# Universal Magnetic Properties of Frustrated Quantum Antiferromagnets in Two Dimensions

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We present a theory of frustrated, two-dimensional, quantum antiferromagnets in the vicinity of a quantum transition from a noncollinear, magnetically ordered ground state to a quantum-disordered phase. Using a sigma model for bosonic, spin-$\frac{1}{2}$, spinon fields, we obtain universal scaling forms for a variety of observables. Our results are compared with numerical data on the spin-$\frac{1}{2}$ triangular antiferromagnet.

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A useful classification of two-dimensional, quantum, Heisenberg antiferromagnets is provided by the structure of the magnetically ordered ground state: the spin condensates on the sites can either be collinear or noncollinear to each other. Collinear magnets have been extensively studied in recent years and many of their properties are reasonably well understood. They possess an O(3)/O(2) order parameter whose fluctuations describe the low temperature ($T$) properties of the magnetically ordered state [1]. The quantum-disordered state has only integer spin excitations (the spinons are confined) and spin-Peierls order is expected for certain values of the single-site spin [2]. The finite-$T$ crossover between these two states has also been studied in some detail [3].

Less is known, however, about noncollinear antiferromagnets, which are the subject of this paper. Examples include the triangular, kagomé, and square (with first, second, and third neighbor interactions) lattices. The magnetically ordered state completely breaks the spin-rotation symmetry, yielding an SO(3) order parameter [4]. Space and time dependent twists of this order parameter then define three independent spin stiffnesses, spin susceptibilities, and associated spin-wave velocities. For simplicity, we will restrict our attention here to magnets with *coplanar* spins and an internal symmetry (a $C_{3v}$ symmetry on the triangular and kagomé lattices, and a screw axis symmetry for the incommensurate planar spirals on the square lattice), which leads to just two independent stiffnesses ($\rho_\perp$, $\rho_\parallel$), susceptibilities ($\chi_\perp$, $\chi_\parallel$), and spin-wave velocities [$c_\perp = (\rho_\perp/\chi_\perp)^{1/2}$, $c_\parallel = (\rho_\parallel/\chi_\parallel)^{1/2}$]; more complicated noncollinear magnets will have similar properties. The long-wavelength action for the SO(3) order parameter has an SO(3)×O(2) symmetry, the O(2) being a continuum manifestation of the internal symmetry noted above [4]. A spacetime dimension $D = 2 + \epsilon$ study of small fluctuations of the SO(3) order parameter about the magnetically ordered state was performed by Azaria *et al.* [5]; they found that the stiffnesses and susceptibilities became asymptotically equal upon approaching the critical point separating the magnetically ordered and quantum-disordered phases, with the critical theory possessing an enlarged O(4) symmetry. A large $N$ theory based upon Sp($N$) symmetry [6] found a similar magnetically ordered state, but was also able to access the quantum-disordered phase. The latter state was predicted to be a featureless, fully gapped spin fluid, with unconfined, bosonic spin-$\frac{1}{2}$ spinon excitations. We also note that there are alternative approaches to the quantum disordered phase [7] which are quite disconnected from the structure of the ordered state.

In this paper, we shall present a theory of the universal, finite-$T$ properties of noncollinear antiferromagnets in the vicinity of the critical point. We will describe the crossover from the magnetically ordered state, with its low-lying spin-wave excitations, to the fully gapped quantum-disordered state via an intermediate quantum-critical region. Our results are in complete agreement with some previous studies of the magnetically ordered state [5] and the quantum disordered state [6], and establish a fundamental connection between the O(4)-symmetric critical point of Ref. [5] and the deconfined bosonic spinons of Ref. [6]; a related connection was noted recently in Ref. [8]. We will also obtain new results for the low $T$ behavior of the dynamic structure factor and uniform susceptibility of magnetically ordered antiferromagnets.

Our motivation for this study is similar to that for the analogous recent study of collinear antiferromagnets [3]. A given $S = \frac{1}{2}$ antiferromagnet may be either magnetically ordered (as is expected for the triangular lattice) or quantum disordered (the kagomé lattice) [9]. At low $T$, the magnetically ordered magnet has thermally excited classical spin-wave fluctuations [the renormalized-classical (RC) region], while the quantum-disordered magnet has only activated deviations from its ground-state properties. At higher $T$, however, both of these magnets are expected to cross over to a quantum-critical [1] (QC) region where classical and thermal fluctuations are equally important. Many properties of this region are universal, and are thus amenable to numerical and experimental tests. In particular, there are significant quantitative differences between the QC behavior of collinear and noncollinear magnets, which are a consequence of the presence of deconfined spinons in the latter.

