Magnetic properties across the metal-insulator transition (invited)

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We review recent theoretical and experimental work on the metal to insulator transition in doped semiconductors. The spin excitations on the insulating side of the transition can be described in terms of a spin-$\frac{1}{2}$ Heisenberg antiferromagnet with the spins randomly located in space. Numerical analysis of this Hamiltonian has led to a fairly complete understanding of the static spin susceptibility and the electron spin resonance spectrum. The physics on the metallic side far from the transition is also fairly well understood in terms of a recently developed theory of the disordered Fermi liquid. The physics near the transition point is, however, still not clear. All the experimental evidence indicates the presence of local electronic moments on the metallic side of the transition. Recent theoretical work and open problems in the description of such a metallic phase are briefly discussed.

I. INTRODUCTION

This paper presents a review of recent theoretical and experimental work on the study of the metal-insulator transition in doped semiconductors. We will focus, in particular, on phosphorus silicon (Si:P) as a prototype system on which a large body of experiments have been performed recently.[1] Although the original studies on this system were performed two decades ago,[2] there remain gaps in our understanding of the physics, especially at doping concentrations n which are close to the metal-insulator transition density n_c.

We begin by presenting an overview of the properties of doped semiconductors as a function of doping concentration. The physics is best understood by considering three regions of dopant concentration: (i) insulating, (ii) transitional, and (iii) metallic. These regions are discussed briefly below, with the details being relegated to the subsequent sections.

At very low phosphorus doping concentrations the extra electron carried by phosphorus remains bound to the excess positive charge carried by the phosphorus nucleus. The strong Coulomb interactions between electrons coupled with the strong positional disorder prevent any charge transport and the system is an insulator.[3] However, the weak orbital overlap between neighboring electrons leads to an antiferromagnetic interaction which dominates the magnetic properties.[4] A good model for the low-lying excitations is therefore a spin-$\frac{1}{2}$ Heisenberg Hamiltonian with the spins located randomly in space. At high temperatures the electron spins fluctuate independently of one another and yield a Curie susceptibility. As the temperature is lowered, the spin fluctuations are gradually quenched and only a smaller number of spin degrees of freedom remain active. In Sec. II we briefly discuss a simple picture of condensation of electrons into tightly bound singlet pairs which gives a satisfactory explanation of the experimental data.

With an increase in phosphorus density, the overlap between the electron orbitals increases and the system undergoes an insulator to metal transition. Stress-tuning experiments[5] have established that this transition is continuous, i.e., the conductivity σ increases continuously from zero as n goes above n_c: $\sigma \sim (n - n_c)^{\mu}$. The exponent μ is expected to be a *universal* feature of the transition: in other words, it is independent of the details of the microscopic interactions and just a property of the scale-invariant critical point describing the transition. Experimentally μ has been found to be near 0.5 in uncompensated semiconductors like Si:P but its value in compensated semiconductors (e.g., silicon doped with a donorlike phosphorus and an acceptorlike boron) is near 1.[6] A number of theoretical works have addressed the question of the exponents but have not been successful in clarifying the experimental situation. In comparing theory to experiments it is important to keep in mind that the finite-temperature experiments may not be measuring the true asymptotic critical behavior of the putative zero temperature scale–invariant theory. If this is the case, then it is clearly necessary to focus on the "nonuniversal" finite temperature thermodynamic and magnetic properties in the transitional region for a complete understanding of the physics.

As one increases the density further, the system goes into a noncritical metallic phase. This region can be described by a disordered version of Landau's Fermi liquid theory.[7–10] The properties of this Fermi liquid theory are very similar to those of the conventional Fermi liquid theory of clean metals. Charge, spin, and energy diffusion occurs by the excitation of long-lived quasiparticles. Theoretical calculations of the thermodynamic and magnetic properties of the system are possible in this region: the dimensionless parameter measuring the strength of the disorder is small and can form the basis of an expansion in perturbation theory. Further details on this point of view and comparison with experiments are discussed in Sec. III.

