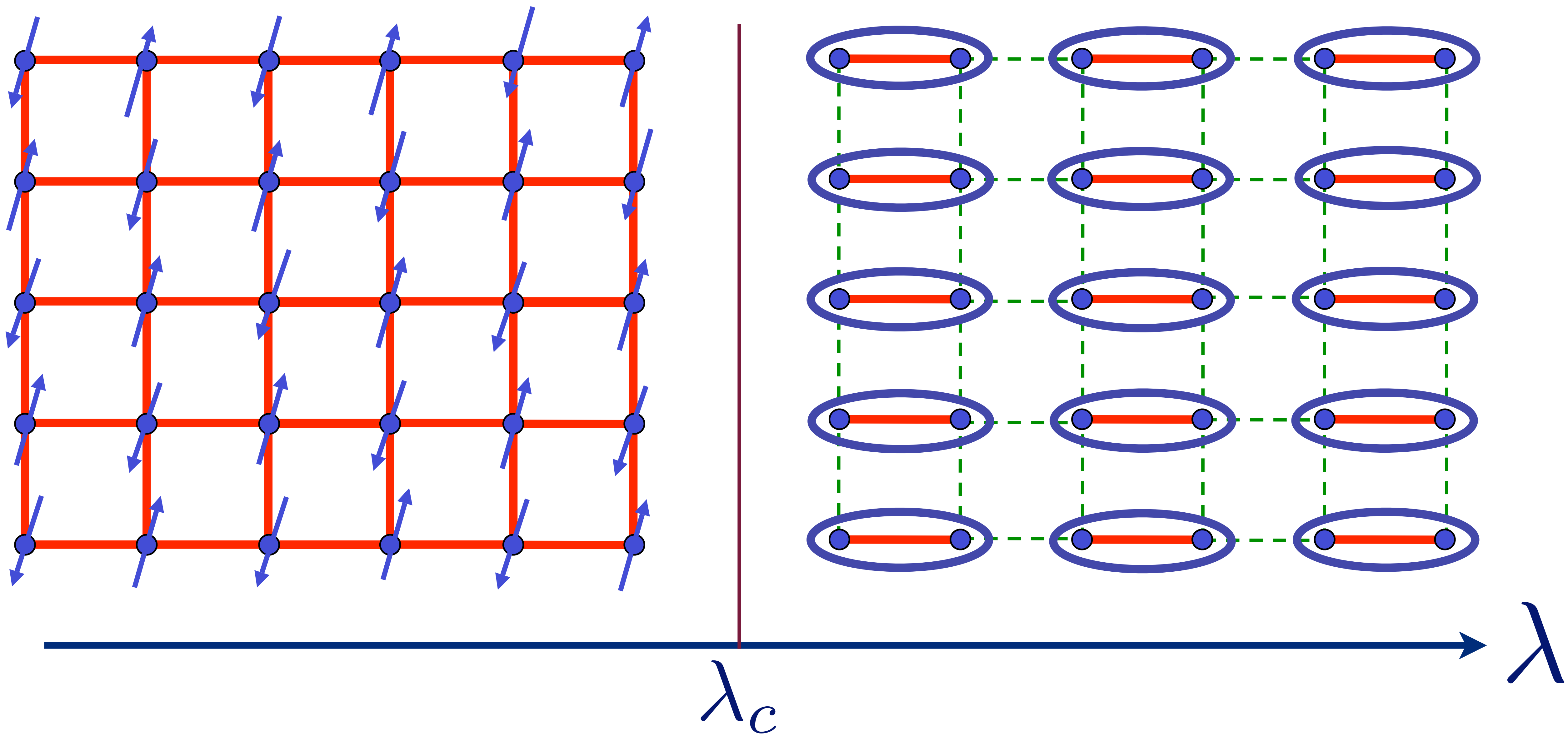
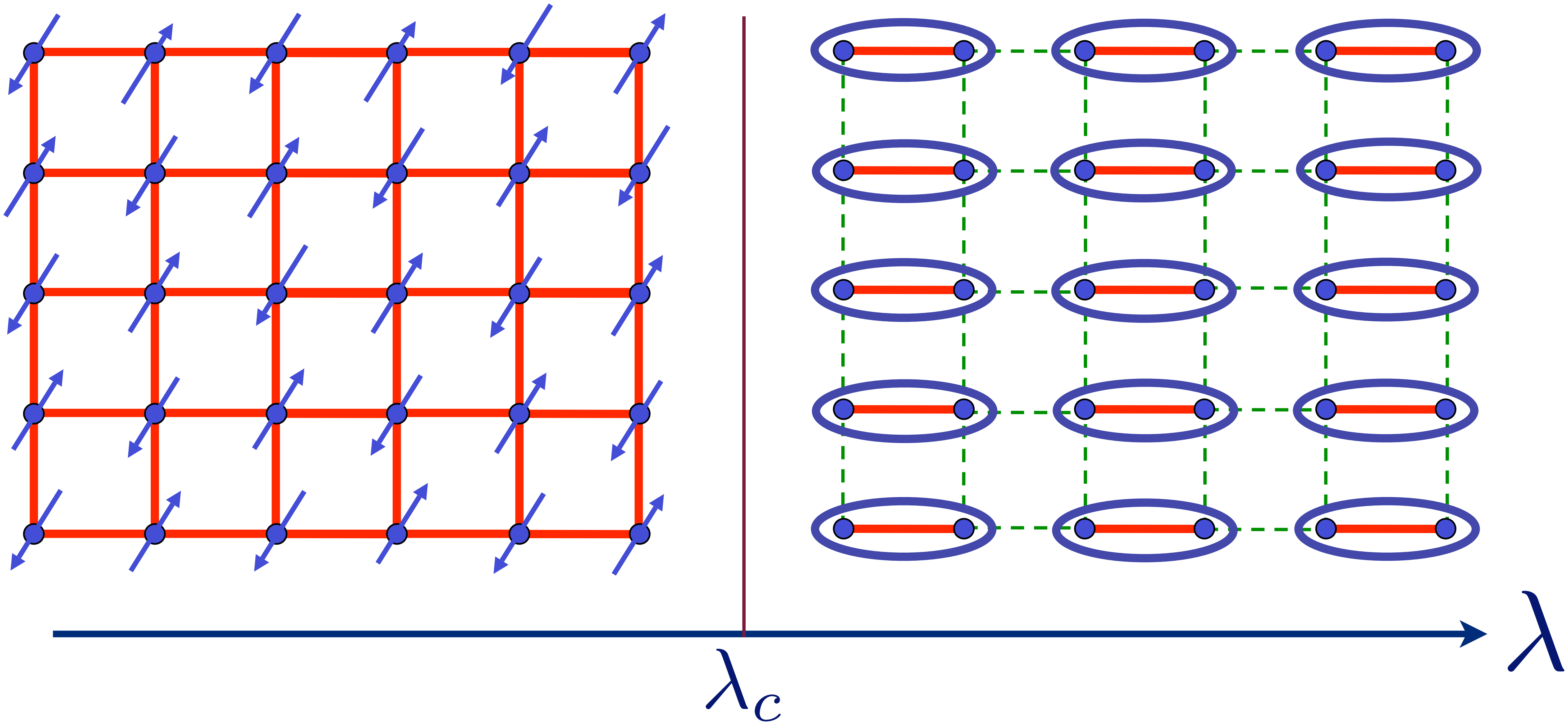