We begin by presenting our effective action. We choose to describe the local spin configuration by an SU(2) rotation about a reference ordered state. The choice of SU(2) rather than SO(3) is significant, and has the immediate consequence of suppressing the vortices [10] associated with $\pi_1(\mathrm{SO}(3)) = Z_2$ for which the SU(2) field is double valued. This choice is motivated partly by the results of Ref. [6], where vortices were suppressed in the quantum-disordered phase by a Higgs condensate. We parametrize the SU(2) matrix by two complex numbers $z_1$, $z_2$ with $|z_1|^2 + |z_2|^2 = 1$, and write down the most general, long-wavelength action with an SU(2)×O(2) invariance:

$$\mathcal{S} = \int d^2x\, d\tau \sum_{\mu=x,\tau} \frac{1}{g_\mu}\left[\partial_\mu z^\dagger \partial_\mu z - \frac{\gamma_\mu}{4}\left(z^\dagger \partial_\mu z - \partial_\mu z^\dagger z\right)^2\right].$$

It is easy to show that $g_x = 1/2\rho_\perp^0, g_\tau = 1/2\chi_\perp^0, \gamma_x = (\rho_\parallel^0 - \rho_\perp^0)/\rho_\perp^0, \gamma_\tau = (\chi_\parallel^0 - \chi_\perp^0)/\chi_\perp^0$, where the superscript 0 denotes bare values; note that if the $\gamma_\mu = 0$, $\mathcal{S}$ has an enlarged O(4) symmetry, and is also Lorentz invariant. The action $\mathcal{S}$ can be explicitly derived by a long-wavelength analysis of the models of Refs. [5] and [6]; we have also learned of a recent study of $\mathcal{S}$ by Azaria *et al.* [11]. The staggered spin-structure factor (wave vectors measured as deviations from the ordering wave vector $\mathbf{G}$) can be shown to be the Fourier transform of $\frac{1}{2}\mathrm{Re}(z^\dagger(x_1,\tau_1)z(x_2,\tau_2))^2$. Note that this is *quartic* in the $z$, consistent with the identification of the $z$ quanta as spin-$\frac{1}{2}$ bosonic spinons.

We studied $\mathcal{S}$ by generalizing $z$ to an $N$-component, unit-length, complex vector, and performing a $1/N$ expansion; $\mathcal{S}$ then has a SU($N$)×O(2) invariance, while for $\gamma_\mu = 0$ it is invariant under O($2N$). This method allows us to work directly in $D = 2+1$ and access both the QC and RC regions. Note that the extension to large $N$ is different from that used in Refs. [8,12].

We expect that $\mathcal{S}$ possesses quantum-disordered and magnetically ordered phases (with the $z$ quanta condensed) as the couplings (say $g_x$) are varied. A key property of the present large $N$ expansion is that the long-distance physics at the critical point at $g_x = g_c$ is O($2N$) symmetric and Lorentz invariant. This is manifested in the magnetically ordered phase ($g_x < g_c$) by the critical behavior of the stiffnesses. Josephson scaling is obeyed by the fully renormalized $\rho_\parallel$, $\rho_\perp$, $\chi_\parallel$, $\chi_\perp$, all of which vanish as $(g_c - g_x)^\nu$, where $\nu$ is the correlation length exponent [$\nu = 1 - 16/3N\pi^2 + \mathcal{O}(1/N^2)$]. However, the relative differences between the stiffnesses also vanish at the critical point: we defined $\Delta_1 = (\rho_\parallel - \rho_\perp)/\rho_\perp$, $\Delta_2 = (\chi_\parallel - \chi_\perp)/\chi_\perp$, and found

$$\Delta_1 = \gamma_1(\xi_J)^{-\phi_1} + \gamma_2(\xi_J)^{-\phi_2},$$
$$\Delta_2 = \gamma_1(\xi_J)^{-\phi_1} - 2\gamma_2(\xi_J)^{-\phi_2}, \qquad (1)$$