The theoretical challenge that remains is to come up with a theory of the transition region which is capable of explaining the transport (conductivity), thermodynamic (specific heat and spin susceptibility), and dynamic spin (electron spin resonance and nuclear magnetic resonance) properties. The transition region is bracketed by two differ-

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ent types of behavior, both of which are amenable to description by a simple intuitive picture: antiferromagnetically interacting electron local moments on the insulating side and non-plane-wave quasiparticles on the metallic side. The transition region exhibits features of both quasiparticle and local moment like behavior. In particular, as we shall discuss in Sec. IV, the experiments now appear to present unambiguous evidence of the presence of local electron moments on the metallic side of the transition. The interaction of the itenerant quasiparticles with the local moments needs to be understood in greater detail. The problem is clearly enormously complicated: the interaction of a signal local moment with electrons in a clean metal is by itself nontrivial[11] and the present situation involves many local moments interacting with electrons in a dirty metal. Nevertheless, some progress can be made at a phenomenological level, leading to experimentally testable predictions. Recent attempts by one of us[12] in this direction are described in Sec. IV.

The remainder of the paper is organized as follows: Sec. II reviews work on the insulating side of the transition while Sec. III does the same for the metallic side of the transition. Section IV presents a discussion of the theoretical implications of experiments in the transition region and a brief discussion of a phenomenological theory of this phase.[12]

II. THE INSULATING PHASE

This section will be brief because the situation on the insulating side of the transition has recently been reviewed in separate articles.[13,14] We only present enough details to place the discussion on the metallic side of the transition in context.

Since there is no charge transport in the insulating phase, it is useful to focus solely on the spin excitations of the system. These can be described by the Hamiltonian

$$H = \sum_{i \neq j} J_{ij}\, \mathbf{S}_i \cdot \mathbf{S}_j \,, \tag{1}$$

where $\mathbf{S}_i$ is a spin-$\frac{1}{2}$ operator for the electron localized around the impurity phosphorus nucleus at $\mathbf{R}_i$. The exchange constant J_{ij} is positive and depends exponentially upon the distance between $\mathbf{R}_i$ and $\mathbf{R}_j$.[4] The positions of the impurity nuclei $\mathbf{R}_i$ are assumed to be distributed randomly in the silicon crystal. This random Heisenberg Hamiltonian has formed the basis of our understanding of many experiments on the insulator.

An important experimental probe is electron spin resonance (ESR). The total weight under the ESR peak gives the electron spin paramagnetic susceptibility. In Fig. 1 we show the electron spin susceptibility measured in this manner for a variety of dopant concentrations n. We focus on concentrations $n < n_c$ in this section. The temperature dependence of the susceptibility $\chi(T)$ is described very well by the phenomenological equation

$$\chi(T) = A(n) T^{-\alpha} \,, \tag{2}$$

where $A(n)$ depends on the density and the exponent $\alpha \approx 0.6$.

This temperature dependence of the paramagnetic susceptibility can be understood in terms of a gradual freezeout

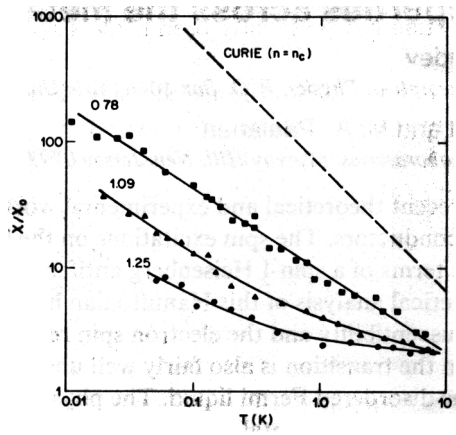

FIG. 1. Temperature dependence of the paramagnetic spin susceptibility of Si:P for different values of the phosphorus concentration n/n_c. The metal-insulator transition occurs at n_c.

of the electron moments due to their mutual exchange interactions. As the temperature is lowered an increasing number of electrons condense into tightly bound singlet pairs with their nearest neighbors; only the unbound spins contribute a Curie-type susceptibility ($\chi \sim 1/T$). The decrease in the number of unbound spins with temperature is therefore the reason for the slower than $1/T$ temperature dependence of Eq. (2). A computer renormalization group calculation by Bhatt and Lee[15] has put this picture on a more rigorous foundation and also yielded results for $\chi(T)$ which have temperature dependence similar to Eq. (2).

