Estimating the mass of vortices in the cuprate superconductors

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Abstract

We explore the experimental implications of a recent theory of the quantum dynamics of vortices in superfluids proximate to Mott insulators. The theory predicts modulations in the local density of states in the regions over which the vortices execute their quantum zero point motion. We use the spatial extent of such modulations in scanning tunnelling microscopy measurements (Hoffman *et al*, Science **295**, 466 (2002)) on the vortex lattice of $Bi_2Sr_2CaCu_2O_{8+\delta}$ to estimate the inertial mass of a point vortex. We discuss other, more direct, experimental signatures of the vortex dynamics.

1 Introduction

It is now widely accepted that superconductivity in the cuprates is described, as in the standard Bardeen-Cooper-Schrieffer (BCS) theory, by the condensation of charge $-2e$ Cooper pairs of electrons. However, it has also been apparent that vortices in the superconducting state are not particularly well described by BCS theory. While elementary vortices do carry the BCS flux quantum of $hc/2e$, the local electronic density of states in the vortex core, as measured by scanning tunnelling microscopy (STM) experiments, has not been explained naturally in the BCS framework. Central to our considerations here are the remarkable STM measurements of Hoffman *et al.* [1] (see also Refs. [2,3]) who observed modulations in the local density of states (LDOS) with a period of approximately 4 lattice spacings in the vicinity of each vortex core of a vortex lattice in $Bi_2Sr_2CaCu_2O_{8+\delta}$.

This paper shall present some of the physical implications of a recent theory of two-dimensional superfluids in the vicinity of a quantum phase transition

to a Mott insulator [4,5] (see also Ref. [6]). By 'Mott insulator' we mean here an incompressible state which is pinned to the underlying crystal lattice, with an energy gap to charged excitations. In the Mott insulator, the average number of electrons per unit cell of the crystal lattice, n_{MI}, must be a rational number. If the Mott insulator is not 'fractionalized' and if n_{MI} is not an even integer, then the Mott insulator must also spontaneously break the space group symmetry of the crystal lattice so that the unit cell of the Mott insulator has an even integer number of electrons. There is evidence that the hole-doped cuprates are proximate to a Mott insulator with $n_{MI} = 7/8$ [7], and such an assumption will form the basis of our analysis of the STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$. The electron number density in the superfluid state, n_S, need not equal n_{MI} and will be assumed to take arbitrary real values, but not too far from n_{MI}.

A key ingredient in our analysis will be the result that the superfluid carries a subtle quantum order, which is distinct from Landau-Ginzburg order of a Cooper pair condensate. In two dimensions, vortices are point-like excitations, and are therefore bona fide quasiparticle excitations of the superfluid. The quantum order is reflected in the wavefunction needed to describe the motion of the vortex quasiparticle. For n_{MI} not an even integer, the low energy vortices appear in multiple degenerate flavors, and the space group symmetry of the underlying lattice is realized in a projective unitary representation that acts on this flavor space. Whenever a vortex is pinned (either individually due to impurities, or collectively in a vortex lattice), the space group symmetry is locally broken, and hence the vortex necessarily chooses a preferred orientation in its flavor space. As shown in Ref. [4], this implies the presence of modulations in the LDOS in the spatial region over which the vortex executes its quantum zero point motion [8]. The short-distance structure and period of the modulations is determined by that of the Mott insulator at density n_{MI}, while its long-distance envelope is a measure of the amplitude of the vortex wavefunction (see Fig. 1). Consequently, the size of the region where the modulations are present is determined by the inertial mass of the vortex. Here we will show how these ideas can be made quantitatively precise, and use current experiments to obtain an estimate of the vortex mass, m_v. There have been a number of theoretical discussions of m_v using BCS theory [9,10,11,12,13,14,15], and they lead to the order of magnitude estimate $m_v \sim m_e(k_F\xi)^2$, where m_e is the electron mass, k_F is the Fermi wavevector, and ξ is the BCS coherence length.

