Quantum field theory of metallic spin glasses

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We introduce an effective field theory for the vicinity of a zero-temperature quantum transition between a metallic spin glass ("spin-density glass") and a metallic quantum paramagnet. Following a mean-field analysis, we perform a perturbative renormalization-group study and find that the critical properties are dominated by static disorder-induced fluctuations, and that dynamic quantum-mechanical effects are dangerously irrelevant. A Gaussian fixed point is stable for a finite range of couplings for spatial dimensionality $d > 8$, but disorder effects always lead to runaway flows to strong coupling for $d \leq 8$. Scaling hypotheses for a *static* strong-coupling critical field theory are proposed. The nonlinear susceptibility has an anomalously weak singularity at such a critical point. Although motivated by a perturbative study of metallic spin glasses, the scaling hypotheses are more general, and could apply to other quantum spin glass to paramagnet transitions.

I. INTRODUCTION

Electronic systems with strong randomness and strong interactions[1] have been studied in a number of experimental systems including doped semiconductors, metallic alloys, and most recently in the doped cuprate and doped heavy-fermion compounds. Some of the most interesting physics in these materials arises from the complex interplay of the fermionic, charge-carrying excitations and the spin fluctuations. A number of distinct equilibrium thermodynamic phases are possible, even at zero temperature (T). In the charge sector, we may have metallic and insulating phases (within the insulator we may also distinguish further between a Mott insulator, with a true $T = 0$ charge gap, and a Fermi glass, which has localized, gapless, charged excitations). In the spin sector, the ground state can either be a spin glass, in which each spin has an infinite-time memory of its spatially random moment, or a quantum paramagnet, in which the spin correlations decay to zero in the long-time limit. There does not appear to be any fundamental principle constraining the relative positions of the transitions in the charge and spin sectors, leading to a rich phenomenology of possible $T = 0$ phases and critical points.

Previous work has examined the quantum paramagnet phase both in the Mott insulator[2] (where the spin fluctuations can be described by a quantum Heisenberg spin model) and the metal.[3] Studies of the spin-glass phase and its onset have, however, been mostly restricted to the insulating phase. In an infinite-range Heisenberg model of the Mott insulator an instability of the quantum paramagnet to a possible spin-glass was noted.[4] Greater progress has been made in elucidating the quantum paramagnet to spin-glass transition in insulating models of Ising spins in a transverse field (which may be appropriate for insulators with strong crystalline anisotropy) and quantum rotors.[5–7] The methodology of a recently developed Landau theory for this transition in the Ising and rotor models[7] will be very useful to us below.

In this paper, we shall analyze systems in the vicinity of a $T = 0$ transition between a spin glass and a quantum paramagnet occurring while the charge sector is *metallic*. Our motivation for this study comes partly from recent experiments in heavy-fermion compounds like $Y_{1-x}U_xPd_3$,[8] which appear to show a paramagnet to spin-glass transition with increasing doping (x) in a metallic regime. We shall make a few remarks about these experimental systems at the end of the paper.

We will begin by introducing in Sec. II a quantum field theory, $\mathcal{A}$, for metallic spin glasses; our approach suggests the identification "spin-density glass" for such systems. In Sec. III we will determine the mean-field phase diagram of $\mathcal{A}$ as a function of a quantum coupling, temperature, and an external magnetic field. Fluctuations about mean field will be studied in Sec. IV, first by a perturbative renormalization group (RG) analysis (Sec. IV A), which finds flows to strong coupling for spatial dimensions $d \leq 8$. These results will then be used (Sec. IV B) to motivate a scaling scenario for the strong-coupling region in which the critical fixed point involves only *static* fluctuations induced by the quenched randomness. Dynamic, quantum fluctuations are dangerously irrelevant at this static fixed point, and their effects are controlled by a crossover exponent $-\theta_u \leq 0$. The critical singularity of the nonlinear susceptibility, $\chi_{\rm nl}$, is weakened by a positive θ_u, thus a nondivergent, cusplike, critical singularity in $\chi_{\rm nl}$ is possible. This scaling scenario generalizes one proposed earlier for insulating Ising and rotor spin glasses[7] which had $\theta_u = 0$. Indeed, there is no fundamental reason why the insulating spin glasses should not also have $\theta_u > 0$.

Static, or $\hbar = 0$, fixed points have also arisen in studies of some other spin systems. A model of quantum rotors in a random field was studied some time ago by Boyanovsky and Cardy,[9] and their results can be interpreted in terms of such a fixed point. However, they did not realize that the crossovers, and positions of phase boundaries, at finite T are modified by a positive θ_u; the required modification is related to that discussed by Weichmann *et al.*[10] and Millis[11] in a rather different physical context, and will also be discussed in this paper. More recently, Fisher[12] has studied the random Ising model in a transverse field in $d = 1$ and shown that the results are consistent with a $\hbar = 0$ fixed point: his scaling results, however, involve an exponential relationship between energy and length scales, unlike the more usual power-law relationship which we shall find. Finally, in very recent work, Kirkpatrick and Belitz[13] have proposed a scaling scenario for the metal-insulator transition which appears to have many similarities to our results below on the metallic spin glass to paramagnet transition.

II. EFFECTIVE ACTION

An initial analysis of metallic spin glasses was performed some time ago by Hertz,[14] although he did not focus on the vicinity of the $T = 0$ quantum transition. We will study models, similar to those in Ref. 14, described by the following class of Hamiltonians:

$$H = -\sum_{i<j,\alpha} t_{ij} c^{\dagger}_{i\alpha} c_{i\alpha} - \sum_{i<j,\mu} J^{\mu}_{ij} S_{i\mu} S_{j\mu} + H_{\rm int}, \quad (2.1)$$

where $c_{i\alpha}$ annihilates an electron on site i with spin $\alpha = \uparrow, \downarrow$, and the spin operator $S_{i\mu} \equiv \sum_{\alpha\beta} c^{\dagger}_{i\alpha} \sigma^{\mu}_{\alpha\beta} c_{i\beta}/2$ with σ^{μ} the Pauli matrices. The sites i, j lie on a d-dimensional lattice, the hopping matrix elements t_{ij} are short-ranged and possibly random, and the J^{μ}_{ij} are Gaussian random exchange interactions, possibly with spin anisotropies. The remainder $H_{\rm int}$ includes other possible short-range interactions between the electrons: we constrain them so that the ground state of H is metallic. A version of H with infinite-range hopping and exchange was studied recently[15] using a static ansatz for the order parameter but with no additional approximations. We will provide below a theory which includes dynamic, quantum effects and also applies to models with finite-range interactions in finite dimensions. Our analysis of H will be similar in spirit to the Stoner model approaches to the appearance of spin-density-wave order in clean metallic systems,[16] except that we now consider condensation into a density wave with a random orientation of spins, or a "spin-density glass."