In addition to the variation in the total spin susceptibility, the ESR linewidth and resonance field also exhibit nontrivial dependence on temperature and frequency.[16] This can be understood in terms of a slowing down of the diffusion of spin excitations which must accompany the condensation of the singlet pairs.[17] Over the timescale probed by ESR, the small diffusion coefficient localizes the electronic excitations on small clusters (sizes $\leqslant 20$). On such small clusters the secular approximation for evaluating the ESR spectrum breaks down. It is necessary to resort to computer diagonalizations of the spin Hamiltonian; these are fortunately tractable because of the small cluster sizes. The results fit the experiments well, confirming the localized nature of the spin excitations.

III. THE METALLIC PHASE

On the metallic side of the metal-insulator transition powerful analytic methods are available for the calculation of the magnetic, transport, and thermodynamic properties of the system.[7,8] All these methods rely upon a perturbation expansion in the disorder present in the system. Deep on the metallic side of the transition, where the large number of dopant electrons raises the Fermi energy E_F, it can be shown that E_F is the largest energy in the problem rather than disorder and electron-electron interactions; consequently the perturbation expansion may be expected to work reasonably. As we shall discuss in this section, it is in this region that the analytic methods have been most sucessful. The perturbation methods can also be combined with renormalization-

group methods to make predictions all the way up to the metal-insulator transition but these have been far less successful.

The zeroth-order description for the metallic side of the transition is the system with no disorder. This system is assumed to be described completely by Fermi liquid theory. Spin, charge, and energy transport in the system occurs via movement of quasiparticle excitations. The quasiparticles have mass m^* interact with each other via the Fermi liquid constants A_l^s and A_l^a in the density and spin channels respectively. The usual Landau interaction constants F_l^s and F_l^a are related to these by

$$F_l^s \quad A_l^s/[1-A_l^s/(2l \quad 1)] \tag{3}$$

and similarly for F_l^a. For technical reasons, the A parameters are more convenient in disordered systems. Performing and physically interpreting a consistent perturbation theory about this state is not easy; although the initial insights for Altshuler and Aronov[18] came eight years ago, it is only recently that a proper physical understanding of the structure of the perturbation theory has emerged.[7,8,10] The perturbation expansion is easiest to organize in two dimensions where one obtains logarithmically divergent terms involving the infrared cutoff (the larger of the temperature T or measurement frequency ω). Keeping terms to leading order in powers of $|\ln T\tau|/(E_F\tau)$, where τ is the elastic scattering time, it is then argued that a resummation of these same terms capture the important physics in $2+\epsilon$ dimensions. Following the example of classical phenomena, it is hoped that this will yield useful results in three dimensions. Such a procedure yields a description of the metallic state in terms of a disordered Fermi liquid. This Fermi liquid has several features which are identical with the clean system: the low-lying excitations are quasiparticle and quasihole-like states which are responsible for charge, spin, and energy *diffusion*. The quasiparticle wave functions, however, are no longer planewave-like: the elastic scattering from the impurities yields wave functions which average over the nonzero angular momenta interaction constants and only the A_0^a and A_0^a terms survive. Another important feature of this disordered Fermi liquid is that the interaction constants and the quaisparticle weights are now *scale* dependent; the temperature T (frequency ω) defines relevant length scale of the system $\sqrt{D/T}$ ($\sqrt{D/\omega}$) where D is the diffusion constant.