Spin waves

Excitation spectrum in the Néel phase



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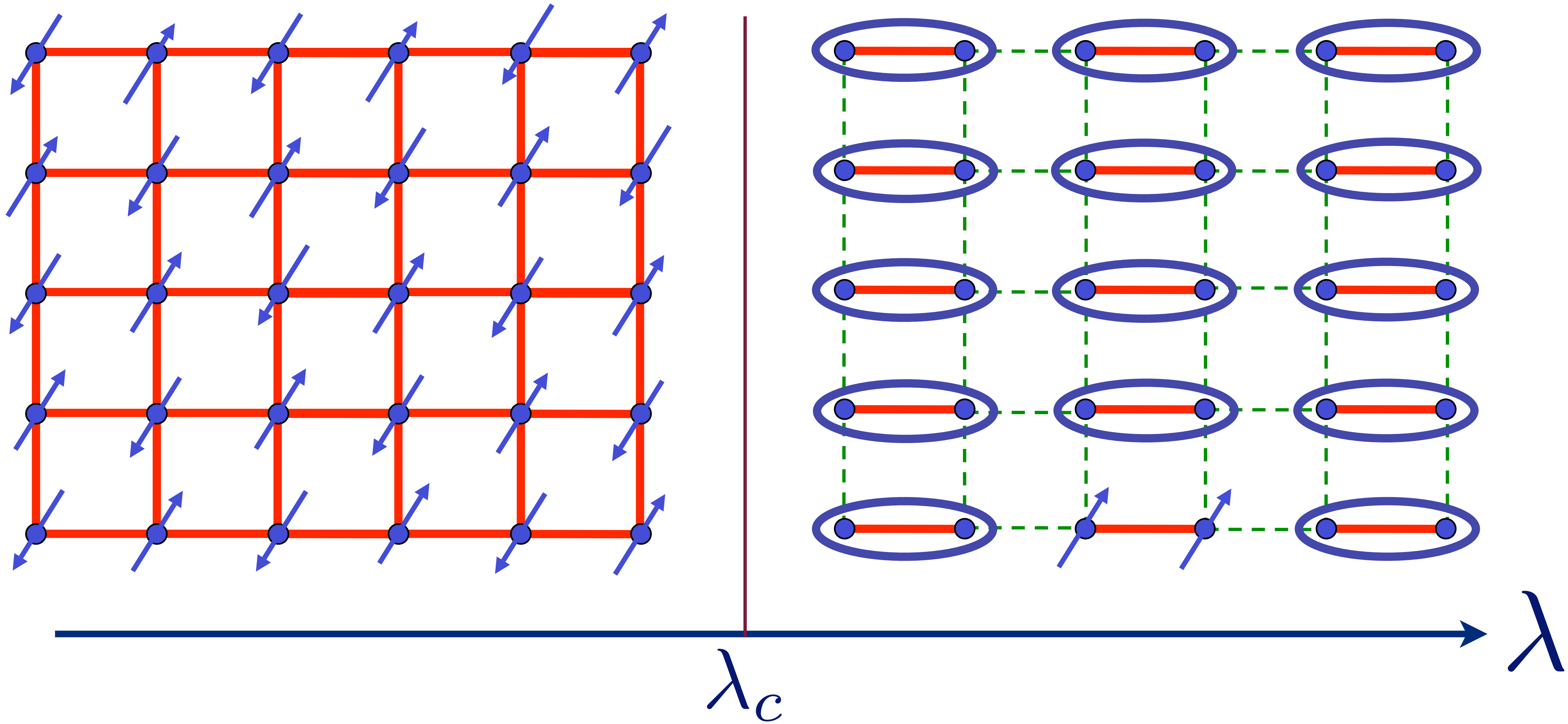
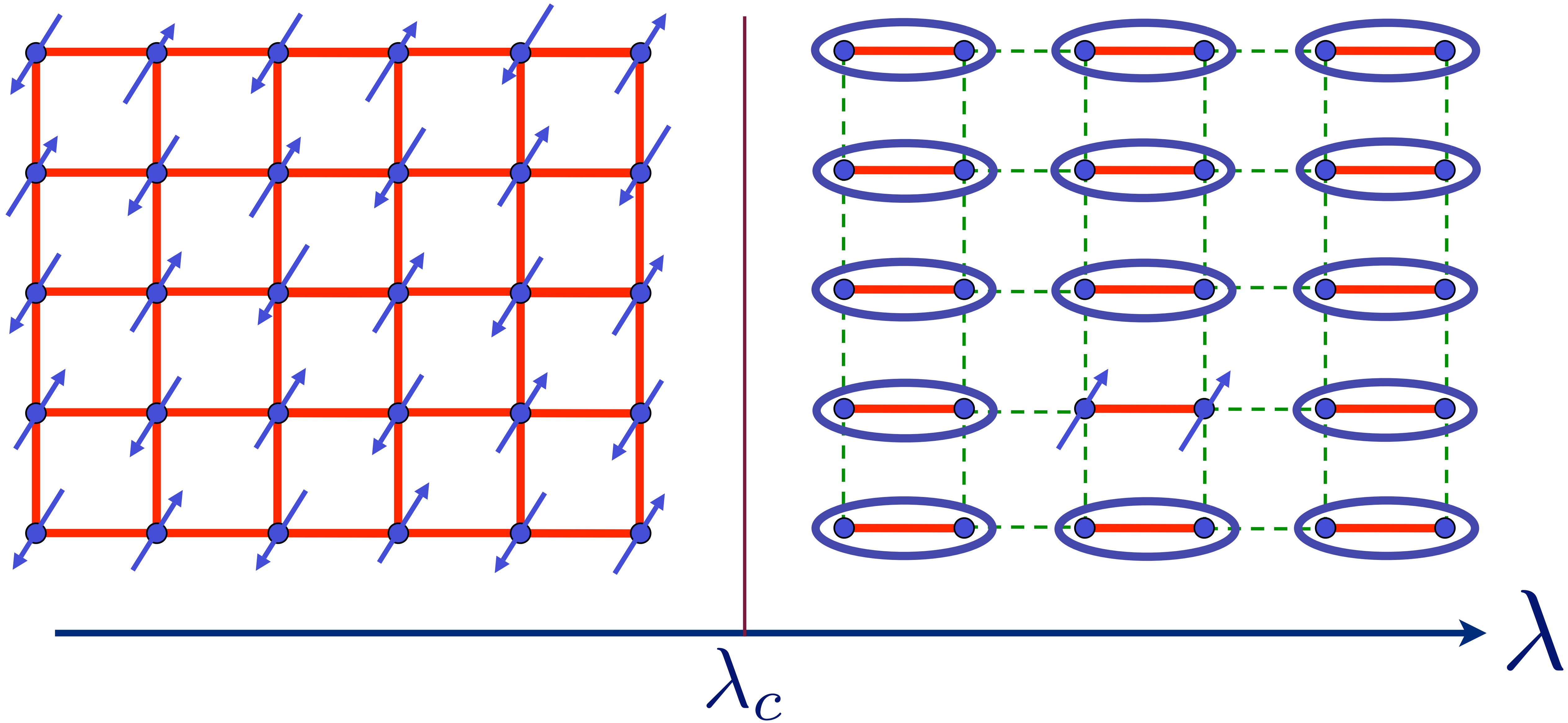
Excitation spectrum in the paramagnetic phase



Excitation spectrum in the paramagnetic phase



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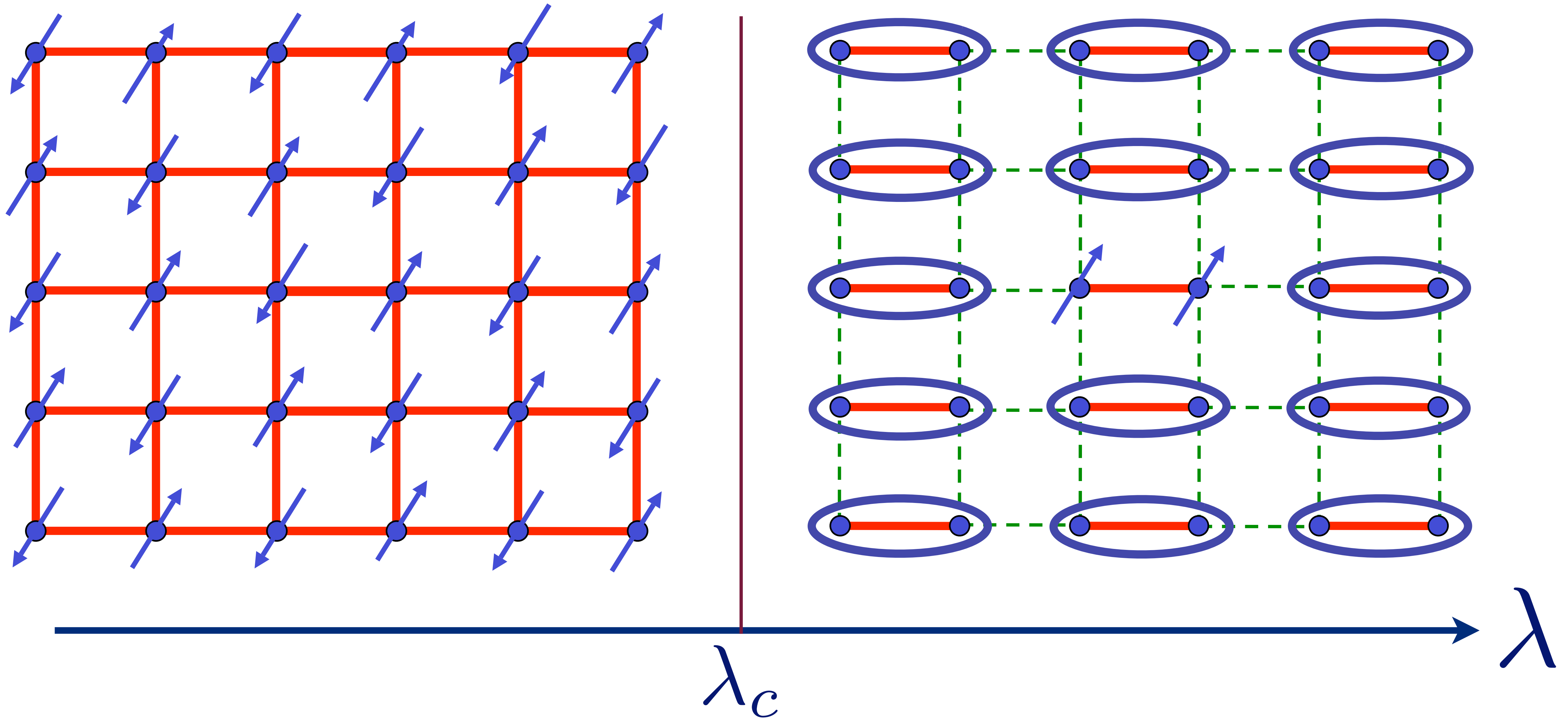

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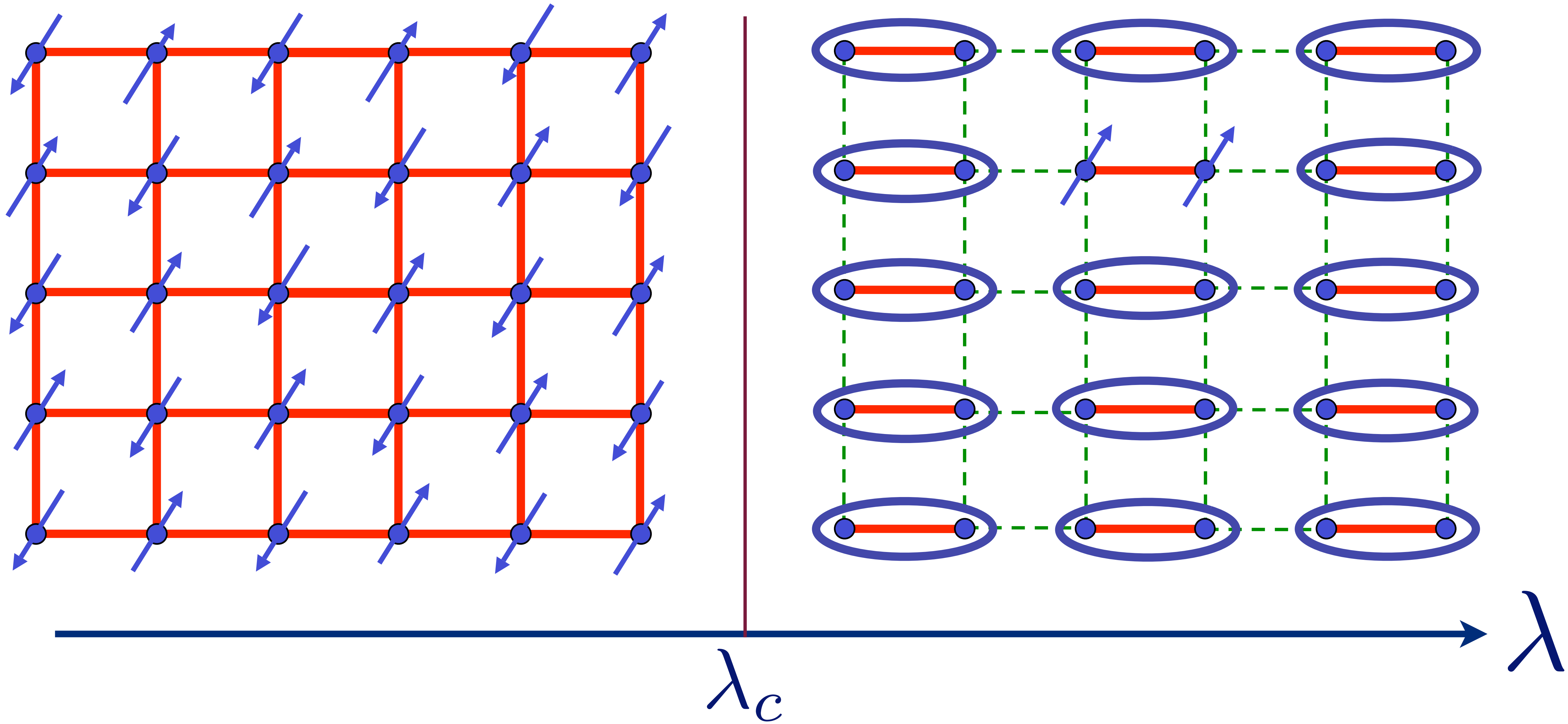