We now derive a low-energy field theory for H in the vicinity of the spin glass to paramagnet transition. The procedure is similar to that of Ref. 7. The metallic nature of the system expresses itself mainly through the modification of a single term, which, however, has important consequences. We begin by defining the spin-glass order parameter

$$Q^{ab}_{\mu\nu}(x, \tau_1, \tau_2) = \sum_{i \in \mathcal{N}(x)} S^{a}_{i\mu}(\tau_1) S^{b}_{i\nu}(\tau_2), \quad (2.2)$$

where $a, b = 1 \ldots n$ are replica indices (the limit $n \to 0$ is taken at the end), τ_1, τ_2 are Matsubara times, and $\mathcal{N}(x)$ is a coarse-graining region around x. At $T = 0$, the Edwards-Anderson spin-glass order parameter, $q_{\rm EA}$ is the expectation of the replica-diagonal part of Q in the limit $|\tau_1 - \tau_2| \to \infty$; however, it is necessary to retain the time dependence and all replica components of Q to capture the full structure of the field theory.[7] The effective action for Q is obtained by averaging over the J_{ij} randomness in H after introducing replicas, decoupling the resulting eight Fermi term by a Hubbard Stratonovich field Q, and integrating out the fermions. This procedure has been carried out in Ref. 7 for the quantum rotor spin glass, and in Ref. 16 for spin-density-wave formation in metallic systems. It is simple to combine these methods and we omit all intermediate steps. The final effective action, $\mathcal{A}$, is expressed in terms of a shifted field $Q \to Q - C_{\mu\nu}\delta^{ab}\delta(\tau_1 - \tau_2)$ where the subtraction only modifies the uninteresting short-time behavior, and the constants $C_{\mu\nu}$ are chosen so that the resulting Q is small at low frequencies near the critical point:[7]

$$\begin{aligned}
\mathcal{A} = \frac{1}{t}\int d^d x \Bigg\{ & \frac{1}{\kappa}\int d\tau \sum_{a\mu} r_{\mu} Q^{aa}_{\mu\mu}(x,\tau,\tau) - \frac{1}{\pi\kappa}\int d\tau_1 d\tau_2 \sum_{a\mu} \frac{Q^{aa}_{\mu\mu}(x,\tau_1,\tau_2)}{(\tau_1-\tau_2)^2} \\
& + \frac{1}{2}\int d\tau_1 d\tau_2 \sum_{ab\mu\nu} \left[\nabla Q^{ab}_{\mu\nu}(x,\tau_1,\tau_2)\right]^2 - \frac{\kappa}{3}\int d\tau_1 d\tau_2 d\tau_3 \sum_{abc\mu\nu\rho} Q^{ab}_{\mu\nu}(x,\tau_1,\tau_2) Q^{bc}_{\nu\rho}(x,\tau_2,\tau_3) Q^{ca}_{\rho\mu}(x,\tau_3,\tau_1) \\
& + \frac{1}{2}\int d\tau \sum_{a\mu\nu} \left[u\, Q^{aa}_{\mu\nu}(x,\tau,\tau) Q^{aa}_{\mu\nu}(x,\tau,\tau) + v\, Q^{aa}_{\mu\mu}(x,\tau,\tau) Q^{aa}_{\nu\nu}(x,\tau,\tau)\right] \Bigg\} \\
& - \frac{1}{2t^2}\int d^d x \int d\tau_1 d\tau_2 \sum_{ab\mu\nu} Q^{aa}_{\mu\mu}(x,\tau_1,\tau_1) Q^{bb}_{\nu\nu}(x,\tau_2,\tau_2) + \cdots . \quad (2.3)
\end{aligned}$$

We have only displayed the small subset of terms which will be important near the critical point. We have allowed a μ dependence in r_{μ} to reflect possible spin anisotropies; the less-important μ dependence of other couplings has been suppressed. The metallic nature of the system is reflected in the second term which has a long-range $1/(\tau_1 - \tau_2)^2$ interaction in time; the power-law decay is a consequence of the gapless particle-hole spin excitations

which lead to the dependence

$$[\langle S_{i\mu}(\tau_1)S_{i\nu}(\tau_2)\rangle] \sim \delta_{\mu\nu}(\tau_1-\tau_2)^{-2} \tag{2.4}$$

in any metallic paramagnet[1,16] (the angular brackets represent an average over quantum and thermal fluctuations, and the square brackets are an average over randomness). This behavior is of course only valid at large $|\tau_1-\tau_2|$ and we cut it off at short-time differences such that its Fourier transform is $\sim -|\omega|$ for small ω. The remaining terms in $\mathcal{A}$ are identical to those obtained in Ref. 7 for the rotor model. These are the most general terms, of low order in Q, which are local in space-time and consistent with underlying symmetries. In particular, the time arguments of Q associated with different replica indices must always be integrated independently because the disorder is static. "Quantum-mechanical" interactions occur only within the same replica, allowing a gradient expansion about the equal-time point for such terms. A more detailed discussion of these criteria can be found in Ref. 7. The particle-hole continuum will induce nonlocal corrections to these terms, but none are as important as that in the term linear in Q. The symmetries also allow a "mass" term $\sim [Q^{ab}_{\mu\nu}(x,\tau_1,\tau_2)]^2$, but such a term is redundant as it can be removed by the shift $Q \to Q - C_{\mu\nu}\delta^{ab}\delta(\tau_1-\tau_2)$; the shift has a delta function in time and thus does not modify the long-time, low-frequency behavior that we are interested in.

We now discuss the physical significance of the couplings in $\mathcal{A}$; for more details the reader is referred again to Ref. 7 and we highlight only the main points here. First note that there are more terms than coupling constants, but it is easy to check that rescalings of x and τ always allow one to reach the form chosen. The coupling r_μ multiplies what turns out to be the "thermal operator" which tunes the system across the transition. The important nonlinearity is the cubic κ coupling which is induced by disorder effects and involves no exchange of energy between the Q fields; there is a $1/\kappa$ in the linear term to ensure that the bare Q propagator is independent of κ. The only terms involving energy exchange are the quadratic u and v terms which represent the "quantum-mechanical" interactions between the fermions. Finally, the $1/t^2$ term in $\mathcal{A}$ represents disorder fluctuation effects and arises from fluctuations in the local position of the critical point as determined by r_μ.