2   Vortex equations of motion

We begin with a simple model computation of the vortex dynamics. Consider a system of point vortices moving in a plane at positions $\mathbf{r}_j$, where j is a
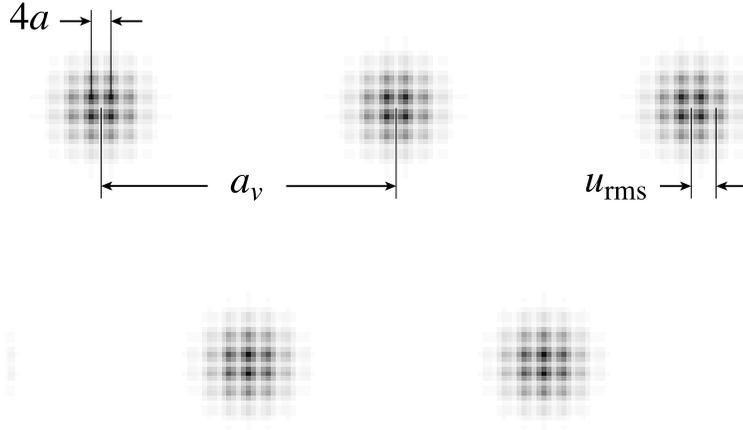
Fig. 1. Schematic of the modulations in the LDOS of a vortex lattice. The short distance modulations in each vortex halo are determined by the orientation of the vortex in flavor space, as discussed in Ref. [4]. The envelope of these modulations is $|\Psi(\mathbf{r}_j)|^2$ where Ψ is the wavefunction of the vortices, and its characteristics are computed in the present paper.

label identifying the vortices. We do not explicitly identify the orientation of each vortex in flavor space, because we are interested here only in the long-distance envelope of the LDOS modulations; the flavor orientation does not affect the interactions between well-separated vortices, and so plays no role in determining the wavefunction of the vortex lattice. In a Galilean-invariant superfluid, the vortices move under the influence of the Magnus force

$$m_v \frac{d^2 \mathbf{r}_j}{dt^2} = \frac{h n_S}{2a^2} \left(\mathbf{v}_s(\mathbf{r}_j) - \frac{d\mathbf{r}_j}{dt} \right) \times \hat{\mathbf{z}}, \tag{1}$$

where t is time, $h = 2\pi\hbar$ is Planck's constant, $\mathbf{v}_s(\mathbf{r})$ is the superfluid velocity at the position $\mathbf{r}$, and n_S/a^2 is the electron number density per unit area (a^2 is the area of a unit cell of the underlying lattice). One point of view is that the force in Eq. (1) is that obtained from classical fluid mechanics after imposing the quantization of circulation of a vortex. However, Refs. [16] emphasized the robust topological nature of the Magnus force and its connection to Berry phases, and noted that it applied not only to superfluids of bosons, but quite generally to superconductors of paired electrons. Here, we need the modification of Eq. (1) by the periodic crystal potential and the proximate Mott insulator. This was implicit in the results of Ref. [4], and we present it in more physical terms. It is useful to first rewrite Eq. (1) as

$$m_v \frac{d^2 \mathbf{r}_j}{dt^2} = \mathbf{F}_E(j) + \mathbf{F}_B(j), \tag{2}$$

where $\mathbf{F}_E$ is the first term proportional to $\mathbf{v}_s$ and $\mathbf{F}_B$ is the second term. Our notation here is suggestive of a dual formulation of the theory in which the vortices appear as 'charges', and these forces are identified as the 'electrical'

and 'magnetic' components. In the Galilean invariant superfluid, the values of $\mathbf{F}_E$ and $\mathbf{F}_B$ are tied to each other by a Galilean transformation. However, with a periodic crystal potential, this constraint no longer applies, and their values renormalize differently as we now discuss.

The influence of the crystal potential on $\mathbf{F}_E$ is simple, and replaces the number density of electrons, n_S, by the superfluid density. Determining $\mathbf{v}_s(\mathbf{r}_j)$ as a sum of contributions from the other vortices, we obtain [17]

$$\mathbf{F}_E(j) = 2\pi\rho_s \sum_{k \neq j} \frac{\mathbf{r}_j - \mathbf{r}_k}{|\mathbf{r}_j - \mathbf{r}_k|^2}, \tag{3}$$

where ρ_s is the superfluid stiffness (in units of energy). It is related to the London penetration depth, λ, by

$$\rho_s = \frac{\hbar^2 c^2 d}{16\pi e^2 \lambda^2}, \tag{4}$$

where d is the interlayer spacing.