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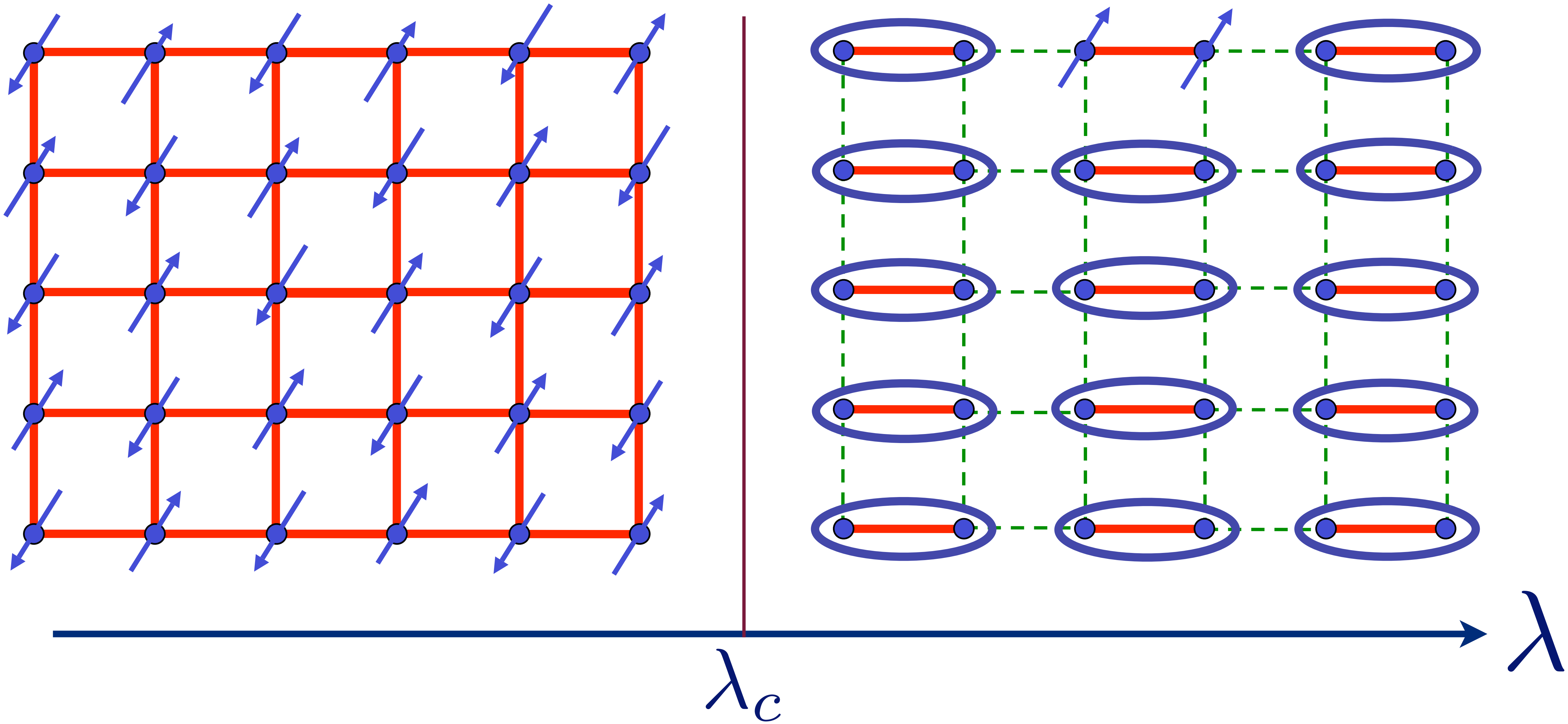
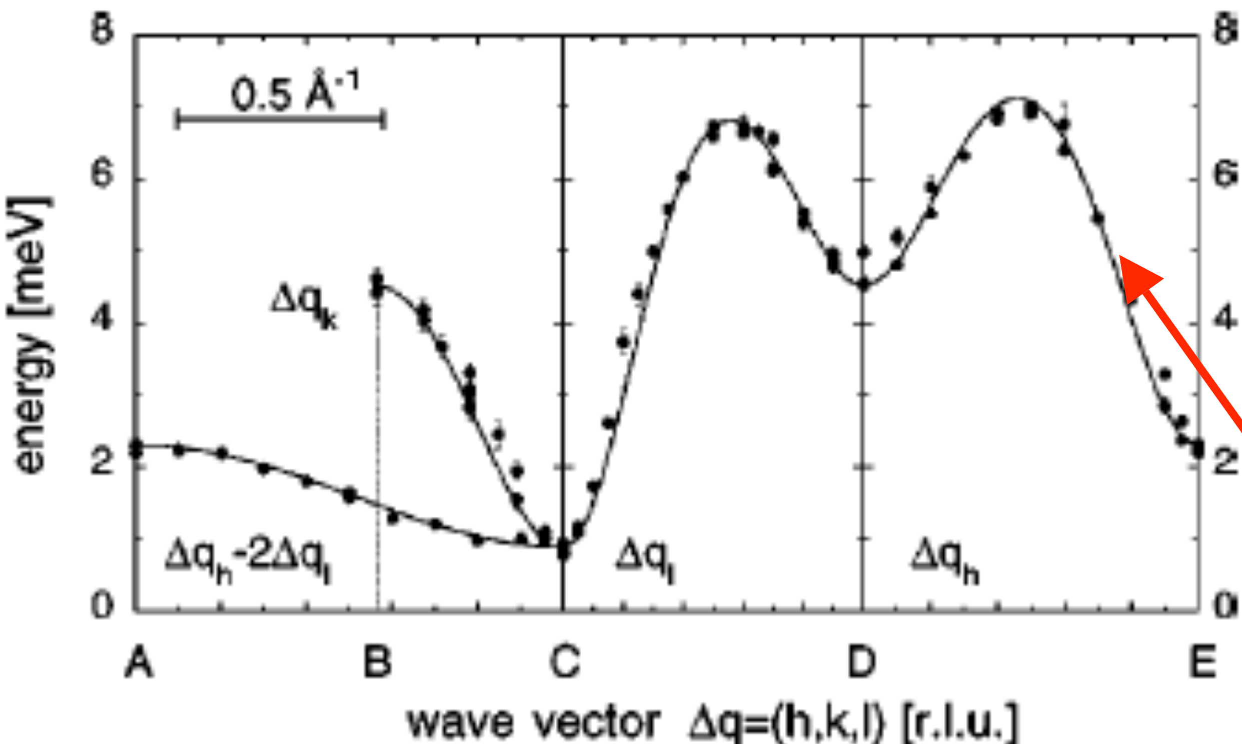
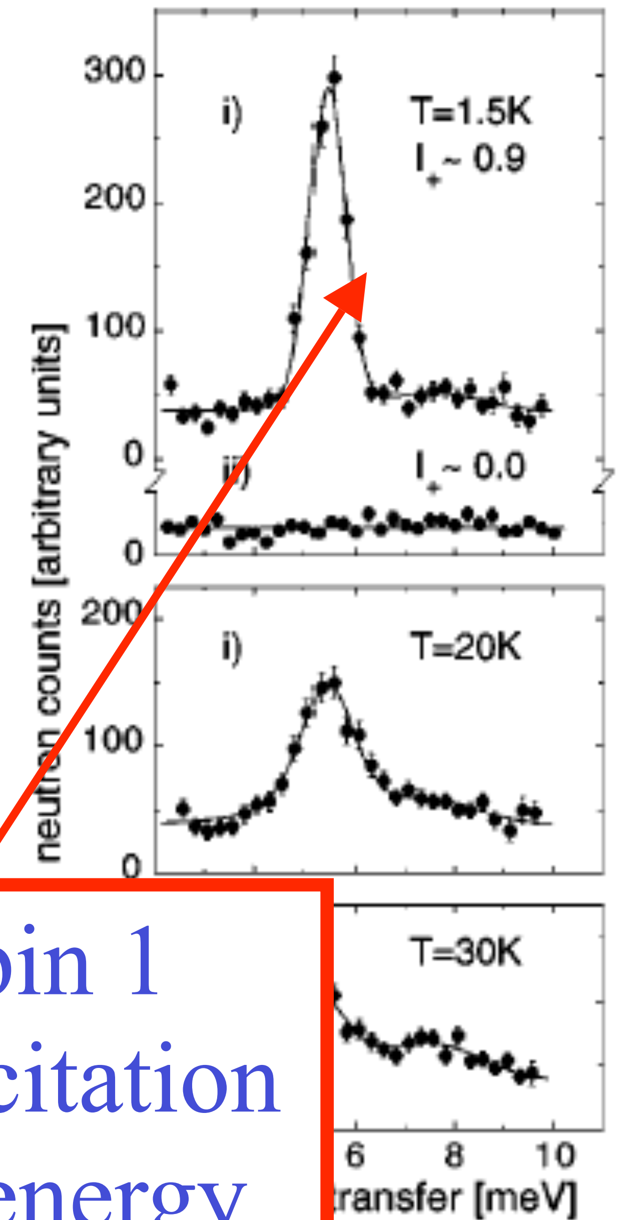


Sharp spin 1 particle excitation above an energy gap (spin gap)