III. MEAN-FIELD THEORY

We now consider the mean-field (or tree-level) properties of $\mathcal{A}$. We will only consider two extreme limits of the spin anisotropy—Ising-like, when $r_1 \ll r_2, r_3$, and Heisenberg-like, when $r_1 = r_2 = r_3$. We will drop the vector μ index except where necessary, and represent the effective number of components by M ($= 1$ for the Ising case, and $= 3$ for the Heisenberg case).

In Ref. 7 we found that, as in the classical spin glass, a theory with only a cubic nonlinearity was replica symmetric, even in the spin-glass phase; replica symmetry breaking appeared only upon including a certain quartic coupling which was formally irrelevant at the quantum critical point. Much of this structure carries over unchanged to the metallic spin glass, and we shall not dwell on it here. We will restrict our considerations to the replica symmetric theory which contains all of the dynamic effects which lead to the important differences between metallic and insulating spin glasses. We therefore make the following replica-symmetric, x-independent ansatz for the mean-field value of Q: in Matsubara frequencies (which are integral multiples of $2\pi k_B T/\hbar$ as usual) we write

$$Q^{ab}(\omega_1,\omega_2) = \beta^2\delta_{\omega_1,0}\delta_{\omega_2,0}q + \beta\delta^{ab}\delta_{\omega_1+\omega_2,0}\chi(i\omega_1), \tag{3.1}$$

where $\beta = \hbar/k_B T$, and χ is the local dynamic spin susceptibility (we will absorb a factor of $k_B/\hbar$ into T from here on). The first term is the spin-glass order parameter—note that it is independent of replica indices, and, therefore, the replica diagonal and off-diagonal components of q are equal and $q = q_{EA}$. In a theory with only a cubic nonlinearity (to which we shall restrict ourselves here) the equality of all the replica components of q holds at all T; upon including higher-order terms in $\mathcal{A}$, the equality between all the replica components persists at $T = 0$, but there are thermal corrections at any nonzero T which distinguish the diagonal and off-diagonal components and which also break replica symmetry.[7]

We now insert (3.1) into $\mathcal{A}$ and determine the saddle point. This determines the spin glass to paramagnet phase boundary at $r = r_c(T)$ and the order parameter

$$q = \begin{cases} [r_c(T)-r]/[\kappa(u+Mv)] & \text{for } r < r_c(T) \\ 0 & \text{otherwise,} \end{cases} \tag{3.2}$$

with

$$r_c(T) = r_c - c(u+Mv)T^{3/2}, \tag{3.3}$$

where $c = \sqrt{\pi/2}\zeta(3/2)$, $r_c \equiv r_c(0) \sim \Lambda_\omega^{3/2}$ is dependent on the large frequency cutoff Λ_ω (see Fig. 1). The local, dynamic spin susceptibility has imaginary part

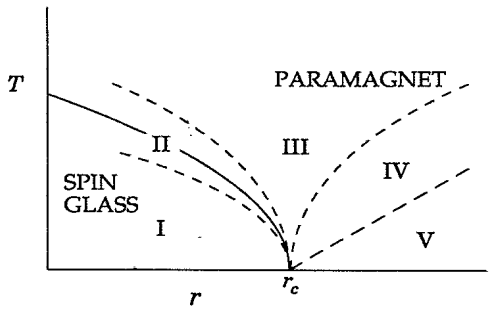

FIG. 1. Phase diagram of the action $\mathcal{A}$ [Eq. (2.3)] as a function of temperature T and r which measures the strength of quantum fluctuations. The full line is the only phase transition and dashed lines denote crossovers between different regimes, which are described in the text.

$$\chi''(\omega) = -\frac{1}{\kappa}\mathrm{Im}\sqrt{-i\omega+\Delta} = \frac{1}{\sqrt{2}\kappa}\frac{\omega}{\sqrt{\Delta+\sqrt{\omega^2+\Delta^2}}} \tag{3.4}$$

Note that the spin fluctuations are gapless, and the crossover from paramagnetic to critical fluctuations occurs at a frequency scale Δ; Δ also determines the correlation length $\xi \sim \Delta^{-1/4}$ that appears in spatial correlation functions, which can be found as Gaussian fluctuations around the saddle point. The value of Δ has a rather complicated dependence on T and r and we describe its limiting behavior in the five different regimes of Fig. 1—there are smooth crossovers between these regimes. Within the spin-glass phase, we have $\Delta = 0$, $\chi'' \sim \mathrm{sgn}(\omega)\omega^{1/2}$ everywhere and there are no crossovers in the present approximation. However, we expect that there will be a crossover between a region characterized by the quantum ground state (I) to a region dominated by thermal, critical fluctuations (II) and such a crossover boundary has been shown in Fig. 1. In the paramagnetic phase the present approximation is much richer, and shows all the expected crossovers. The scale Δ is determined by the equation

$$\Delta = r - (u+Mv)\frac{1}{\beta}\sum_{\omega}(|\omega|+\Delta)^{1/2}. \tag{3.5}$$

Solution of (3.5) yields four different regimes of behavior (II–V) which we describe in turn.

(II) $|r-r_c(T)| \ll (u+Mv)^2T^2$: this is closest to the phase boundary, and is the region with classical fluctuations. We have $\Delta = \{[r-r_c(T)]/[T(u+Mv)]\}^2$—note that Δ depends on the *square* of the distance $r-r_c(T)$ from the transition (as shown in Ref. 7, this is crucial for obtaining the classical exponent $\nu = 1/2$).

(III) $[|r-r_c|/(u+Mv)]^{2/3} \ll T$: this is the "quantum-critical" region in that T is the most significant energy scale and the system behaves as if it is at the critical coupling $r = r_c$. Here we find, to lowest order in u, v that $\Delta = c(u+Mv)T^{3/2}$. Note that the expected quantum-critical scaling $\Delta \sim T$ (Ref. 17) is violated. This is a consequence of the fact that all T dependent corrections are controlled by quantum interactions u, v which are irrelevant at the critical point (see below)—in other words $\theta_u > 0$ spoils the naive $\Delta \sim T$ scaling. A similar interpretation can be given to the position of the phase boundary $r_c(T)$.

(IV) $r-r_c \ll T \ll [(r-r_c)/(u+Mv)]^{2/3}$ *and* (V) $T \ll r-r_c$: these are the "quantum-disordered" regions in that T dependent corrections are secondary and to leading order in T, u, v we have $\Delta = r - r_c$. The subleading terms in Δ are different in the two regimes: in regime IV, $\Delta(T)-\Delta(0) = c(u+Mv)T^{3/2}$, while in regime V, $\Delta(T)-\Delta(0) = (u+Mv)\pi T^2/(6\sqrt{r-r_c})$. This subdivision of the quantum-disordered region is similar to that found in a different context in Ref. 11, and is also a consequence of the dangerous irrelevancy of u, v.