where $\gamma_1 = (2\gamma_x + \gamma_\tau)/3$; $\gamma_2 = (\gamma_x - \gamma_\tau)/3$ (these are the spin-0 and spin-2 combinations under the Lorentz group), and $\xi_J$ is the Josephson length measured in lattice units. The positive crossover exponents $\phi_{1,2}$ measure the irrelevancy of the $\gamma_\mu$ terms in $\mathcal{S}$; the $\gamma_\mu$ are actually "dangerously" irrelevant as $\Delta_{1,2}$ control long-wavelength physics for $g_x < g_c$. To order $1/N$, we found $\phi_1 = 1 + 32/3\pi^2 N$, $\phi_2 = 1 + 112/15\pi^2 N$ [13].

We now present our scaling results for the wave vector ($k$) and frequency ($\omega$) dependent staggered ($\chi_s$) and uniform ($\chi_u$) spin susceptibilities in the vicinity of $g_x = g_c$. We restrict ourselves to $g_x < g_c$, although more complete results have been obtained [14]. We found

$$\chi_s(k,\omega) = \frac{2\pi N_0^2}{N\rho_\perp}\left(\frac{\hbar c_\perp}{k_B T}\right)^2 \left(\frac{N k_B T}{4\pi\rho_\perp}\right)^{\overline{\eta}} \times \Phi_s\left(\overline{k}, \overline{\omega}, x, \Delta_1, \Delta_2\right),$$
$$\chi_u(k,\omega) = \left(\frac{g\mu_B}{\hbar c_\perp^2}\right)^2 k_B T\, \Phi_u\left(\overline{k}, \overline{\omega}, x, \Delta_1, \Delta_2\right), \qquad (2)$$

where $N_0$ is the on-site magnetization at $T = 0$, $\Phi_{1s}$, $\Phi_{1u}$ are universal functions of the dimensional variables $\overline{k} = \hbar c_\perp k/k_B T$, $\overline{\omega} = \hbar\omega/k_B T$, $x = N k_B T/4\pi\rho_\perp$. We found the exponent $\overline{\eta} = 1 + 32/3\pi^2 N$. The prefactor of $\Phi_s$ remains nonsingular at $g_x = g_c$ as $N_0 \sim (g_c - g_x)^{\overline{\beta}}$ with $2\overline{\beta} = (1+\overline{\eta})\nu$. All scaling functions are defined such that they remain finite as $x \to \infty$. As before [3], the argument $x$ determines whether the system is in the QC ($x \gg 1$) or RC ($x \ll 1$) region.

An important difference in the above scaling forms from those for collinear magnets [3] is in the value of $\overline{\eta}$. Here we have $\overline{\eta}$ close to unity, while the analogous exponent for collinear magnets was close to zero. This is a consequence of the presence here of deconfined spinons: it is the $z$ quanta which behave like almost free particles (at $T = 0$, $\langle z^\dagger z\rangle \sim 1/k^{2-\eta}$ with $\eta$ close to 0) while the staggered susceptibility is a correlator of a composite operator of two spinons ($\chi_s \sim 1/k^{2-\overline{\eta}}$ with $\overline{\eta}$ close to 1).

We have computed $\Phi_s$, $\Phi_u$ in a $1/N$ expansion to linear order in $\Delta_{1,2}$. We describe our results as they relate to various observables.

*Correlation length.*—As in collinear magnets, we define the correlation length, $\xi$, from the long-distance $e^{-r/\xi}$ decay of the equal-time spin-spin correlation function. We found that, to order $1/N$, there is a simple relationship between the values of $\xi$ for collinear and noncollinear magnets. For all values of $x$, the noncollinear $\xi$ is precisely $\frac{1}{2}$ the previously computed $\xi$ [3] for the *isotropic* O($2N$) sigma model. The factor of $\frac{1}{2}$ is a signature of deconfined spinons. The collinear expression for $\xi$ [3], however, must be used with the effective values $\rho_s = \rho_\perp[1 + N\Delta_1/(2N^2-2)]$, $\chi = \chi_\perp[1 + N\Delta_2/(2N^2-2)]$, and $c = (\rho_s/\chi)^{1/2}$; notice also the factor of 4 difference in the coupling constant in $\mathcal{S}$ and in [3]. For the physical case $N = 2$, we have to first order in $\Delta_{1,2}$ that $\rho_s = (2\rho_\perp + \rho_\parallel)/3$, $c = (2c_\perp + c_\parallel)/3$, and our RC result for $\xi$ is then consistent with that of Ref. [5].