$TlCuCl_3$ at ambient pressure





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Discussion of bond operator method

http://qpt.physics.harvard.edu/leshouches/bondoperators.pdf

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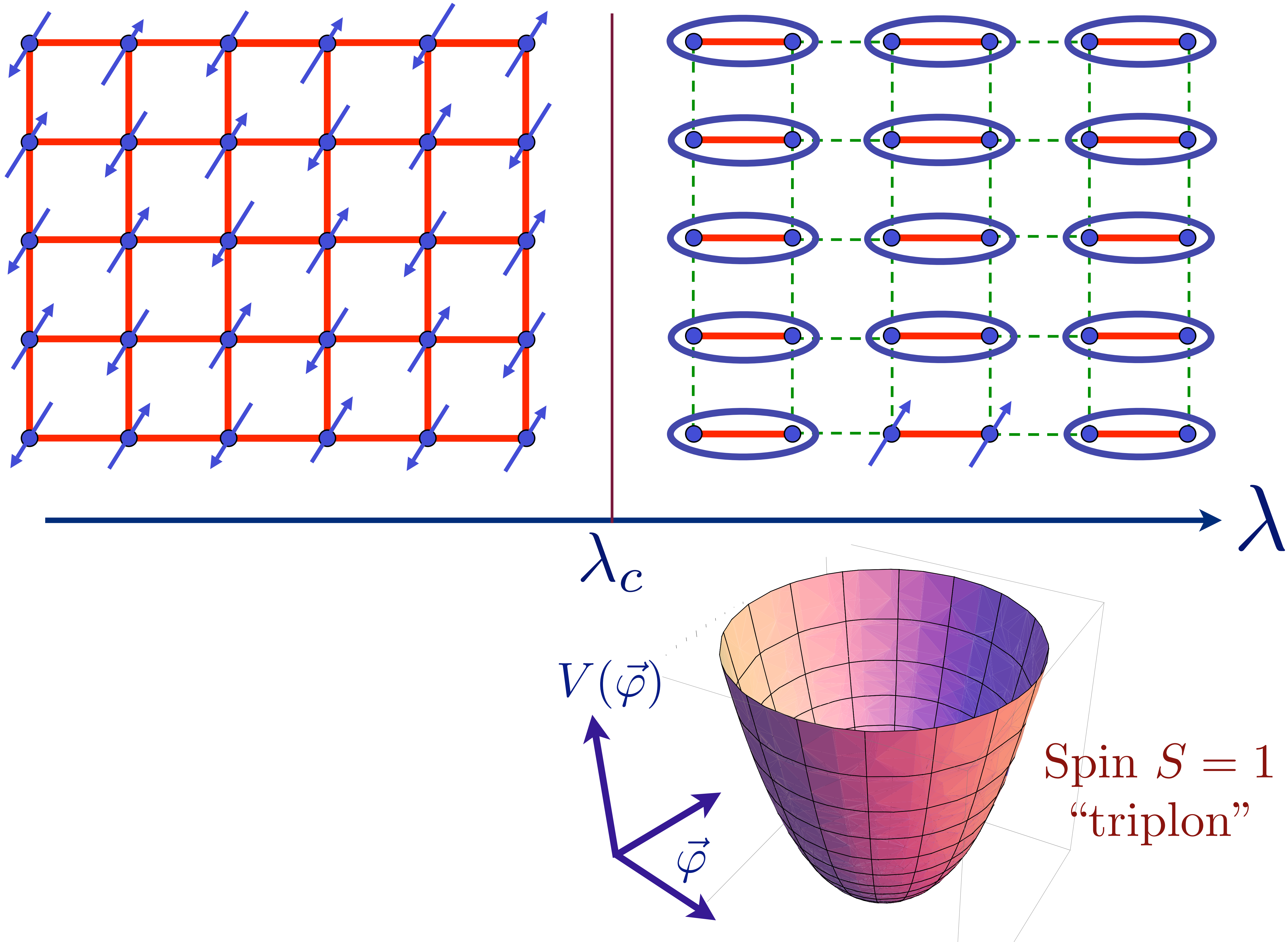

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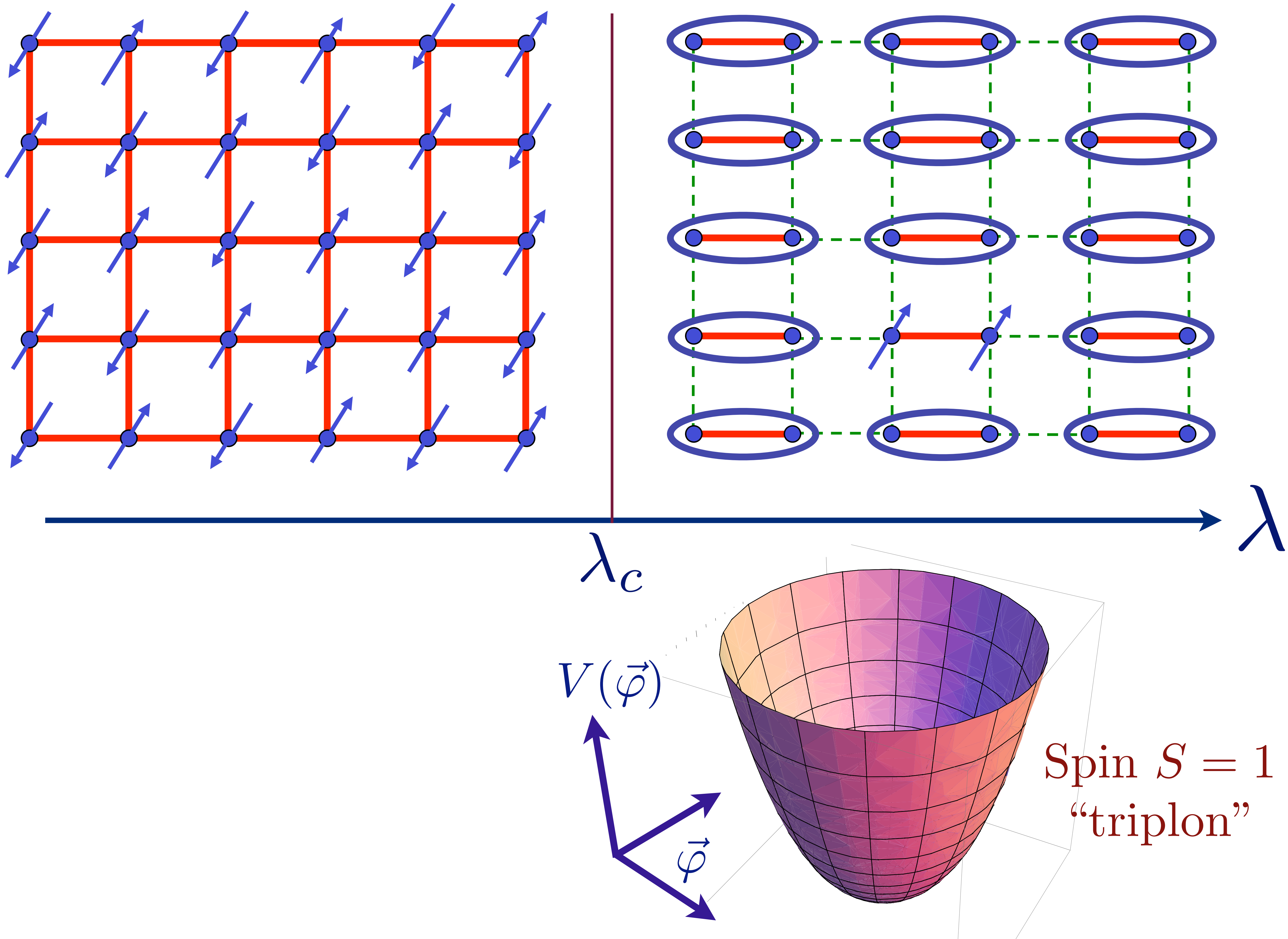

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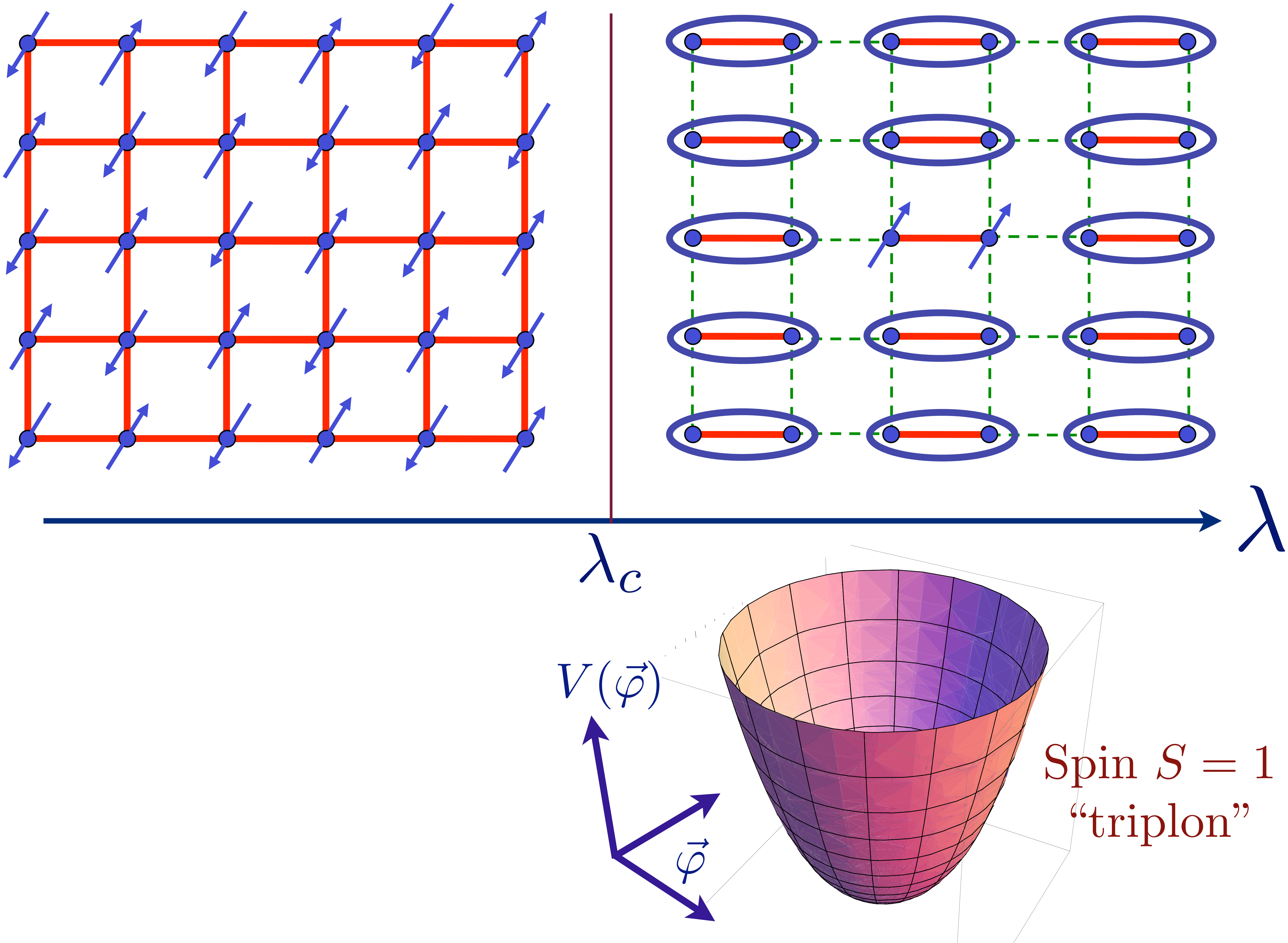