All of the above crossover boundaries and exponents are of course characteristics of the mean-field theory, which can, in general, be modified by fluctuations—we will indicate the nature of these modifications in the discussion in Sec. IV B.

Phases in a magnetic field

We complete our discussion of mean-field theory by discussing the effect of an external magnetic field, H_μ on $\mathcal{A}$. The additional terms induced by H can be determined following Refs. 18 which examined the effects of H on spin-density wave formation in clean systems—by this method we found

$$\begin{aligned}\mathcal{A} \to \mathcal{A} &- \frac{g}{t}\int d^dx d\tau_1 d\tau_2 \sum_{ab} Q^{ab}_{\mu\nu}(x,\tau_1,\tau_2)H_\mu H_\nu \\ &- \frac{1}{t}\int d^dx d\tau \sum_a \left(i\alpha_1 \epsilon_{\mu\nu\lambda}H_\lambda \left.\frac{\partial Q^{aa}_{\mu\nu}(x,\tau_1,\tau_2)}{\partial\tau_1}\right|_{\tau_1=\tau_2=\tau} + \alpha_2 H_\lambda H_\lambda Q^{aa}_{\mu\mu}(x,\tau,\tau) \right. \\ &\left. + \alpha_3(H_\mu H_\nu - H_\lambda H_\lambda \delta_{\mu\nu})Q^{aa}_{\mu\nu}(x,\tau,\tau)\right).\end{aligned} \tag{3.6}$$

The field has several different competing effects. The first term, proportional to the coupling g, is the static paramagnetic susceptibility of the fermions which polarizes the spins along the field, and which always dominates at small H. The α terms account for the precession of the spins in the plane perpendicular to $\vec{H}$ and the energetic contribution of quantum fluctuations about the static spin directions: as in clean antiferromagnets[19] we expect these terms to prefer magnetic order in a plane perpendicular to H (and so $\alpha_3 > 0$). We now consider some cases separately:

(*i*) *Ising spins, H along easy axis*—only the term proportional to g in (3.6) need be considered as the α terms are never important. The finite field phase diagram, the field dependence of observables, and the position of the Almeida-Thouless boundary,[20] are essentially identical to that for the insulating Ising model considered earlier,[7] and will therefore not be considered here. The only difference in the metallic case is that no logarithms are present—e.g., the free energy at $r = r_c$ and $T = 0$ that depended on H as $H^{8/3}/\ln^{1/3} H$ in Ref. 7, here varies as $H^{8/3}$.

(*ii*) *Ising spins, H perpendicular to easy axis*—now the g term in (3.6) couples only to noncritical components of

Q and is not important; after integrating out the noncritical Q we find that the main consequence of the α terms is to induce a shift $r_1 \to r_1 + \alpha' H^2$ in the position of the critical point.

(*iii*) *Heisenberg spins*—in finite field we now have to allow for the possibility of spin-glass order appearing in the plane perpendicular to H; the onset of this order is the Gabay-Toulouse transition at $H = H_{\rm GT}$.[20] Let us take a field H pointing along the $\mu = 3$ direction. The subsequent mean-field theory is most convenient in a circularly-polarized basis for the vector components of Q: we take $Q_{33} = Q_L$, $Q_{11} = Q_{22} = (Q_{+-} + Q_{-+})/2$, $Q_{12} = -Q_{21} = i(Q_{+-} - Q_{-+})/2$, and all other vector components of $Q = 0$. We make the same ansatz as in (3.1) for the frequency and replica dependence of Q_L, Q_{+-} and Q_{-+} by introducing the quantities q_L, χ_L, q_{+-}, etc. It is then not difficult to solve the resulting mean-field equations. It is slightly more convenient to approach the Gabay-Toulouse boundary from the Gabay-Toulouse phase with spin-glass order in the transverse direction: $q_{+-} = q_{-+} \neq 0$. The solution of the mean-field equations for this case are

$$\begin{aligned} q_{+-} &= q_{-+} = q_T, \\ \chi_{+-}(i\omega) &= \chi^*_{-+}(i\omega) = -\frac{1}{\kappa}(|\omega| + i\alpha_1 H\omega)^{1/2}, \\ q_L &= \frac{gH^2}{4\sqrt{\Delta}}, \\ \chi_L(i\omega) &= -\frac{1}{\kappa}(|\omega| + \Delta)^{1/2}, \end{aligned} \tag{3.7}$$

where the Gabay Toulouse spin-glass order parameter q_T and the frequency scale Δ are determined by the solutions of the two equations

$$\begin{aligned} \Delta &= r + \alpha_2 H^2 + (u+v)\left(\frac{\kappa g H^2}{4\sqrt{\Delta}} - \frac{1}{\beta}\sum_\omega (|\omega| + \Delta)^{1/2}\right) + 2v\left(\kappa q_T - \frac{1}{\beta}\sum_\omega (|\omega| + i\alpha_1 H\omega)^{1/2}\right), \\ 0 &= r + (\alpha_2 - \alpha_3)H^2 + v\left(\frac{\kappa g H^2}{4\sqrt{\Delta}} - \frac{1}{\beta}\sum_\omega (|\omega| + \Delta)^{1/2}\right) + (u+2v)\left(\kappa q_T - \frac{1}{\beta}\sum_\omega (|\omega| + i\alpha_1 H\omega)^{1/2}\right). \end{aligned} \tag{3.8}$$

The Gabay-Toulouse boundary is determined by imposing the condition $q_T = 0$ on these two equations, which gives us a line $H = H_{\rm GT}(r)$ in the $r - H$ plane. The result of such a computation at $T = 0$, is shown in Fig. 2. For small H, the first term in (3.6) dominates and we find $H_{\rm GT} \sim (r_c - r)^{3/4}$. For large H, the α terms take over, and for $\alpha_3 > 0$ we find that $H_{\rm GT}$ turns over and extends to $r > r_c$ as $H_{\rm GT} \sim (r - r_c)^{1/2}$ (see Fig. 2); for sufficiently negative α_3 this turn over will not occur.

IV. FLUCTUATIONS

We begin by a perturbative RG analysis of fluctuations, which will unfortunately not be of much direct utility as there is runaway flow to strong coupling below $d = 8$. Nevertheless, the structure of this analysis will help motivate a general scaling scenario which we will describe subsequently.