*Static uniform susceptibility.*—The result for $\chi_u$ is obtained by evaluating the response to a vector potential coupled to the conserved charge of the SU($N$) symmetry.

In the RC region ($Nk_BT \ll 4\pi\rho_s$) we obtained

$$\chi_u = \left(\frac{g\mu_B}{\hbar}\right)^2 \left(\frac{N\chi_\perp + \chi_\parallel}{(N+1)\chi_\perp}\right) \left[\frac{2\chi_\perp}{N} + \frac{N-1}{N}\frac{k_BT}{2\pi c^2}\right].$$

It is worth emphasizing that although we are considering an essentially classical regime, the $T$ dependence of $\chi_u$ is a purely quantum effect—it disappears if the spin waves had a classical, thermal distribution.

In the QC region ($Nk_BT \gg 4\pi\rho_s$), we found to order $1/N$,

$$\chi_u = \left(\frac{g\mu_B}{\hbar c}\right)^2 k_BT\frac{\sqrt{5}\Theta}{4\pi}\left[\left(1-\frac{0.31}{N}\right) + \alpha\,\bar{x}^{-1/\nu} + \cdots\right],$$

where $\Theta = 2\ln[(\sqrt{5}+1)/2]$, $\bar{x} = Nk_BT/4\pi\rho_s$, and $\alpha = 0.8 + \mathcal{O}(1/N)$. Note that the slope of the linear in $T$ term is precisely $\frac{1}{2}$ of that in the O($2N$) sigma model [3]. The factor of $\frac{1}{2}$ is again a signature of spin-$\frac{1}{2}$ spinons and should be amenable to experimental tests.

*Staggered dynamic susceptibility and structure factor.*—In the RC region, the scaling form (2) for $\chi_s$ collapses into a reduced scaling form in which the physical $\xi$, rather than $c/k_BT$, is the most important length scale [1,3]:

$$\chi_s(k, i\omega_n) = \frac{N_0^2}{\rho_s(N-1)}\left[\frac{k_BT(N-1)}{4\pi\rho_s}\right]^{(N+1)/(N-1)} \times \xi^2 f(k\xi, \omega_n\xi/c), \tag{3}$$

where $f$ is a scaling function. Note that computations were in fact done only to order $1/N$—the form at arbitrary $N$ follows from a reasonable guess about the wave function renormalization of the composite field. The overall factor in (3) is chosen such that $f(0,0) \doteq 1 + \mathcal{O}(1/N)$. The behavior of $f(x,y)$ at intermediate $x, y = \mathcal{O}(1)$ is rather complicated, chiefly because spin-wave velocity also acquires a substantial downturn renormalization at $k\xi = \mathcal{O}(1)$ [1]. However, at $k\xi \sim \omega\xi/c \gg 1$, velocity renormalization is irrelevant and we obtained

$$f(x,y) = \left(\frac{N-1}{N+1}\right)\frac{1}{x^2+y^2}\left[\frac{\ln(x^2+y^2)}{2}\right]^{\frac{N+1}{N-1}}. \tag{4}$$

One can demonstrate that this result for $f(x,y)$ yields a $\chi_s(k,\omega)$ which is precisely the rotationally averaged spin-wave result for the ordered antiferromagnet, as it of course should be at $k\xi \gg 1$ but $k\xi_J \ll 1$.

We also computed Im$\chi_s(k,\omega)$ for real $\omega$. In the RC region, we describe the results using the dynamic structure factor $S(k,\omega)$ which satisfies

$$S(k,\omega) = N_0^2\left(\frac{k_BT}{\rho_s}\right)^2\left(\frac{\bar{\Xi}(k,\omega)}{\omega}\right), \tag{5}$$

where $\bar{\Xi}$ is straightforwardly related to $\Phi_s$ introduced in (2). For experimental comparisons, it is sufficient to consider the frequency range $\omega \le c/\xi$. We then found $\bar{\Xi}(k,\omega) = \omega/2\pi ck^3$ for $ck \gg \omega$ (in this region of $k$, collisionless Landau damping is dominant), and $\bar{\Xi}(k,\omega) \propto (\omega\xi^3/c)\;[(N-1)k_BT/4\pi\rho_s]^{(5-N)/2(N-1)}$ for $ck \sim \omega$ (the dominant contribution is the damping of quasiparticles).