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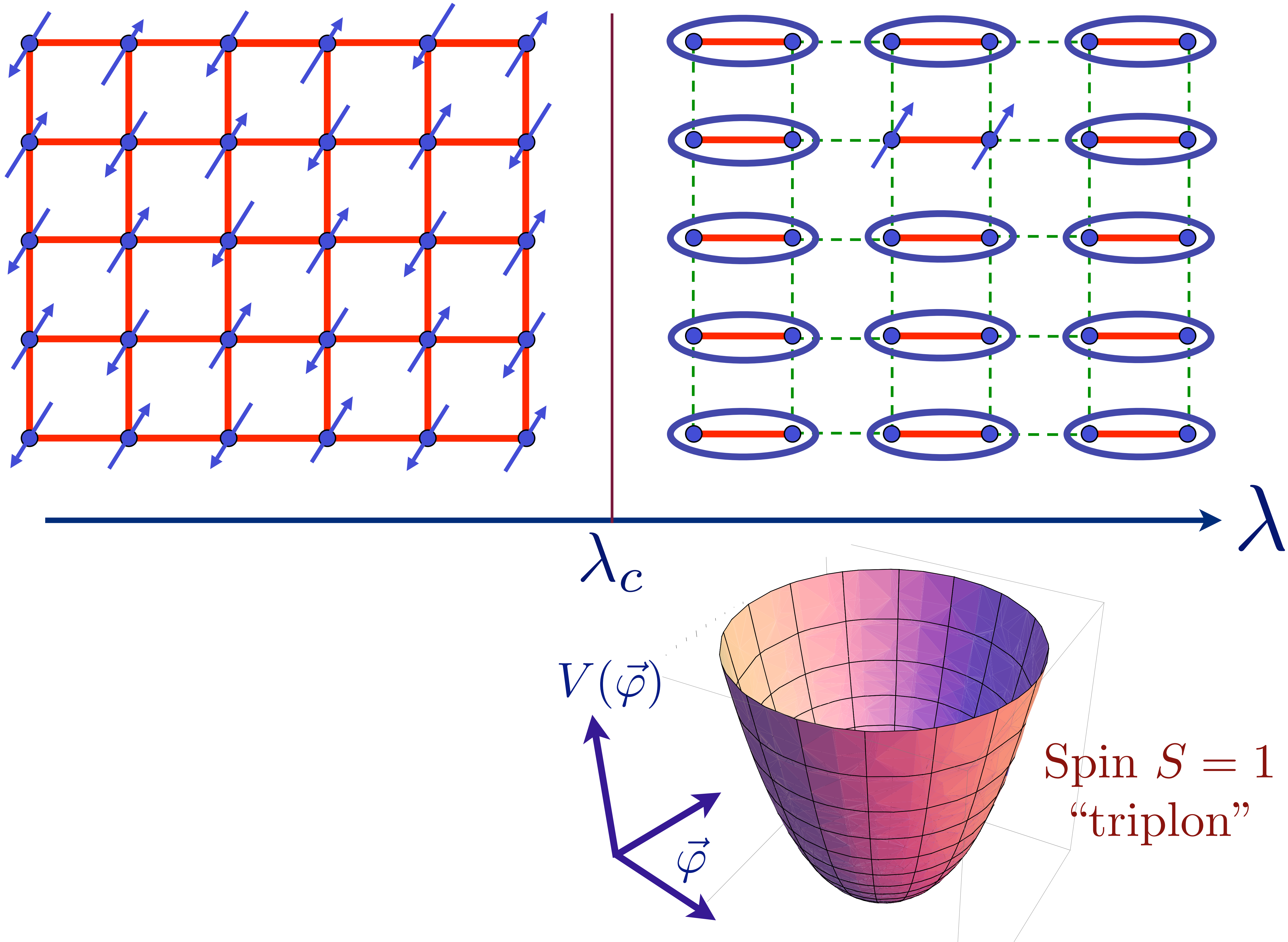

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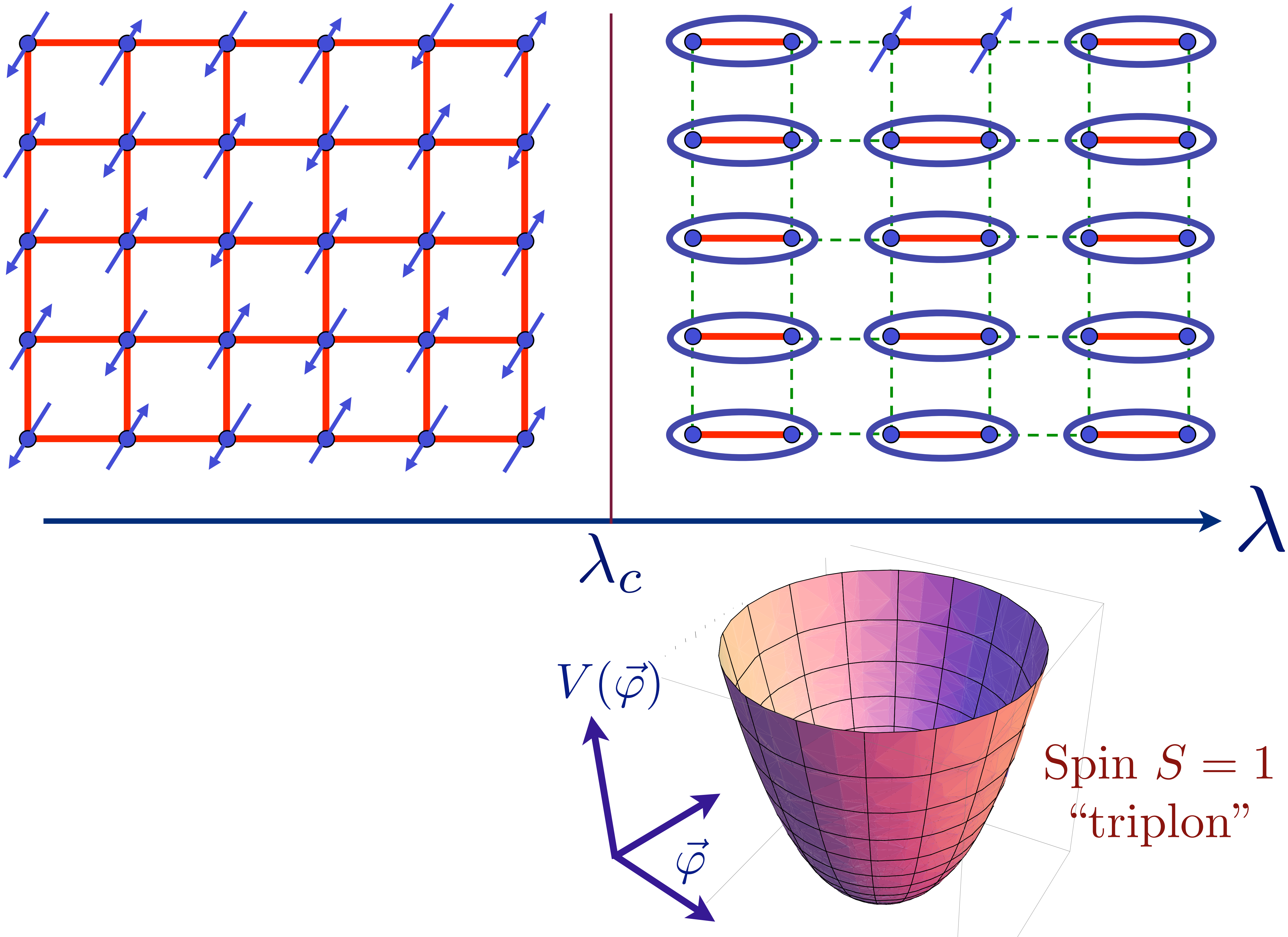

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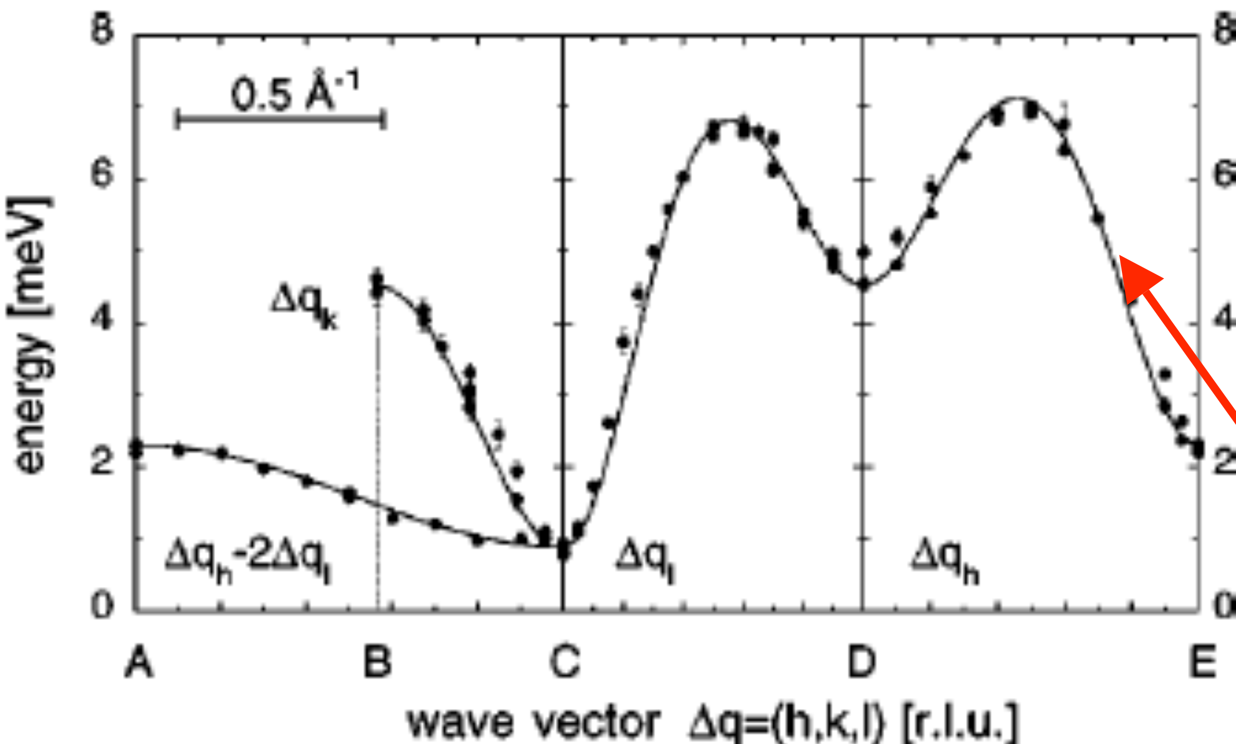