Far away from the metal-insulator transition this theory can be used to make definite predictions. The conductivity $\delta(T)$ is predicted to have the temperature dependence[19–21]

$$\frac{\sigma(T)}{\sigma_{\rm fl}} = 1 + \frac{\beta}{(2E_F\tau)^{1+\epsilon}}\left(4 \quad 3\frac{1+|A_0^a|}{|A_0^a|}\ln(1+|A_0^a|)\right)(T\tau)^{\epsilon/2} \tag{4}$$

in a space of dimension $2+\epsilon$ where $\sigma_{\rm fl}$ is the conductivity and A_0^a a Fermi liquid interaction constant of the clean Fermi liquid, and β is phase-space factor of order unity. We replace ϵ by 1 in the above expression to get an estimate of the conductivity in three dimensions. The resulting $\sqrt{T}$ term in the conductivity can have either sign depending upon the magnitude of A_0^a. This expression has been fit successfully to the temperature dependence of the conductivity in Ge:Sb.[22] The coefficient of the $\sqrt{T}$ term is arbitrary in the fit and can be used to determine the value of the Fermi liquid interaction constant. A more stringent test would require the use of this known interaction constant in some other experiment, but this has not been done for Ge:Sb. In Si:P, the physics is complicated by the presence of six valleys in the conduction band and the scattering of electrons between the valleys. The valley scattering rate is an additional parameter which affects the conductivity and makes a determination of the coefficient of the $\sqrt{T}$ term from first principles difficult.[23]

In a similar manner the spin susceptibility $\chi_s(T)$ is found to have the enhancement[8,24–26]

$$\frac{\chi_s(T)}{\chi_{\rm fl}} - 1 + \frac{\gamma}{(2E_F\tau)^{1+\epsilon}}2|A_0^a|(T\tau)^{\epsilon/2} \tag{5}$$

$$= \frac{1}{(\tau_s)_{\rm fl}}\left(1 + \frac{2\gamma}{(2E_F\tau)^{1+\epsilon}}[2\ln(1+|A_0^a|) \quad |A_0^a|](T\tau)^{\epsilon/2}\right) \tag{6}$$

For small A_0^a Eqs. (5) and (6) predict that the enhancement of the susceptibility and the linewidth are identical. Experimentally this prediction is borne out by the experiments on Si:P.[27] In actuality the proportionality between the electron spin susceptibility and the ESR linewidth persists all the way to n_c; this cannot be understood in the framework of this perturbation theory. This proportionality is an important constraint which should be satisfied by any theory of the

where γ is a phase space factor of order unity. Examining Fig. 1 again we see that in the region $n \gg n_c$ the spin susceptibility does have a weak upturn at low temperatures which can be fit to a $\sqrt{T}$ term. However, an n approaches n_c, the spin susceptibility increases much more rapidly at low temperature, and smoothly crosses over the $T^{-\gamma}$ dependence upon the insulating side of the transition. This rapid increase is clearly beyond the applicability of the perturbation theory expression in Eq. (5). The spin diffusivity has corrections which are exactly the inverse of the spin susceptibility, as might be expected in any Fermi liquid.

Another magnetic property which is correlated with the spin susceptibility is the ESR linewidth.[27] As on the insulating side of the transition, the hyperfine interaction of the electrons with the phosphorus nuclei is an important source of spin dephasing, although certain spin-orbit scattering processes may also be significant. Here we only treat the hyperfine term for simplicity. In the clean Fermi liquid the linewidth is given by a spin relaxation time $1/\tau_s = 2\pi N_0 A^2$, where N_0 is the density of states at the Fermi level. In the disordered system the suppresion of spin diffusivity leads to a decrease in the amount motional narrowing and a broadening of the line. A direct calculation shows[25,26]

transitional phase in uncompensated semiconductors.

Mention was made earlier of the valley structure of the silicon conduction band which complicated effects in the comparison of the theory with the data on Si:P. It was shown in Ref. 28, however, that this valley structure can be turned into an advantage by making experimentally testable predictions which are a nontrivial consequence of the structure of the disordered Fermi liquid theory. Using an analogy between the valley and spin quantum numbers, it was noted that the decrease in spin diffusivity with falling temperatures should have a parallel in the temperature dependence of the *valley* diffusivity. The valley diffusivity measures the rate at which a polarization of the electron population in the valley configuration would diffuse. Such a polarization is dynamically created by an ultrasonic wave propagating through the silicon crystal. It is therefore not surprising that the attenuation rate of the ultrasonic wave is proportional to a valley relaxation time. This valley relaxation time is the analog of the spin relaxation time that is measured in an ESR experiment. Reasoning in analogy with the spin relaxation time we can therefore conclude that the ultrasonic attenuation in multivalley doped semiconductor like Si:P should have a temperature dependent increase at low temperatures which can be correlated with the broadening of the ESR line. This temperature dependence of the ultrasonic attenuation has a clear signature in its dependence on the direction of propagation and polarization of the ultrasonic wave. An experimental measurement of the ultrasonic attenuation should therefore provide a rather unambiguous test of the applicability of the disordered Fermi liquid theory in Si:P.