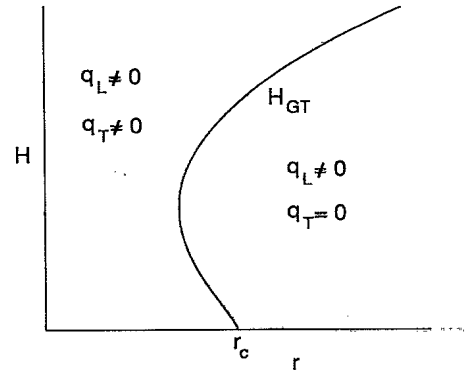

FIG. 2. $T = 0$ phase diagram of $\mathcal{A}$ for the Heisenberg case, in a field H. $H_{\rm GT}$ is the Gabay-Toulouse boundary, $q_L = \left[\langle S_\mu \rangle^2\right]$ for μ along the field direction, and similarly $q_T = \left[\langle S_\mu \rangle^2\right]$ for μ perpendicular to the field.

A. Perturbative RG

The perturbative RG analysis is quite similar to that of Ref. 7. The main difference at tree level will be that the dynamic exponent z is $z = 4$ rather than $z = 2$. This difference has the important consequence of now making the u, v couplings dangerously irrelevant, which in turn leads to a positive θ_u.

The RG begins with the rescalings

$$x' = x/s, \tau' = \tau/s^z,$$

$$t' = ts^{-\theta}, \quad Q' = Qs^{(d-\theta+2z-2+\eta)/2}. \tag{4.1}$$

The exponents z, η have their usual meaning, while θ is introduced to allow for violations of hyperscaling: we will have $\theta > 0$ causing t to flow to 0, and behave as a dangerously irrelevant variable. The irrelevant coupling t and its exponent θ should not be confused with the couplings controlling quantum-mechanical effects and the exponent θ_u which will be discussed momentarily. At tree level (or equivalently, at the Gaussian fixed point), the above rescalings leave $\mathcal{A}$ invariant provided we modify the couplings

$$\begin{aligned} r' &= rs^z, & \kappa' &= \kappa s^{(6+\theta-d-3\eta)/2}, \\ u' &= us^{2-z-\eta}, & v' &= vs^{2-z-\eta}, \end{aligned} \tag{4.2}$$

and choose the exponents

$$z = 4, \qquad \eta = 0, \qquad \theta = 2. \tag{4.3}$$

Thus the cubic nonlinearity κ becomes relevant for d below 8, and the rescaling of r gives us the Gaussian exponent $\nu = 1/4$; note the correlation length is given by $\xi \sim \Delta^{-1/4}$ in the notation of Sec. III. The most important point, and the key difference from Ref. 7, is that u and v are now irrelevant with exponent -2. As these are the only couplings associated with quantum effects, we introduce a new crossover exponent, $-\theta_u$ which will control the "dangerously irrelevant" consequences of quantum fluctuations. At tree level we clearly have $\theta_u = 2$. Finally, note that for d above 8, κ also becomes dangerously irrelevant about the Gaussian fixed point.

It is straightforward to extend the above analysis to include one-loop diagrams. Using the diagrams discussed in Ref. 7, we find ($s = e^{\ell}$)

$$z(\ell) = 4 + 8\kappa^2(\ell) \quad , \quad \eta = 2\kappa^2(\ell) \quad , \quad \theta = 2, \tag{4.4}$$

and the flow equations

$$\frac{dr(\ell)}{d\ell} = zr(\ell) - a\kappa^2(\ell) \quad ; \quad \frac{d\kappa(\ell)}{d\ell} = \frac{8-d}{2}\kappa(\ell) + 9\kappa^3(\ell). \tag{4.5}$$

We have absorbed various phase-space factors into the couplings (see Ref. 7), and a is an uninteresting positive constant. The irrelevant couplings u, v were set equal to 0 at the outset. There is no stable fixed point of (4.5) for real κ below $d = 8$. Above $d = 8$, the Gaussian fixed point is stable, but its domain of attraction is limited to a region which vanishes as d approaches 8 from above. For $d \leq 8$, and for all physical initial conditions, the coupling κ flows to strong coupling, making quantitative computation of exponents impossible in the present approach.

B. Scaling hypotheses

We will now discuss a nonperturbative scaling scenario for quantum spin glasses, assuming that the structure of the dangerously irrelevant variables remains similar to that found in the perturbative analysis above. We will consider a *static* strong-coupling critical theory with two dangerously irrelevant directions: one associated with a coupling analogous to t which controls disorder fluctuation effects and has exponent $-\theta$, and a second associated with dynamic, quantum-mechanical effects (couplings u, v) and exponent $-\theta_u$. In the previous analysis of insulating spin glasses[7] only t, the first of these dangerously irrelevant couplings, was present; our present scaling relations reduce to the earlier ones upon putting $\theta_u = 0$. As we shall see below, a positive θ_u has important physical consequences. Although the analysis below is clearly motivated by our mean-field theory of metallic spin glasses above, there is no fundamental reason why the insulating models considered in Ref. 7 should not also have $\theta_u > 0$.

It is helpful to discuss nonperturbative effects by considering the scaling behavior of observable correlation functions. Among two-point correlators of Q, there are three independent observables:[7] in the paramagnetic phase these are the spin-glass susceptibility, G,

$$\begin{aligned} G(x-y, \tau_1-\tau_2, \tau_3-\tau_4) &\equiv \sum_{\mu\nu} [\langle S_{i\mu}(\tau_1) S_{j\mu}(\tau_2)\rangle \langle S_{i\nu}(\tau_3) S_{j\nu}(\tau_4)\rangle] \\ &= \lim_{n\to 0} \frac{1}{n(n-1)} \sum_{a\neq b,\mu\nu} \left\langle\!\left\langle Q^{ab}_{\mu\nu}(x,\tau_1,\tau_3) Q^{ab}_{\mu\nu}(y,\tau_2,\tau_4)\right\rangle\!\right\rangle \end{aligned} \tag{4.6}$$