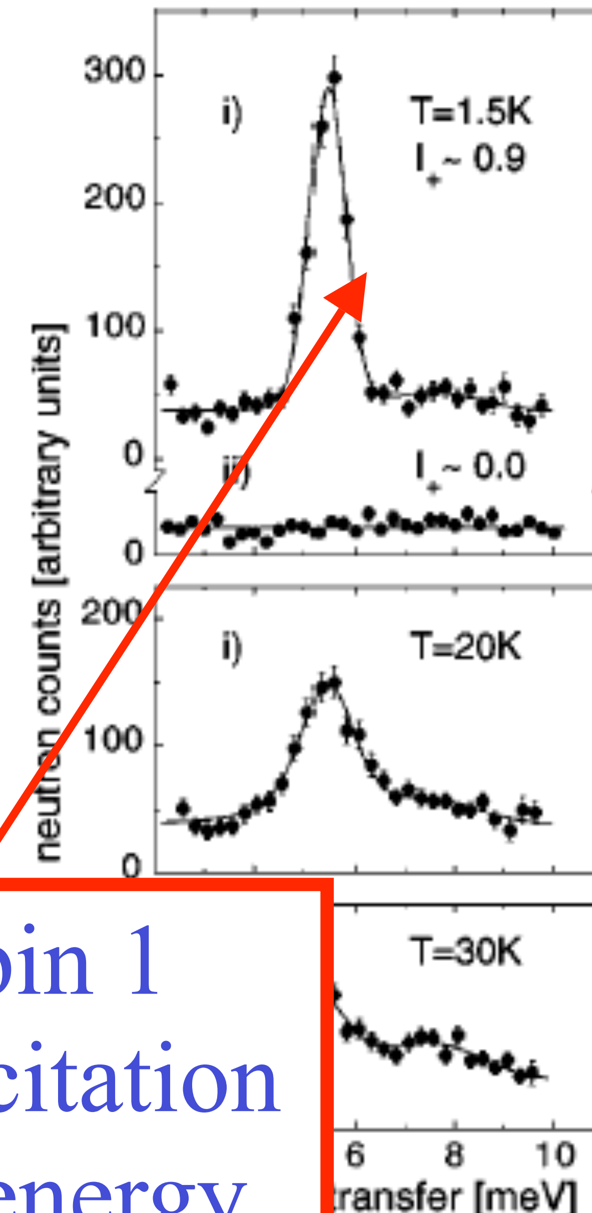

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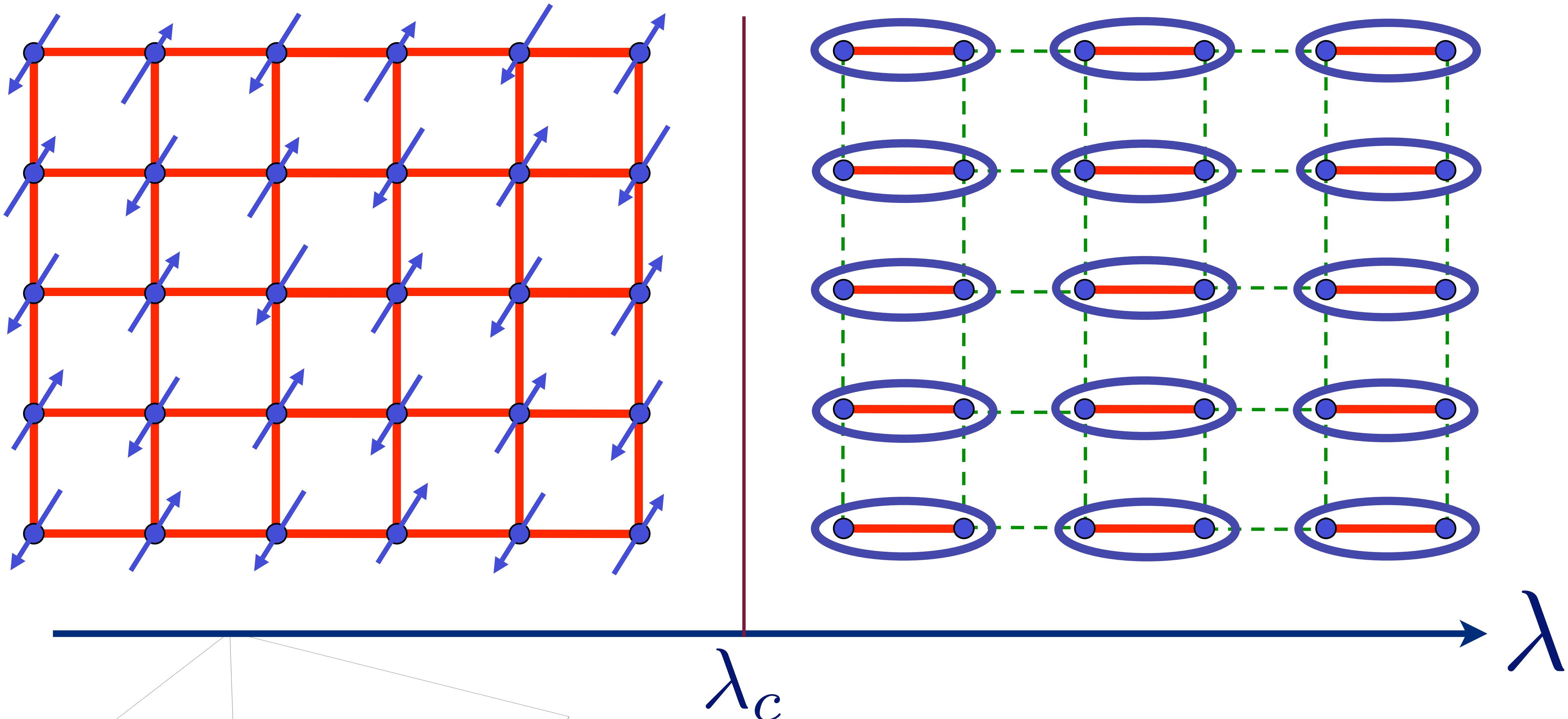