Now the QC region. Here we restrict our results to the critical point $x = \infty$, and negligible anisotropy ($\Delta_{1,2} = 0$). For $\hbar ck, \hbar\omega \gg k_BT$ we obtained

$$\Phi_s = \frac{A_N}{16[\overline{k}^2 - (\overline{\omega}+i\delta)^2]^{1-\overline{\eta}/2}}, \tag{6}$$

where $A_N = 1 + \mathcal{O}(1/N)$. At small $k$ and $\omega$, we have Re$\Phi_s = (\sqrt{5}/16\pi\Theta)\{1 - [\overline{k}^2(1+2\Theta/\sqrt{5}) - \overline{\omega}^2]/12\Theta^2 + \cdots\}$ where $\cdots$ stand for higher powers of $\overline{k}, \overline{\omega}$ and for regular corrections in $1/N$. For Im$\Phi_s$ we obtained the following asymptotic limits for large $N$:

$$\mathrm{Im}\Phi_s = \begin{cases} \dfrac{A_N\sin(\pi\overline{\eta}/2)}{16}\,\dfrac{\theta(\overline{\omega}^2-\overline{k}^2)}{(\overline{\omega}^2-\overline{k}^2)^{1-\overline{\eta}/2}}, & \overline{\omega} \gg 1, \\ \dfrac{\overline{\omega}}{8\sqrt{\pi}}\,\dfrac{e^{-\overline{k}/2}}{\overline{k}^{3/2}}, & \overline{\omega} \ll 1,\ \ \overline{k} \gg 1, \end{cases}$$

where $\theta(x)$ is a step function. In both cases, Landau damping is dominant. Finally, when both $\overline{\omega} \ll 1$ and $\overline{k} \ll 1$, quasiparticle excitations are overdamped and we only know that Im$\Phi_s \propto \overline{\omega}$.

In the $T = 0$ quantum-disordered phase, Im$\Phi_s$ shows a clear signature of deconfined spinons—the spectral weight at fixed $k$ is a broadband continuum rather than the delta-function peak present in collinear magnets [14].

*Local susceptibility and spin-lattice relaxation.*—The local dynamic structure factor $S(\omega)$ is given by $S(\omega) = \int d^2k\; S(k,\omega)/4\pi^2$. Simple inspection then shows that for $\omega \sim c\xi^{-1}$, the integration over momentum is also confined to $k \sim \xi^{-1}$ and therefore $S(\omega)$ is a universal observable. The small frequency limit of $S(\omega)$ is directly related to the spin-lattice relaxation rate of nuclear spins coupled to electronic spins in the antiferromagnet: $1/T_1 \propto S(\omega \to 0)$. In the RC region, we find, using our previous results for the scaling functions, that for $\omega \sim c\xi^{-1}$

$$S(\omega) \propto \frac{N_0^2\xi}{c}\left(\frac{(N-1)k_BT}{4\pi\rho_s}\right)^{(3N+1)/2(N-1)}. \tag{7}$$

For $N = 2$, we then have $1/T_1 \propto T^{7/2}\,\xi$.

Deep in the QC region, we found

$$S(\overline{\omega}) = \frac{4\pi\hbar N_0^2}{N\rho_\perp}\left(\frac{Nk_BT}{4\pi\rho_\perp}\right)^{\overline{\eta}}\frac{K(\overline{\omega})}{1-e^{-\overline{\omega}}}, \tag{8}$$

where $K(\overline{\omega}) = \overline{\omega}B_N\sin(\pi\overline{\eta}/2)/32\pi$ at $\overline{\omega} \gg 1$, and $K(\overline{\omega}) = \overline{\omega}C_N(\sqrt{5}-1)/64\pi$ at $\overline{\omega} \ll 1$. The factors $B_N$ and $C_N$ both behave as $1+\mathcal{O}(1/N)$. Clearly then, $1/T_1 \propto T^{\overline{\eta}}$.