(the double angular brackets represent averages with a replicated, translationally invariant action like $\mathcal{A}$); the quantum mechanically disconnected correlator, G^d,

$$\begin{aligned} G^d(i-j, \tau_1-\tau_2, \tau_3-\tau_4) &\equiv [\langle S_{i\mu}(\tau_1) S_{i\mu}(\tau_2)\rangle \langle S_{j\nu}(\tau_3) S_{j\nu}(\tau_4)\rangle] - \text{subtractions} \\ &= \lim_{n\to 0} \frac{1}{n(n-1)} \sum_{a\neq b} \left\langle\!\left\langle Q^{aa}_{\mu\mu}(x,\tau_1,\tau_2) Q^{bb}_{\nu\nu}(y,\tau_3,\tau_4)\right\rangle\!\right\rangle - \cdots; \end{aligned} \tag{4.7}$$

and the connected correlation function, G^c,

$$\begin{aligned} G^c_{\mu\nu\rho\sigma}(i-j, \tau_1-\tau_4, \tau_2-\tau_4, \tau_3-\tau_4) &\equiv [\langle S_{i\mu}(\tau_1) S_{i\nu}(\tau_2) S_{j\rho}(\tau_3) S_{j\sigma}(\tau_4)\rangle] - \text{subtractions} \\ &= \lim_{n\to 0} \frac{1}{n} \sum_{a} \left\langle\!\left\langle Q^{aa}_{\mu\nu}(x,\tau_1,\tau_2) Q^{aa}_{\rho\sigma}(y,\tau_3,\tau_4)\right\rangle\!\right\rangle - \cdots. \end{aligned} \tag{4.8}$$

The nonlinear susceptibility, $\chi_{\rm nl}$, is given by the integral over space and time of G^c. The correlator G^c is nonzero only because of the "quantum interactions" u, v, and therefore carries a prefactor of u, v; in contrast G and G^d are nonzero even in a purely static theory. Under the rescalings (4.1) we may conclude from arguments similar to those in Ref. 7 that G^d and G scale as

$$G^d(x,\tau,\tau) \sim x^{-(d+2z-\theta-2+\eta)}, \tag{4.9}$$

$$G(x,\tau,\tau) \sim x^{-(d+2z-2+\eta)}, \tag{4.10}$$

for fixed τ/x^z at criticality. The scaling dimensions of G and G^d differ because G carries a prefactor of the dangerously irrelevant variable t, while G^d does not. Finally,

G^c carries a prefactor of t, and an additional prefactor of the irrelevant quantum interactions: hence

$$G^c(x,\tau,\tau,\tau) \sim x^{-(d+2z+\theta_u-2+\eta)}. \tag{4.11}$$

Indeed, one can consider the three equations (4.9)–(4.11) as the definition of the three independent exponents η, θ, and θ_u. In the previous analysis,[7] $\theta_u = 0$ and hence G and G^c had the same scaling dimension. By taking the space-time integral of (4.8), we can deduce that the nonlinear susceptibility $\chi_{\rm nl}$ behaves as

$$\chi_{\rm nl} \sim |r - r_c|^{-(2-\eta-\theta_u)\nu} \tag{4.12}$$

near the $T = 0$ quantum critical point. For sufficiently large θ_u, $\chi_{\rm nl}$ need not diverge.

The existence of a static critical theory, and the associated positivity of θ_u, has important consequences for the finite T behavior away from the critical point. Recall that T only appears as a finite-size length, $1/T$, along the time direction, and hence scaling[9] implies that its scaling dimension is z. However, some power of a combination of interactions like u, v must appear in any frequency scale and hence we expect that the naive scaling of finite ω correlators as functions of ω/T (Ref. 17) will now be modified. A related modification of naive scaling has been discussed in Refs. 10, 11 for some clean systems, and we will now present a similar analysis. It is useful to consider a simple model of the renormalization-group flows near the quantum-critical point at low temperatures. Let us move away from the quantum critical point ($r = r_c$, $T = 0$) by perturbing the system along the single, relevant eigendirection by the amount $r - r_c$, and along the least irrelevant eigendirection which involves terms with frequency exchange by the amount u. For small r, u, and T we expect flow equations like

$$\begin{aligned}
\frac{dT(\ell)}{d\ell} &= zT(\ell), \\
\frac{dr(\ell)}{d\ell} &= \frac{1}{\nu}[r(\ell) - r_c] + uf[T(\ell)], \\
\frac{du(\ell)}{d\ell} &= -\theta_u u(\ell)\ ,
\end{aligned} \tag{4.13}$$

where $f(T)$ [$f(0) = 0$] is some function arising from thermal occupation of the short distance modes of the order parameter fluctuations which are being integrated out. The key property of (4.13) is that a T dependence is induced into the flow of the relevant coupling r only via the irrelevant coupling u. The integral of (4.13) is

$$\begin{aligned}
r(\ell) - r_c = (r - r_c)e^{\ell/\nu} \\
+ue^{\ell/\nu}\int_0^\ell d\ell' e^{-(\theta_u+1/\nu)\ell'} f(Te^{z\ell'})
\end{aligned} \tag{4.14}$$

[as is customary, we have abbreviated $r(0) = r$, $u(0) = u$, and $T(0) = T$]. We now change integration variables to $\zeta = Te^{z\ell'}$, and integrate to the correlation length $\xi = e^{\ell=\ell^*}$ at which $r(\ell^*) - r_c = 1$ to obtain

$$\begin{aligned}
1 = \xi^{1/\nu}\Bigg[r - r_c + \frac{uT^{(\theta_u+1/\nu)/z}}{z} \\
\times \int_T^{T\xi^z} \frac{d\zeta}{\zeta}\zeta^{-(\theta_u+1/\nu)/z} f(\zeta)\Bigg].
\end{aligned} \tag{4.15}$$

It is now possible to deduce scaling properties provided it is permissible in the critical region to set the lower and upper limits of the integral in (4.15) to zero and infinity, respectively. As $f(T)$ represents thermal contribution of short distance modes we expect it to vanish as $T \to 0$; these modes, however, do mix with the particle-hole continuum of the metal, leading us to expect a linear density of states at low energies even at short distances, and therefore $f(T) \sim T^2$ for small T. In the opposite large T limit, all modes must become classical, and therefore $f(T) \sim T$. For these asymptotic behaviors in $f(T)$, the limits on the integration can be extended provided $z\nu < 1 + \theta_u\nu < 2z\nu$. We then obtain at $r = r_c$ but T finite $\xi^{-1} \sim u^\nu T^{(1+\theta_u\nu)/z}$. In this same region, a similar reasoning implies that the local dynamic spin susceptibility will scale as

$$\chi''(\omega) = \omega^{(d-\theta-2+\eta)/2z}\phi\left(\frac{K\omega}{u^{z\nu}T^{1+\theta_u\nu}}\right), \tag{4.16}$$

for some universal scaling function ϕ, and nonuniversal constant K. At tree level this gives us a frequency scale $\sim T^{3/2}$ which is consistent with the results of Sec. III. Similarly, the position of the finite temperature spin glass to paramagnet boundary [at $r = r_c(T)$] will scale near the quantum critical point at $r = r_c$ and $T = 0$ as

$$r_c - r_c(T) \sim uT^{(1+\theta_u\nu)/z\nu}. \tag{4.17}$$

Again, this agrees with the tree-level result (3.3). A very similar result applies to the boundary between regions III and IV of Fig. 1 which occurs at $r - r_c \sim uT^{(1+\theta_u\nu)/z\nu}$, while the boundary between regions IV and V is at $r - r_c \sim T^{1/z\nu}$.