The modification of $\mathbf{F}_B$ is more subtle. This term states that the vortices are 'charges' moving in a 'magnetic' field with $n_S/2$ 'flux' quanta per unit cell of the periodic crystal potential. In other words, the vortex wavefunction is obtained by diagonalizing the Hofstadter Hamiltonian which describes motion of a charged particle in the presence of a magnetic field and a periodic potential. As argued in Ref. [4], it is useful to examine this motion in terms of the deviation from the rational 'flux' $n_{MI}/2 = p/q$ (p, q are relatively prime integers) associated with the proximate Mott insulator. The low energy states of the rational flux Hofstadter Hamiltonian have a q-fold degeneracy, and this constitutes the vortex flavor space noted earlier [18]. However, these vortex states describe particle motion in zero 'magnetic' field, and only the deficit $(n_S - n_{MI})/2$ acts as a 'magnetic flux'. This result is contained in the action in Eq. (2.46) of Ref. [4], which shows that the dual gauge flux fluctuates about an average flux determined by $(n_S - n_{MI})$. The action in Ref. [4] has a 'relativistic' form appropriate to a system with equal numbers of vortices and anti-vortices. Here, we are interested in a system of vortices induced by an applied magnetic field, and can neglect anti-vortices; so we should work with the corresponding 'non-relativistic' version of Eq. (2.46) of Ref. [4]. In its first-quantized version, this 'non-relativistic' action for the vortices leads to the 'Lorentz' force in Eq. (2) given by

$$\mathbf{F}_B(j) = -\frac{h(n_S - n_{MI})}{2a^2} \frac{d\mathbf{r}_j}{dt} \times \hat{\mathbf{z}}, \tag{5}$$

If the density of the superfluid equals the commensurate density of the Mott insulator, then $\mathbf{F}_B = 0$; however, $\mathbf{F}_E$ remains non-zero because we can still have $\rho_s \neq 0$ in the superfluid. These distinct behaviors of $\mathbf{F}_{E,B}$ constitute a
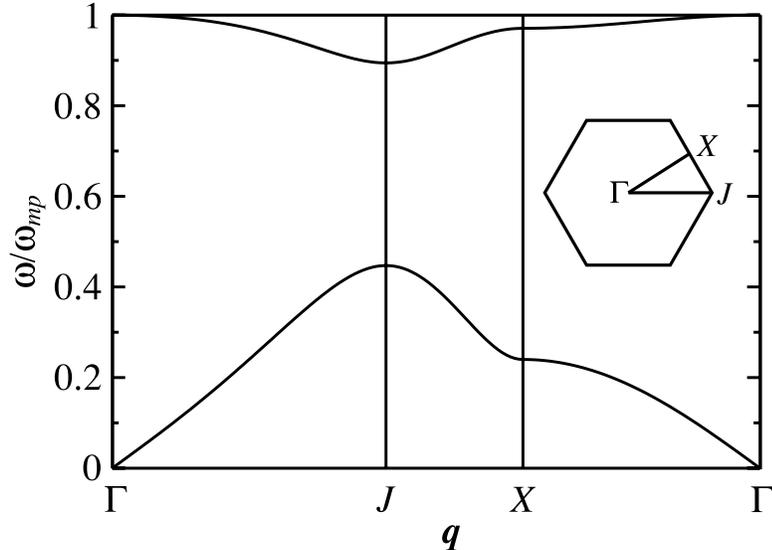
Fig. 2. Dispersion of the 'magnetophonon' modes of the vortex lattice (here we take $\omega_c = 0.5\omega_p$).

key difference from Galilean-invariant superfluids. In experimental studies of vortex motion in superconductors [19], a force of the form of Eq. (5) is usually quoted in terms of a 'Hall drag' co-efficient per unit length of the vortex line, α; Eq. (5) implies

$$\alpha = -\frac{h(n_S - n_{MI})}{2a^2d}. \tag{6}$$

Thus the periodic potential has significantly reduced the magnitude of α from the value nominally expected [20] by subtracting out the density of the Mott insulator. A smaller than expected $|\alpha|$ is indeed observed in the cuprates [19]. It is worth emphasizing that $\mathbf{F}_B$ (but not $\mathbf{F}_E$) is an *intrinsic* property of a single vortex. Moreover, we expect that, taken together, the relation Eq. (6) and the flavor degeneracy q are robust "universal" measures of the quantum order of a clean superconductor, independent of details of the band structure, etc.