*Static structure factor.*—Unlike collinear magnets, the static structure factor $S(k) = \int d\omega\, S(k,\omega)/2\pi$ in noncollinear antiferromagnets is nonuniversal because the frequency integral over quantum fluctuations is divergent. This follows from the behavior at large frequencies where $S(k,\omega) \propto 1/\omega^{2-\overline{\eta}}$ and $\overline{\eta} > 1$. The nonuniversality is, however, more relevant for the QC region, where

$T$ is the only scale for fluctuations; in the RC region, $\xi$ is exponentially large, and there is a universal contribution to $S(k)$ from classical fluctuations which scales as $\xi^2$. In the RC region we then have $S(k) \approx k_B T \chi_s(k,0)$, where $\chi_s(k,0)$ is given by (3). At $k = 0$ this yields $S(0) \propto T^{2N/(N-1)}\ \xi^2$. For $N = 2$, $S(0) \propto T^4 \xi^2$.

*Application to the $S = \frac{1}{2}$ triangular antiferromagnet.*—We performed a $1/S$ expansion on this antiferromagnet to obtain the $T = 0$ values of $\rho_\perp$, $\rho_\|$, $\chi_\perp$, $\chi_\|$ (all to order $1/S$), and $N_0$ (to order $1/S^2$). For $S = \frac{1}{2}$ this gave us $N_0 = 0.266$, $\chi_\perp = 0.09/Ja^2$, $\chi = 0.084/Ja^2$, $\rho_s = 0.086J$, and $c = (\rho_s/\chi)^{1/2} = 1.01Ja$. For the uniform susceptibility in the RC regime we then obtained $\chi_u = (g\mu_B/\hbar a)^2[0.08/J + 0.07k_BT/J^2 + \mathcal{O}(T^2/J^3)]$. On the other hand, in the QC regime, we have $\chi_u = (g\mu_B/\hbar a)^2\,[0.14k_BT/J^2 + 0.07/J\,(k_BT/2\pi\rho_s)^{1-1/\nu} + \cdots]$. The temperature dependence in the subleading term is likely to be quite small in the region of experimental interest ($k_BT \sim 2\pi\rho_s$), and we can well approximate this term by a constant. Note, however, that the factor 0.07 is an $N = \infty$ result—the $1/N$ corrections to this factor have not been computed. Further, the correlation length behaves in the RC regime as $\xi \approx 0.24\,(4\pi\rho_s/k_BT)^{1/2}\ \exp[4\pi\rho_s/k_BT]$ where $4\pi\rho_s \approx 1.08J$, and deep in the QC region as $\xi = 0.51Ja/k_BT$.

Consider now the numerical results for $\chi_u$. The data of recent series expansion studies [15] show that $\chi_u$ obeys a Curie-Weiss law at high $T$, passes through a maximum at $T \approx 0.4J$, and then falls down. The region below the maximum is quite small; nevertheless, we fitted this data by a straight line and found $0.13 \pm 0.03$ for the slope and about 0.06 for the intercept—both results in better agreement with our QC expression than the RC result. Finally, at very low $T$, we expect a crossover to the RC regime, and the corresponding value of $\chi_u$ at $T = 0$ is also consistent with the data. We also compared the data for the correlation length and $S(0)$ at $k_BT \sim 0.4J$ and found rough consistency with our expressions in the crossover region between QC and RC regimes. Note that our interpretation of the numerical data is different from that in Ref. [15].

To conclude, we have presented a theory of the critical properties of noncollinear quantum antiferromagnets in two dimensions. Our key assumption was on the validity of a continuum description in SU(2) variables, which suppressed vortex excitations. However, we were then able to show that our results were consistent with earlier large $N$ [6] and $D = 2+\epsilon$ [5] studies. The quantum disordering transition was described by an anisotropic sigma model for spin-$\frac{1}{2}$, bosonic spinon fields. All physical observables involve a collective mode of two spinons, and we computed explicit scaling forms for a variety of experimentally measurable quantities. Our results for $\chi_u$ in the QC region are consistent with recent numerical data on the spin-$\frac{1}{2}$ triangular antiferromagnet [15]; this may be viewed as some indirect evidence for the presence of deconfined spinons. However, numerical results also seem to indicate that the $T$ range where QC behavior may be observed is rather narrow for this system. More detailed studies, especially in quantum-disordered noncollinear magnets, will be quite useful.

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