All of the results discussed so far in this section have been obtained using only rather general scaling ideas. In particular, they do not rely on the particular form of the action $\mathcal{A}$. We will now obtain a few results which *do* rely on explicit features of $\mathcal{A}$, and their validity is therefore somewhat more questionable.

A simple argument can be given to fix the value of z, using the manner in which time dependence enters into $\mathcal{A}$. Consider a correlator of the Q fields in which all external frequencies have been fixed at the same frequency ω. As the critical field theory is static, and because the Q field is bilocal in time, ω will act simply as an external source which shifts the value of the "thermal" coupling $r \to r + |\omega|$, as is apparent from the first two terms in $\mathcal{A}$; for insulating Ising and rotor models the corresponding shift is $r \to r + \omega^2$. As the scaling dimension of r is $1/\nu$, this gives us the scaling relation

$$z\nu = \begin{cases} 1 & \text{metallic spin glasses} \\ 1/2 & \text{insulating Ising} \\ & \text{and rotor spin glasses}\ . \end{cases} \tag{4.18}$$

We emphasize that both results rely on the assumption of a static critical theory; this assumption was not made in the analysis of Ref. 7. We also note in passing that the present argument fixing the value of z cannot be applied to the random-field quantum rotor model of Ref. 9 (which also had a static critical point), because the same external frequency ω does not flow through all internal propagators in this case, and some propagators are always at zero frequency.

We now ask whether there is a *classical* statistical mechanics field theory which is also described by the static critical point postulated above. We are only able to answer this question within the confines of perturbation theory: a perturbative expansion in κ suggests that the relevant field theory is that describing singularities along the imaginary field, ih, axis in a d-dimensional randomly diluted Ising ferromagnet.[21] The latter model has a Yang-Lee edge singularity[22] at the same value, $h = h_c^0$ as the nondiluted Ising ferromagnet.[23] Note, however, that $h_c^0 = 0$ in random Ising ferromagnets with an unbounded probability distribution for the local randomness. It has been argued[21] that there is critical field, $h = h_c$, such that for $h_c^0 < h < h_c$, the zeros of the partition function are analogous to the localized states in the band tail in Anderson localization. [The "Griffiths effects" (Ref. 23) leading to this region also have a parallel in the paramagnetic phase of the quantum spin glass.] The singularity at $h = h_c$ is then analogous to a mobility edge.[21] It is this singularity at $h = h_c > 0$, called the "pseudo Yang-Lee edge" (Ref. 21), that interests us here. The field theory for this singularity is:[24,25,21]

$$\mathcal{A}_{\rm YL} = \int d^d x \left\{ \frac{1}{t} \sum_a \left[i \frac{r}{\kappa} \phi^a(x) + \frac{1}{2} (\nabla \phi^a)^2 + i \frac{\kappa}{3} [\phi^a(x)]^3 \right] + \frac{1}{2t^2} \sum_{ab} \phi^a(x) \phi^b(x) \right\}, \tag{4.19}$$

where ϕ^a is the replicated order parameter for the Ising model. This field theoretic model was also considered earlier by Parisi and Sourlas,[26] who argued that for κ *imaginary*, $\mathcal{A}_{\rm YL}$ describes the statistics of lattice animals. As we will argue shortly, the perturbative RG equations for $\mathcal{A}_{\rm YL}$ are given precisely by (4.5), and a perturbative fixed point with κ imaginary can indeed be obtained in the $8 - d$ expansion. However, in this paper we are only interested in the case of κ real, which also describes the "pseudo Yang-Lee edge" (Ref. 21) in a random Ising ferromagnet.

Now we discuss the perturbative connection between models defined by $\mathcal{A}_{\rm YL}$ and $\mathcal{A}$. Consider the Feynman graph expansions with the action $\mathcal{A}_{\rm YL}$ for the correlators

$$G_{\rm YL}(x-y) = \frac{1}{n} \sum_a \langle\langle \phi^a(x) \phi^a(y) \rangle\rangle - G^d_{\rm YL}(x-y),$$
$$G^d_{\rm YL}(x-y) = \frac{1}{n(n-1)} \sum_{a \neq b} \langle\langle \phi^a(x) \phi^b(y) \rangle\rangle . \tag{4.20}$$

Compare this with the Feynman graph expansion with the action $\mathcal{A}$ of *zero frequency* correlators G and G^d, respectively. It is not difficult to show, term by term, that these two expansions are identical to all orders in κ and to leading order in t. The fact that $\mathcal{A}$ involves a matrix field (two replica indices) while $\mathcal{A}_{\rm YL}$ has scalar field (one replica index) does not affect any of the multiplicity factors associated with any graph; to leading order in t, all relevant graphs were tree graphs before averaging over the disorder in r_μ that corresponds to the $1/t^2$ vertex, and none of these graphs have numerical factors associated with summation over replica or vector indices. The equality of these perturbative expansions suggests, but does not establish, that the perturbatively inaccessible static fixed points of $\mathcal{A}$ and $\mathcal{A}_{\rm YL}$ may also be identical: if so, any scaling relations satisfied by $\mathcal{A}_{\rm YL}$ should apply also to $\mathcal{A}$.

The nonrandom Yang-Lee edge problem has a simple scaling structure[24]—there is a scaling relation between the exponents η and ν as the order parameter ϕ is also the "thermal operator." The simplest scaling hypothesis for the random case is that the identification of ϕ as the thermal operator continues to hold. This gives us the scaling relation

$$\frac{1}{\nu} = \frac{d - \theta + 2 - \eta}{2}. \tag{4.21}$$

Numerical tests of this scaling relation in the randomly diluted Ising model would be quite useful (numerical studies of the full quantum spin-glass problem are expected to be much more difficult). When combined with (4.18), (4.21) leads to a scaling relation between z, θ, and η which is very similar (or identical if $z\nu = 1$) to one considered recently by Kirkpatrick and Belitz[13] for the metal-insulator transition.