IV. THE TRANSITION REGION

In this section we discuss the region just around n_c. As noted in the introduction, no complete theory of this region exists. We will discuss a few significant experiments and present recent theoretical attempts in understanding this region.

Reexamining Fig. 1 we see that the spin susceptibility gradually crosses over from a weak $\sqrt{T}$ correction at high densities to the almost Curie-type behavior in the insulating phase. A remarkable feature of the data is that on the metallic side of the transition (at a density of $1.09n_c$) the susceptibility at 50 mK is over ten times larger than that at 10 K. This clearly cannot be explained by the expression in Eq. (5). A similar point is made by Fig. 2: in this figure we have plotted the temperature dependent susceptibility as a function of the conductivity for two different metallic samples ($n = 1.09n_c$ and $1.25n_c$). The almost vertical slope of the line clearly indicates that the temperature dependence of the susceptibility is far stronger than that of the conductivity. This disagrees with Eqs. (4) and (5) which predict a change in the conductivity and susceptibility of the same order of magnitude. We also show in Fig. 2 the results of a renormalization group calculation[28,29] for the temperature dependence of the susceptibility and conductivity. This calculation can be interpreted as a resummation of an infinite number of terms in the weak disorder perturbation theory; Eqs. (4) and (5) contain only the leading order terms in this series.

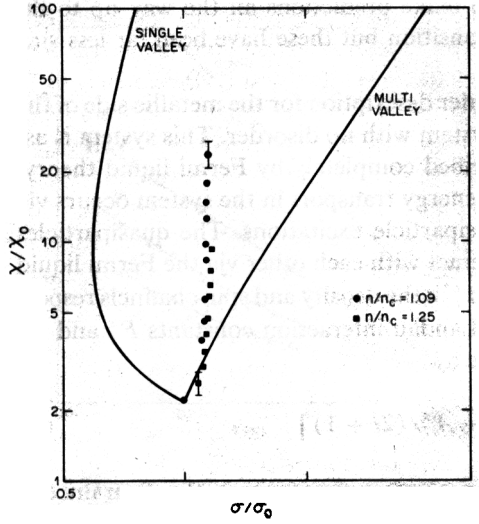

FIG. 2. Spin susceptibility (χ) and conductivity (σ) for two different metallic samples of Si-P. The suceptibility is normalized to the susceptibility (χ_0) of a free Fermi gas, while the conductivity is normalized to the conductivity of the sample at 10 K (σ_0). The solid lines represent theoretical renormalization-group calculations of two versions of the disordered Fermi liquid theory. The two versions differ in their treatment of the valley index of the conduction band. There is only one adjustable parameter in the theoretical calculations, and it is used to fix the high-temperature point common to the two solid curves and the experimental points.

The results of this calculation depend upon whether the disordered Fermi liquid preserves the many-valley structure of the underlying conduction band or averages over it; both cases are presented in Fig. 2. As plotted in Fig. 2, there is only one adjustable Fermi liquid constant which determines the theoretical curves. This constant is used to fix the high-temperature starting point of the renormalization-group calculation; this is the point at which the two theoretical curves intersect in Fig. 2. The lack of any correspondence between the theoretical and experimental curves indicates the inapplicability of the disordered Fermi liquid theory. Neither of the theoretical curves is close to vertical as the conductivity and susceptibility vary on the same temperature scale. To get better agreement with the experiment it is then clearly necessary to introduce a different model which treats the spin excitations and charge transport differently from one another.