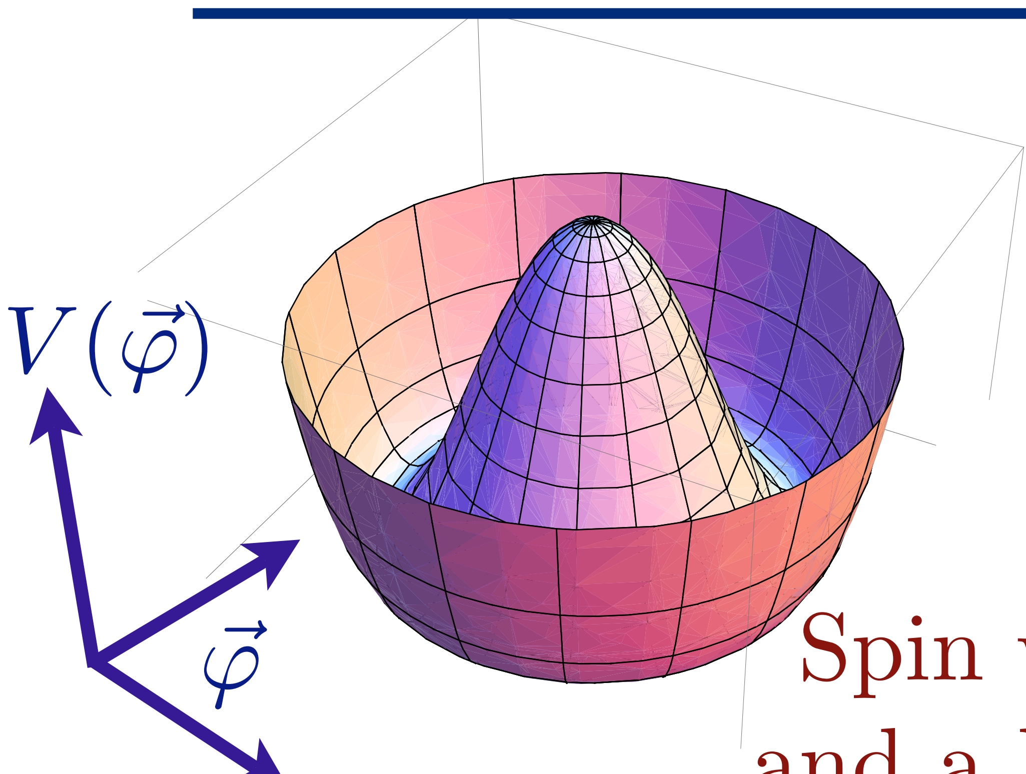

Spin waves (“Goldstone” modes)
and a longitudinal “Higgs” particle

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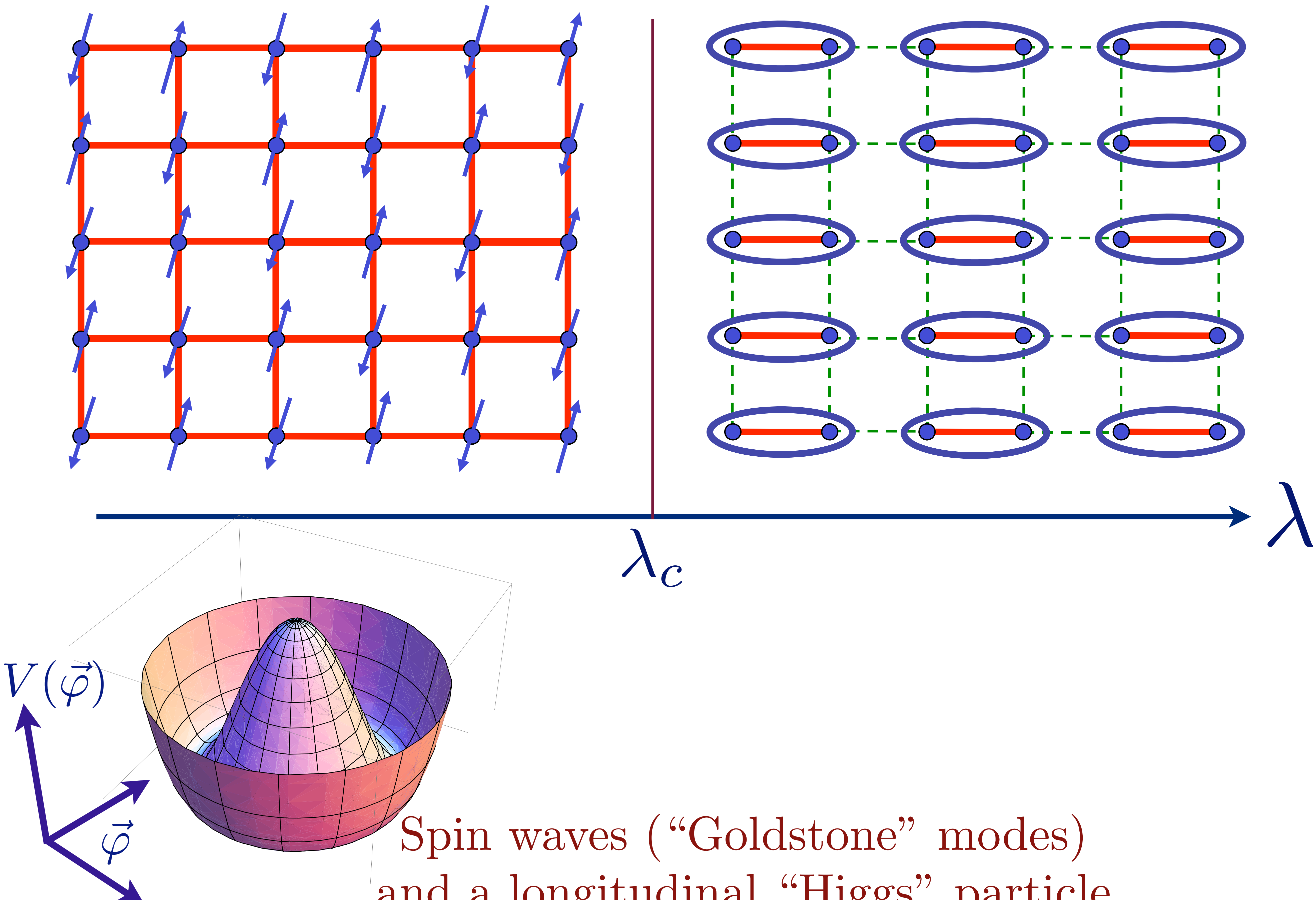

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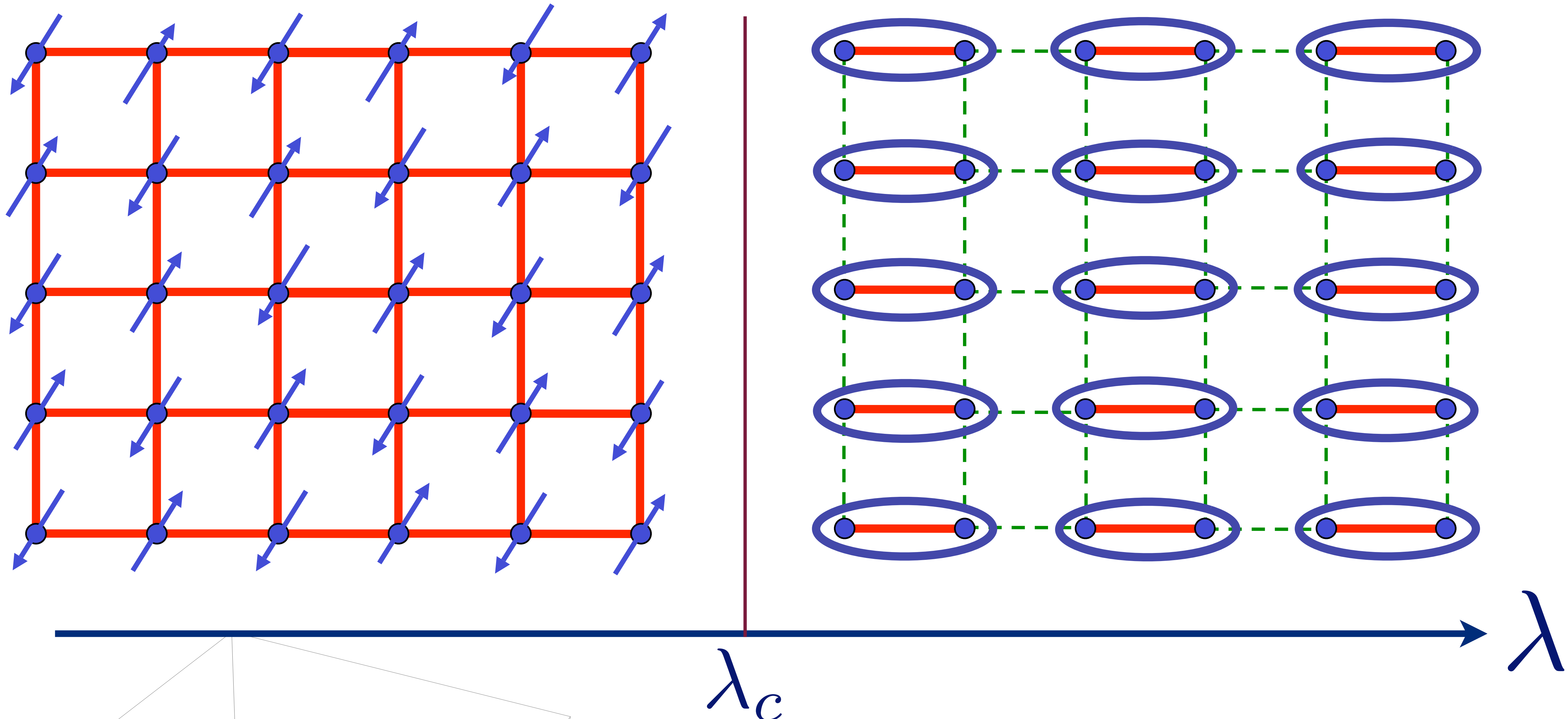
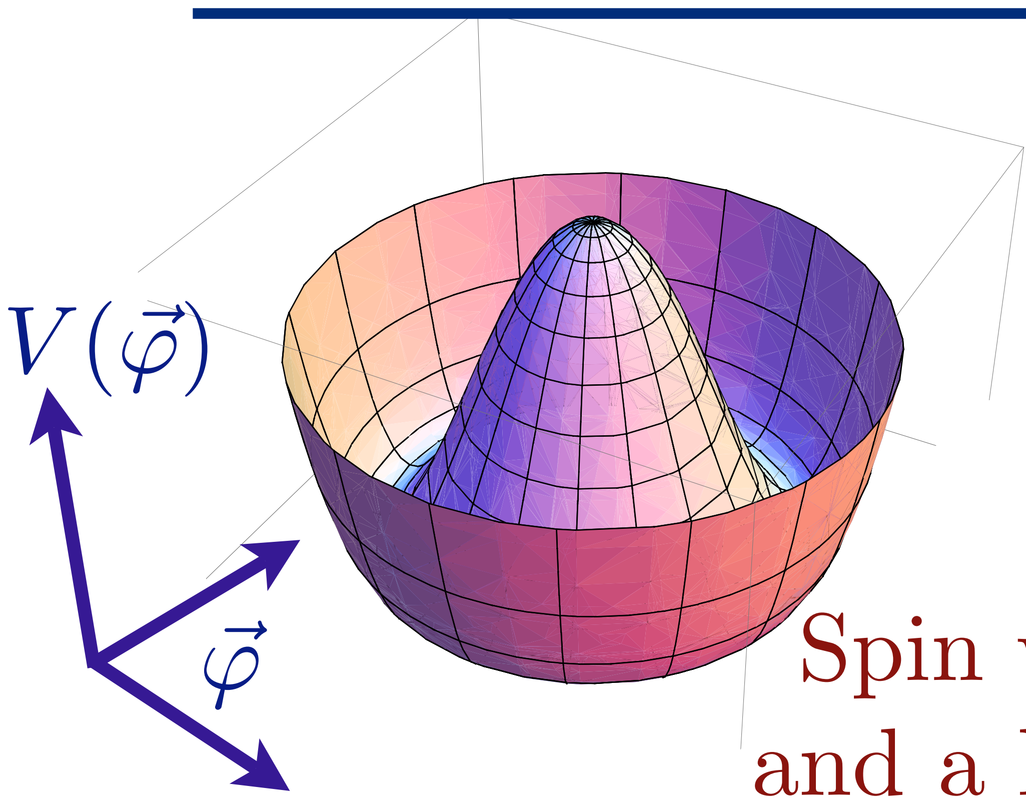
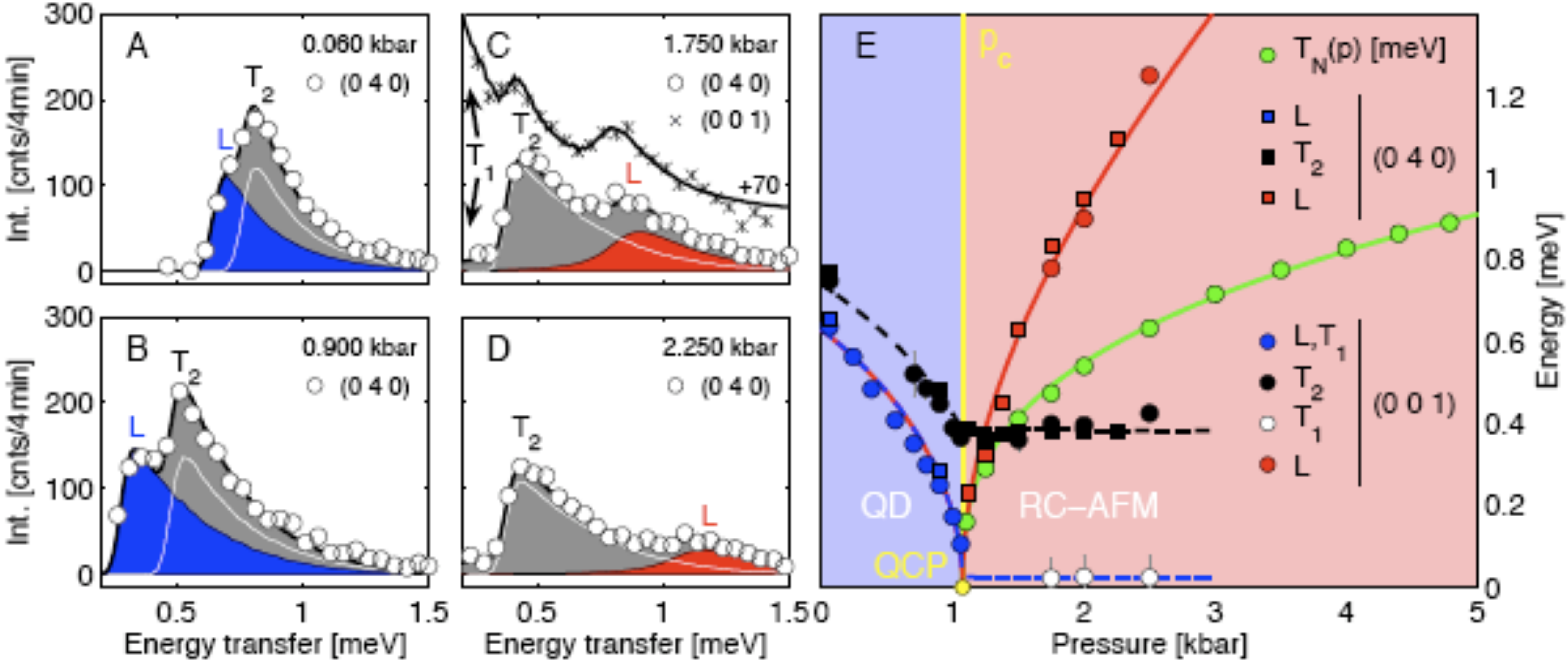