3 Vortex lattice normal modes

We solved Eqs. (2-5) in the harmonic approximation in deviations from a perfect triangular lattice of vortices at positions $\mathbf{R}_j$. This leads to the vortex 'magnetophonon' modes shown in Fig 2. The computation of these modes is a generalization of other vortex oscillation modes discussed previously in superconductors [21,22], rotating superfluids [23,24,25], and, in a dual picture of 'charges', also to oscillations of electronic Wigner crystals in a magnetic field [26]. Quantizing these modes, we determine the mean square displacement of each vortex due to the quantum zero point motion of the vortex lattice, which

5

we denote $u_{\text{rms}}^2 = \langle |\mathbf{r}_j - \mathbf{R}_j|^2 \rangle/2$. We found an excellent fit of this solution to the interpolation formula

$$m_v = \frac{0.03627 a_v^2 \hbar^2}{\rho_s u_{\text{rms}}^4} F(x), \tag{7}$$

where a_v is the separation between nearest neighbor vortices, $x \equiv |\alpha| d u_{\text{rms}}^2/\hbar$, and

$$F(x) \approx 1 - 0.4099 x^2; \tag{8}$$

Eq. (8) holds only as long as the r.h.s. is positive, while $F(x) = 0$ for larger x (we will see below and in Fig. 3 that this apparent upper bound on x is relaxed once we allow for viscous damping). Similarly, for the frequency ω_{mp} in Fig. 2, we obtain $\omega_{mp} = \sqrt{\omega_p^2 + \omega_c^2}$ with

$$\omega_p = \frac{35.45 \rho_s u_{\text{rms}}^2}{\hbar a_v^2 [F(x)]^{1/2}} \quad ; \quad \omega_c = \frac{27.57 \rho_s u_{\text{rms}}^2 x F(x)}{\hbar a_v^2}. \tag{9}$$

For the experiments of Ref. [1] we estimate $\rho_s = 12$ meV [27], $u_{\text{rms}} = 20$ Å [28], $a = 3.83$ Å, and $a_v = 240$ Å. The overall scale for m_v is determined by setting $n_S = n_{MI}$ so that $x = 0$ and $F(x) = 1$. This yields $m_v \approx 8 m_e$ and $\hbar \omega_p \approx 3$ meV (or $\nu_p \approx 0.7$ THz). For a more accurate determination, we need n_S, for which there is considerable uncertainty $e.g.$ for $|n_S - n_{MI}| = 0.015$, we find $x = 1.29$, $m_v \approx 3 m_e$ and $\hbar \omega_p \approx 5$ meV.

4 Limitations

We now consider the influence of a variety of effects which have been neglected in this simple computation:

4.1 Viscous drag

It is conventional in models of vortex dynamics at low frequencies [29] to include a dissipative viscous drag term in the equations of motion, contributing an additional force

$$\mathbf{F}_D(j) = -\eta d \frac{d\mathbf{r}_j}{dt}, \tag{10}$$

to the r.h.s. of Eq. (2). There are no reliable theoretical estimates for the viscous drag co-efficient, η, for the cuprates. However, we can obtain estimates of its value from measurements of the Hall angle, θ_H, which is given by [29,30] $\tan \theta_H = \alpha/\eta$. Harris $et\ al.$ [30] observed a dramatic increase in the value $|\tan \theta_H|$, to the value 0.85, at low T in "60 K" YBa$_2$Cu$_3$O$_{6+y}$ crystals, suggesting a small η, and weak dissipation in vortex motion. For our purposes, we
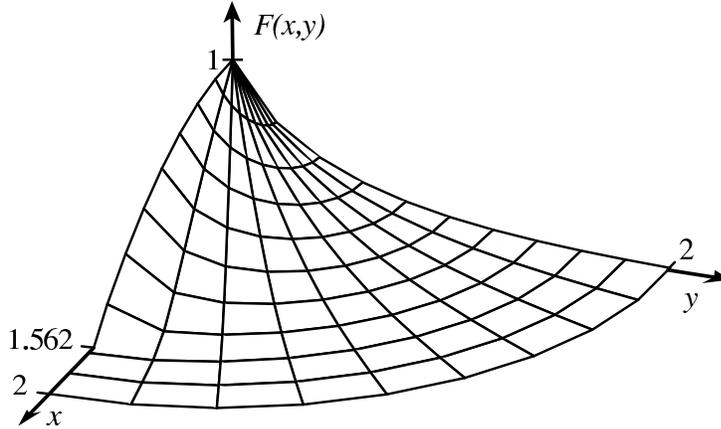
Fig. 3. Plot of the function $F(x, y)$ which replaces $F(x)$ in Eqs. (7,8,9) upon including viscous drag, η ($y \equiv \eta d u_{\text{rms}}^2/\hbar$). The argument x measures the Hall drag α ($x \equiv |\alpha| d u_{\text{rms}}^2/\hbar$) and the Hall angle is determined by $|\tan \theta_H| = x/y$.