Before abandoning the Fermi liquid description at densities close to n_c, it is of interest to examine whether its general physical framework is useful, independent of the validity of perturbation theory. The disordered Fermi liquid theory framework allows one to make phenomenological scaling ansatzes for the frequency, density and temperature dependences of the thermodynamic quantities of the system. For example, it states that asymptotically close to n_c, the conductivity should satisfy the following equation[10]:

$$\sigma(n,T) = (n - n_c)^{\mu} \Phi\left[T^{x}(n - n_c)\right], \qquad (7)$$

where Φ is smooth function of its arguments. Attempts to fit the data in Si:P to Eq. (7) have not been successful[29] indicating that the corrections to scaling terms in the experimental region of interest are so large that the scaling concept is not very useful.

Two recent nuclear magnetic resonance (NMR) experiments[30,31] have provided important clues about the nature of this intermediate phase. The first by Paalanen *et al.*[30] measured T_1 relaxation times of Si^{29} nuclei. They found strong enhancements over the Korringa relaxation rate present in a clean metal. In addition, the T_1 relaxation rates were strongly dependent upon the magnitude of the magnetic field. These results were explained by Gan and Lee[32] using a simple two fluid model of the electrons. The electron liquid was assumed to be made divided into two mutually independent components: (i) a disordered Fermi liquid which was responsible for the charge transport and (ii) electron local moments, which were precursors of the spins on the insulating side of the transition and were responsible for the large spin susceptibility. The local moments were necessary in the model of Gan and Lee[32] to explain the magnetic field dependence of T_1. The second experiment by Alloul *et al.*[31] measured the NMR signal from P nuclei. The interaction of the P nuclei with the electrons leads to a shift in resonance frequency of NMR (the Knight shift). If the electron is in a localized bound state near a nucleus, then the shift is so large that the NMR signal from the nucleus becomes part of the background. The NMR experiment therefore only measures the signal from the P nuclei which interact with itinerant electrons. The experiment found an abrupt decrease in the Knight shift as the doping density was changed from $1.1n_c$ to n_c. This can be interpreted as the transformation of the itinerant quasiparticle spectral weight into local moments. This transformation occurs while the system still has a nonzero conductivity at zero temperature.

The two-fluid model discussed by Gan and Lee[32] is actually a very old one and was also used by Quirt and Marko[33] in their early experiments on the magnetic properties of doped semiconductors. At its simplest level the model can be taken as a phenomenological description of the electronic states as being made up of two independent fluids: a Fermi fluid with extended quasiparticle-like excitations which carry charge, spin, and energy and a configuration of localized moments which consist of electrons singly occupying localized states and contributing a large spin susceptibility. To go beyond this model it is obviously necessary to introduce an interaction between the two components. Such a program would have to ultimately show the correlation between the decrease in the quasiparticle weight in Fermi fluid and the increase in the weight of the local moment spectrum. In the remaining part of this section we summarize the result of a recent attempt by one of us[12] to develop a theory of the interaction between the local moments and the Fermi liquid.

The Hamiltonian describing the interactions of the Fermi liquid and the local moments can be written as follows:

$$H = H_{\rm fl} + \sum_{i \neq j} J_{ij}\, \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{\mathbf{k},\mathbf{q}} \sum_{\alpha,\beta} \sum_i K_i\, \mathbf{S}_i\, c^{\dagger}_{\mathbf{k}+\mathbf{q},\alpha}\, \sigma_{\alpha,\beta}\, c_{\mathbf{k},\beta}\, e^{i\mathbf{q}\cdot\mathbf{R}_i}, \quad (8)$$