Spin waves ("Goldstone" modes) and a longitudinal "Higgs" particle

$TlCuCl_3$ with varying pressure

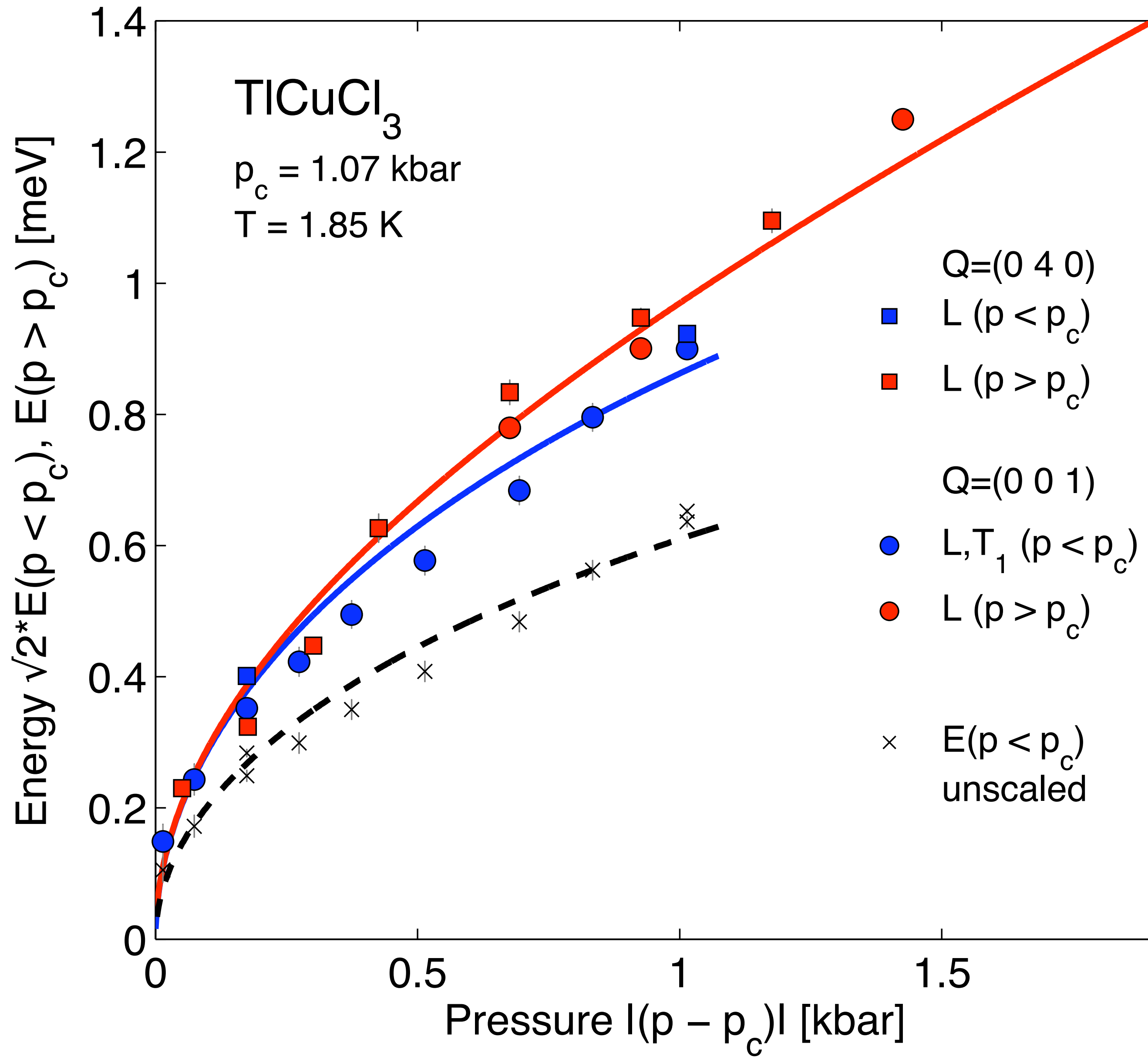


Observation of 3 → 2 low energy modes, emergence of new longitudinal mode (the "Higgs boson") in Néel phase, and vanishing of Néel temperature at quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

$$\frac{\text{Energy of "Higgs" particle}}{\text{Energy of triplon}} = \sqrt{2}$$



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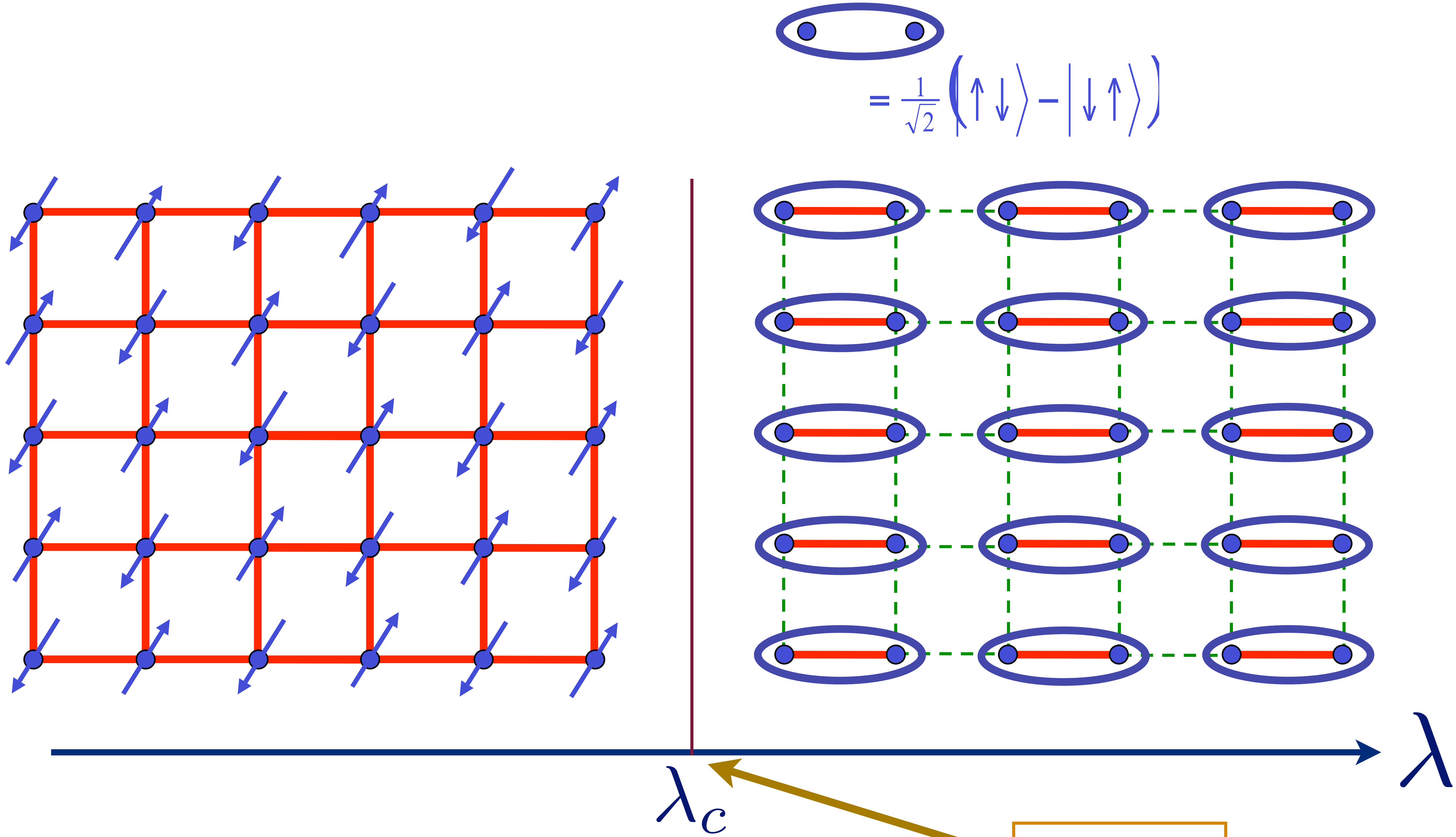

O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s \vec{\varphi}^2 + u \left(\vec{\varphi}^2\right)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [. . .] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096 - \sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096 + \sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107[d]	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230 - \sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
$1.8230 + \sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

S. Wenzel and W. Janke, arXiv:0808.1418
M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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Field-theoretic RG of CFT3 E. Vicari *et al.*

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