need the value of η for frequencies of order ω_p, and not just in the d.c. limit. The very naïve expectation that $\eta(\omega)$ behaves like the quasiparticle microwave conductivity would suggest it decreases rapidly beyond a few tens of GHz, well below ω_p [31]. Lacking solid information, we will be satisfied with an estimate of the influence of viscous drag obtained by neglecting the frequency dependence of η (a probable overestimate of its influence). The resulting corrections to Eqs. (7-9) are easily obtained (as in Ref. [11]), and can be represented by the replacement

$$F(x) \rightarrow F(x, y), \text{ where } y \equiv \frac{\eta d u_{\text{rms}}^2}{\hbar} \tag{11}$$

and

$$F(x, 0) = F(x). \tag{12}$$

The sketch of the function $F(x, y)$ is in Fig. 3; as long as $y > 0$, we have $F(x, y) > 0$. As expected, the viscous damping decreases the estimate of the mass, and this decrease is exponential for large y, e.g. at $x = 0$ we have the interpolation formula

$$F(0, y) \approx (1 + 0.41y + 2.69y^2)e^{-3.43y}. \tag{13}$$

4.2 Meissner screening

The interaction in Eq. (3) is screened at long distances by the supercurrents, and the intervortex coupling becomes exponentially small. This does have an important influence at small momenta in that the shear mode of the vortex lattice disperses as [21,22] $\sim k^2$. However, as long as $a_v \ll \lambda$, there will not be a significant influence on u_{rms} or ω_p.

4.3   Retardation

The interaction Eq. (3) is assumed to be instantaneous; in reality it is retarded by the propagation of the charged plasmon mode of the superfluid. We have estimated the corrections due to this mode in a model of superfluid layers coupled by the long-range Coulomb interaction. Briefly, we compare the energy per unit area of a 'phase fluctuation' at the wavevector of the vortex lattice Brillouin zone boundary ($\sim \rho_s/a_v^2$) with its electrostatic energy ($\sim \hbar^2 \omega_p^2/(e^2 d)$); this shows that such corrections are of relative order $\sim (\hbar^2/m_v)/(e^2 d)$. For the parameters above and $d = 7.5$ Å this is ~ 0.009, and hence quite small.

4.4   Nodal quasiparticles

We expect that nodal quasiparticles contribute to the viscous drag, and so their contribution was already included in the experimentally determined estimate of η in (i). The quasiparticle contribution to m_v has been estimated [9,10] in a BCS theory of a continuum of core levels, valid in the limit $k_F \xi \gg 1$, and a $\sim 1/\sqrt{H}$ dependence (H is the magnetic field) was obtained. The applicability of such a model to the cuprates is questionable, but nevertheless the estimates of m_v above should provisionally be assumed to be specific to the H field in the STM experiments.

4.5   Disorder

We have assumed here a triangular lattice of vortices. In reality, STM experiments show significant deviations from such a structure, presumably because of an appreciable random pinning potential. This pinning potential will also modify the vortex oscillation frequencies and its mean square displacement. Both pinning and damping η tend to reduce vortex motion. For this reason, the estimates of m_v above in which these effects are neglected must be regarded as upper bounds.

The above considerations make it clear that new experiments on cleaner underdoped samples, along with a determination of the spatial dependence of the hole density (to specify α), are necessary to obtain a more precise value for m_v; determining the H dependence of m_v will enable confrontation with theory.

5 Implications

An important consequence of our theory is the emergence of ω_p as a characteristic frequency of the vortex dynamics. It would therefore be valuable to have an inelastic scattering probe which can explore energy transfer on the scale of $\hbar\omega_p$, and with momentum transfer on the scale of h/a_v, possibly by neutron [32] or X-ray scattering. A direct theoretical consideration of magnetoconductivity in our picture would have implications for far-infrared or THz spectroscopy, allowing comparison to existing experiments [33]; further such experiments on more underdoped samples would also be of interest.

Another possibility is that the zero point motion of the vortices emerges in the spectrum of the LDOS measured by STM at an energy of order $\hbar\omega_p$. We speculate that understanding the 'vortex core states' observed in STM studies [2,3] will require accounting for the quantum zero point motion of the vortices; it is intriguing that the measured energy of these states is quite close to our estimates of $\hbar\omega_p$.

6 Acknowledgements

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