Comment on "Critical spin dynamics of the 2D quantum Heisenberg antiferromagnets: $Sr_2CuO_2Cl_2$ and $Sr_2Cu_3O_4Cl_2$"

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We compare the neutron measurements of Kim *et al.* (cond-mat/0012239) on two-dimensional, $S = 1/2$ antiferromagnets with the continuum quasiclassical theory of S. Sachdev and O.A. Starykh (Nature, **405**, 322 (2000)). The damping of the lowest energy spin excitations is characterized by a dimensionless number whose temperature dependence was predicted to be determined entirely by that of the uniform spin susceptibility. Theory and experiment are consistent with each other.

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Kim *et al.* [1] have recently provided illuminating neutron scattering measurements of the damping of the lowest energy spin excitations in $S = 1/2$, square lattice antiferromagnets. They characterized this damping by the temperature (T) dependence of a dimensionless number, R_ω. (Related measurements, at higher energies, are those of [2].) Here we will use an existing quasiclassical dynamic theory [3] to relate $R_\omega(T)$ to a thermodynamic observable, $\chi_u(T)$, the uniform spin susceptibility, and show that this provides a unified understanding of the experiments of Kim *et al.*, and earlier studies of NMR relaxation rates [4].

Detailed quantitative predictions have been made for $\chi_u(T)$ (and other static observables) for antiferromagnets with a magnetically ordered ground states with a spin stiffness ρ_s [5,6] . When the antiferromagnet is also near a quantum critical point at which ρ_s vanishes, then these observables become universal functions of T/ρ_s at all T below J (a near-neighbor exchange interactions), while at higher T there is simple decoupled spin behavior. In particular, χ_u obeys [6] ($k_B = \hbar = 1$)

$$\chi_u(T) = (T/c^2)\Omega(T/\rho_s), \tag{1}$$

where c is the spin-wave velocity, and $\Omega(x)$ is a universal function which crosses over from the 'renormalized classical' (RC) regime $\Omega(x \ll 1) = 2/(3x) + 1/(3\pi) + \ldots$, to the 'quantum critical' (QC) regime $\Omega(\infty) \approx 0.27$ [6]. Such results agree well with Monte Carlo simulations on antiferromagnets known to be near a quantum critical point [7].

For the square lattice antiferromagnet, RC behavior is well established as $T \to 0$. However, somewhat surprisingly, it was found that $\chi_u(T)$ was quite close to the QC limit above for $T > 0.3J$, and in clear disagreement with the T dependence predicted by the RC limit, suggesting a crossover from RC to QC behavior even in this unfrustrated, isotropic, and undoped system [6]; the corresponding behavior was not seen for the correlation length, and a theoretical rationale was offered for this [6]. Subsequent precision Monte Carlo studies were consistent with these observations [8].

It is clearly of interest to obtain quantitative predictions of the RC to QC crossover in dynamic properties. However, accurate predictions of damping rates in the QC regime are quite difficult to obtain. An expansion in $\epsilon = 3 - d$ (d is the spatial dimensionality) was developed in [9], but the accuracy of the leading order term in ϵ is not known. In [3], physical arguments were used to motivate a simple continuum quasiclassical model which had the advantages of being expressed directly in $d = 2$, and of also describing the crossover into the RC regime. The scaling arguments of [3] predict that $R_\omega(T)$ obeys

$$R_\omega(T) = A_\omega \sqrt{T/(c^2\chi_u(T))}, \tag{2}$$

where A_ω is a dimensionless number. So in [3], *the crossover in the damping is determined entirely by that in* $\chi_u(T)$. Upon applying a self-consistent, one-loop approximation to the theory of [3], we obtain integral equations closely related to those of Grempel [10]; from his numerical solution of these equations, we deduce $A_\omega = 0.31$.

In the $T \to 0$, RC limit, (2,1) predict that $R_\omega(T) = 0.38\sqrt{T/\rho_s}$, which is precisely the result of [5,10]. In the QC regime, it is a very general prediction that $R_\omega(T)$ is a T-independent constant, as appears to be observed at higher T in [1]; from (2,1) we obtain that $R_\omega(T) \approx 0.60$, a prediction which is consistent with the observations of [1]. It would be interesting to use the measured values of $\chi_u(T)$ to test (2) over the entire temperature range.

Similar comments apply to the computation of NMR relaxation rates in [4]. These papers use a computation very similar to that of Grempel [10], but with a lattice cutoff, and include the full T dependence of $\chi_u(T)$. As we noted earlier, the latter is not described by the RC behavior but displays a RC to QC crossover; in the model of [3], the crossover in $\chi_u(T)$ is sufficient to describe the crossover in the dynamics into the QC regime. The agreement of the results in [4] with experimental observations is therefore consistent with our discussion here.

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