where $H_{\rm fl}$ is the Fermi liquid Hamiltonian for the quasiparticles, $c^{\dagger}$ is a quasiparticle creation operator, $\mathbf{S}_i$ are local electron spins interacting with each other with exchange J_{ij} and with the extended quasiparticles with exchange K_i. An issue that arises at the outset is the lack of an analytical description of the spatial and dynamical correlations in the local moment susceptibility. In Ref. 12 a simple phenomenological description of the local moments is assumed. All spatial correlations are ignored and the local moments are assumed to be randomly located. After averaging over the locations of the moments, this leads to a momentum independent local moment susceptibility $\chi(\omega_n)$. It is assumed that the spectral weight in χ is zero above a spin-fluctuation frequency ω_s, with ω_s satisfying $\omega_s \ll E_F$. Then to second order in coupling K between the local moments and the conduction electrons several important features of the properties of the system can be understood which are independent of the spectral shape of $\chi(\omega_n)$. At any finite temperature T the self-energy $\Sigma(\omega)$ of the electrons in the Fermi liquid has the form

$$\Sigma(\mathbf{k},\omega) = \Sigma_C(\mathbf{k},\omega) + \omega(1-Z) - \frac{\iota}{2\tau_{\rm in}}\,\mathrm{sgn}(\omega)\,. \quad (9)$$

Here Σ_C is the Fermi liquid self-energy from the Coulomb interactions between the itinerant electrons and remaining terms arise from the interaction with the local moments. The only wave-vector dependence in Σ is contained in Σ_C. The frequency renormalization factor Z can be shown to arise from the local moment spectral weight in the region $\omega > T$ and the inelastic scattering from the local moment spectral weight in the region $\omega < T$. It is shown in Ref. 12 that the renormalization factor Z cancels in the expression for the compressibility and conductivity, but yields an enhancement of the spin susceptibility. The local moment inelastic scattering (described by the term $\tau_{\rm in}$), however, leads to a decrease in the conductivity. Using a local sum rule on the magnitude of the local-moment spin at each site it can be shown to exponential accuracy in T/ω_s that

$$1/2\tau_{\rm in} \sim TK^2 N \chi_s(T)\,, \quad (10)$$

where $\chi_s(T)$ is the *static* local moment susceptibility. Assuming that the spin susceptibility of the itenerant electrons is negligible compared to that of the local moments this immediately leads to an experimentally testable connection between the temperature dependence of the spin susceptibility and the conductivity. As discussed elsewhere[12] this correlation is found to be consistent with experiments except *very* close to the metal insulator transition.

As an aside we note that although the inelastic spin-flip rate $\tau_{\rm in}$ will modify all the quantum interference effects in the disordered Fermi liquid, it will not show up as an additional linewidth in the ESR experiment. In particular $\tau_{\rm in}$ will be relevant in choosing between the various renormalization group fixed points of Finkelstein.[7,8] The reason it does not contribute to the ESR linewidth is simple: the ESR experiment measures the *total* electron spin susceptibility while $\tau_{\rm in}$ represents relaxation of spin between different parts of the electron system. An analytic calculation describing this phenomena in detail can be found in Ref. 12.

The results that emerge out of this analysis of the interactions between the local moments and the disordered Fermi liquid are therefore very simple. The dominant effect of the local moments on the Fermi liquid is to introduce a frequency renormalization factor and a strongly temperature depen-

dent inelastic scattering rate. The inelastic scattering leads to a temperature dependent conductivity and also cuts off all the quantum interference effects which were responsible for the scaling behavior.

V. CONCLUSIONS

We have presented a review of experimental and theoretical work on the metal insulator transition in doped semiconductors, especially Si:P. While considerable progress has been made in the understanding of the insulating and metallic phases, there are still many unresolved issues on the physics of the transition between the two regions. There appears to be a decoupling of the charge and spin degrees of freedom in the transition region between the metal and insulator. While the conductivity vanishes at a critical density n_c, there is clear experimental evidence for the appearance of local electron moments at densities greater than n_c. Although a complete understanding of this unusual phase is still lacking, phenomenological approaches[12] have recently had some success in understanding the interaction between extended disordered quasiparticles and local moments.

On the experimental front, most work on magnetic properties has so far concentrated on uncompensated semiconductors. An important future direction is therefore the measurement of magnetic properties of compensated materials. The measurement of the conductivity exponent μ has already indicated differences from the uncompensated semiconductors. The magnetic properties of compensated semiconductors will undoubtedly yield important clues about the value of μ and the physics of metal-insulator transition.

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