V. CONCLUSIONS

This paper has proposed a quantum field theory, defined by the action $\mathcal{A}$, for the low energy properties of metallic spin glasses in the vicinity of a $T = 0$ transition between a metallic paramagnet and a metallic spin glass. The mean-field phase diagram of the model as a function of a quantum coupling, temperature, and applied magnetic field was described. The phase transitions and crossovers in this phase diagram were argued to be characteristic of a zero temperature, *static* critical theory containing no dynamic quantum fluctuations. Quantum effects were shown to be dangerously irrelevant and controlled by a crossover exponent $-\theta_u = -2$.

Next an attempt was made to extend these results beyond mean-field theory, but found runaway flows to strong coupling for all spatial dimensions below $d = 8$. Nevertheless we used the insight gained from the mean-field theory to propose a set of scaling hypotheses. We assumed that the true critical theory also contained only static, randomness induced fluctuations, and the exponent θ_u controlling quantum effects took an unknown positive value. This had some important observable consequences, similar to those found in the mean-field theory:

(*i*) The nonlinear susceptibility had a weaker singularity at the critical point than might have been suggested by the usual scaling arguments. In particular, for a large enough θ_u a nondivergent cusplike singularity was also possible.

(*ii*) In a simple model of the renormalization-group flow equations, finite T dynamic response functions scaled as functions of $\omega/T^{1+\theta_u \nu}$, rather than the usual scaling as functions of ω/T.

(*iii*) Exponents associated with various crossovers in the vicinity of the $T = 0$ critical point were modified by θ_u.

While these results were directly motivated by our analysis of metallic spin glasses, it is possible that some of the scaling ideas are more general and could apply also to insulating Ising and rotor spin glasses and other $T = 0$ transitions in random quantum systems.

We conclude with a few remarks about experiments. In the thermodynamic and transport measurements on $Y_{1-x}U_xPd_3$ and related compounds (for a recent review see Ref. 27), evidence has been presented for "non-Fermi-liquid behavior," where the term "Fermi-liquid behavior" refers to the properties an isolated spin in a metal undergoing a conventional Kondo effect, and to those of region V of our phase diagram, Fig 1. It has been suggested that the non-Fermi-liquid behavior is due to single-ion effects leading to more exotic Kondo-like phenomena.[27] However, as we have shown in this paper, non-Fermi-liquid behavior can also arise from cooperative effects among the spin moments, as is the case in the quantum critical region III of Fig 1. Unfortunately, we do not yet have good estimates of the exponents in $d = 3$, making direct comparisons with experiments difficult. It would be useful to have more detailed experiments in the vicinity of $r = r_c$ which investigate the possible presence of the crossovers in Fig 1.

Note added. In a recent paper,[28] Sengupta and Georges have considered a model closely related to ours, and obtained results in general agreement with the mean-field theory of Sec. III.

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[1] P.A. Lee and T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985); D. Belitz and T.R. Kirkpatrick, *ibid.* **66**, 261 (1994).
[2] S.-k. Ma, C. Dasgupta, and C.-k. Hu, Phys. Rev. Lett. **43**, 1434 (1979); R.N. Bhatt and P.A. Lee, *ibid.* **48**, 344 (1982).
[3] M. Milovanovic, S. Sachdev, and R.N. Bhatt, Phys. Rev. Lett. **63**, 82 (1989); R.N. Bhatt and D.S. Fisher, *ibid.* **68**, 3072 (1992); C. Dasgupta and J.W. Halley, Phys. Rev. B **47**, 1126 (1993).
[4] S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
[5] D.A. Huse and J. Miller, Phys. Rev. Lett. **70**, 3147 (1993); J. Ye, S. Sachdev, and N. Read, *ibid.* **70**, 4011 (1993); J. Ye, Ph.D. thesis, Yale University, 1993.
[6] M. Guo, R.N. Bhatt, and D.A. Huse, Phys. Rev. Lett. **72**, 4137 (1994); H. Rieger and A.P. Young, *ibid.* **72**, 4141 (1994).
[7] N. Read, S. Sachdev, and J. Ye, Phys. Rev. B **52**, 384 (1995).
[8] W.D. Wu, A. Keren, L.P. Le, G.M. Luke, B.J. Sternlieb, Y.J. Uemura, C.L. Seaman, Y. Dalichaouch, and M.B. Maple, Phys. Rev. Lett. **72**, 3722 (1994).
[9] D. Boyanovsky and J.L. Cardy, Phys. Rev. B **27**, 5557 (1983).
[10] P.B. Weichmann, M. Rasolt, M.E. Fisher, and M.J. Stephen, Phys. Rev. B **33**, 4632 (1986).
[11] A.J. Millis, Phys. Rev. B **48**, 7183 (1993).
[12] D.S. Fisher, Phys. Rev. Lett. **69**, 534 (1992); Phys. Rev. B **50**, 3799 (1994).
[13] T.R. Kirkpatrick and D. Belitz, Phys. Rev. Lett. **74**, 1178 (1995).
[14] J.A. Hertz, Phys. Rev. B **19**, 4796 (1979).
[15] R. Oppermann and M. Binderberger, Ann. Phys. (N.Y.) **3**, 494 (1994).
[16] J.A. Hertz, Phys. Rev. B **14**, 525 (1976).
[17] S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
[18] L. Ioffe and A.J. Millis (unpublished); S. Sachdev, A. Chubukov, and A. Sokol, Phys. Rev. B **51**, 14874 (1995).
[19] S. Sachdev, Z. Phys. B **94**, 469 (1994).
[20] K.H. Fischer and J.A. Hertz, *Spin Glasses* (Cambridge University Press, Cambridge, England, 1991).
[21] J.L. Cardy and A.J. McKane, Nucl. Phys. B **257**, 383 (1985).
[22] C.N. Yang and T.D. Lee, Phys. Rev. **87**, 404 (1952); T.D. Lee and C.N. Yang, *ibid.* **87**, 410 (1951).
[23] R.B. Griffiths, Phys. Rev. Lett. **23**, 17 (1969).
[24] M.E. Fisher, Phys. Rev. Lett. **40**, 1610 (1978); J.L. Cardy, *ibid.* **54**, 1354 (1985).
[25] G. Grinstein, Phys. Rev. Lett. **37**, 944 (1976).
[26] G. Parisi and N. Sourlas, Phys. Rev. Lett. **46**, 871 (1981).
[27] M.B. Maple *et al.*, J. Low Temp. Phys. **95**, 225 (1994).
[28] A.M. Sengupta and A. Georges, following paper, Phys. Rev. B **52**, 10295 (1995).