Quantum connections

Sean Hartnoll, Subir Sachdev, Tadashi Takayanagi, Xie Chen, Eva Silverstein and Julian Sonner

Theoretical high-energy and condensed-matter physics share various ideas and tools. New connections between the two have been established through quantum information, providing exciting prospects for theoretical advances and even potential experimental studies. Six scientists discuss different directions of research in this inter-disciplinary field.

New connections

Sean Hartnoll. High-energy and condensed-matter physics are organized around common fundamental concepts such as symmetry breaking and the renormalization group, and share core mathematical machinery including Feynman diagrams and topology. This has led to a historically fruitful interface between the two fields. In the last couple of decades, two new points of connection have emerged.

First, holographic duality has established that the classical evolution of black-hole horizons precisely captures the dissipative dynamics of a strongly quantum phase of matter. In recent years, this connection has moved beyond simple correlation functions (that describe the approach to coarse-grained thermal equilibrium) to more nuanced observables that can probe signatures of many-body quantum chaos. Tied up with this shift has been the emergence of the Sachdev–Ye–Kitaev (SYK) model. This model shares many of the features (and limitations) of fully fledged holographic theories, but is both microscopically closer to conventional condensed-matter Hamiltonians and under greater technical control.

Second, many-body quantum entanglement has simultaneously emerged as an organizing principle in both fields. It appears that quantum states supporting holographically emergent gravity have an entanglement structure that may be analogous to that of topologically non-trivial phases of matter. Fleshing out this connection promises to be a source of future progress.

Quantum matter without quasiparticles

Subir Sachdev. The study of compressible states of electronic matter without quasiparticle excitations began in the early 1980s, motivated by observations of the ‘strange metal’ in copper-oxide-based superconductors, and by the quantum Hall state in the half-filled Landau level of semiconductors in a strong magnetic field. When quasiparticles are present, a quantum many-body system will relax to equilibrium in response to an external perturbation in a time of the order of the mean collision time between the quasiparticles. Without quasiparticles, equilibration proceeds more rapidly and in a time as short as the ‘Planckian time’ $\hbar/(k_B T)$, where T is the temperature of the final state. Remarkably, there is a solvable model of non-quasiparticle quantum dynamics, the SYK model of electrons with all-to-all and random interactions. This model displays Planckian time relaxation and also describes some other aspects of the physics of the copper oxide superconductors.

A distinct physical system that also displays Planckian time relaxation is a black hole, and this becomes clear from the quantum temperature of a black hole computed by Stephen Hawking in 1974. In this case, there is a remarkable and surprisingly close connection between the SYK model and black holes with a net electric charge, the latter described by quantizing Einstein’s classical theory of gravity and Maxwell’s classical theory of electromagnetism. The quantization is described by a path integral over different configurations of spacetime and electromagnetic fields, each weighted by the exponential of the classical action divided by $\hbar$. In general, such an integral is not a well-defined mathematical object, because of the uncontrollably large number of possible configurations. However, in the low-temperature limit of charged black holes, it can be evaluated in the near-horizon region, where the classical theory shows that the spacetime becomes $1+1$ dimensional. And the path integral over these $1+1$-dimensional spacetimes and electromagnetic fields maps exactly as a hologram onto the low-temperature path integral of the $0+1$-dimensional SYK model. In recent work, extensions of such mappings have led to advances in our understanding of the quantum behaviour of Einsteinian theories of gravity, and insights into the black-hole information paradox.

Gravity and entanglement

Tadashi Takayanagi. The wavefunctions of quantum many-body systems are typically complicated from an algebraic point of view. However, it is often possible to extract a simple geometrical structure from a wavefunction, namely quantum entanglement. In other words, the ‘skeleton’ of a many-body state is given by a network of quantum entanglement.

Quantum entanglement also appears as a fundamental degree of freedom in quantum gravity. The amount of quantum entanglement is measured by a quantity called entanglement entropy. With the framework of holographic duality, entanglement entropies in conformal field theories (CFTs) turn out to be equal to areas of minimal surfaces in an anti-de-Sitter space. This holographic entanglement entropy formula was recently generalized into CFTs coupled to a dynamical gravity, providing a remarkable explanation for the black hole information paradox.

Surprisingly, the above two observations in two different subjects suggest the fascinating prospect that gravitational spacetime can emerge from quantum entanglement. This idea has been used successfully over the past several years within the conventional holography for a special class of spacetime whose cosmological constant is negative. I expect this approach will also be crucial when trying to tackle holography for general spacetimes, including de Sitter spaces.

Fracton models

Xie Chen. In the past few years, a new class of quantum many-body models, called the ‘fracton’ models, has captured the imagination of theorists from the condensed- matter and the high-energy communities alike. These models, many of which were discovered as quantum error correction codes, host a number of exotic properties never seen before. They contain point excitations that cannot move on their own, the ground-state degeneracy grows exponentially with system size, and the usual renormalization group transformation does not apply.

Fracton models challenge both condensed-matter and high-energy theory. The immobility of the point excitations results in slow and non-thermal dynamics that needs to be better understood and characterized. The growing ground-state degeneracy indicates an order that is not only beyond the symmetry-breaking paradigm but also beyond the conventional topological paradigm. The renormalization group transformation needs to be augmented with non-trivial resource states to be possible. All these features suggest that these models cannot be properly described using a continuous field theory, at least not in the usual way.

Progress has been made on many aspects, including the discovery of new models, connecting some of the features to systems we are familiar with, and exploring new conceptual ideas to properly capture the underlying order (for a review, see REFS[1,2]). Yet we are only scratching the surface. The exotic physics of the fracton model may fundamentally change and extend the way we approach quantum many-body systems, while bringing the quantum-information, condensed-matter and high-energy communities closer together (FIG. 1).

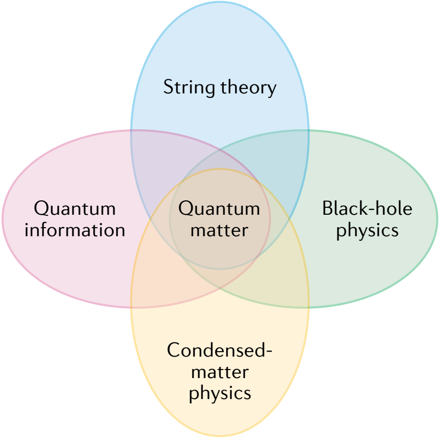

Fig. 1 | **Quantum connections.** The study of quantum matter lies at the intersection of four areas of fundamental physics, providing a fertile ground for the exchange of ideas and tools. Figure adapted with permission from REF.[6], Cambridge University Press.

A new notion of universality

Eva Silverstein. A classic connection between high-energy and condensed-matter physics concerns the renormalization group and the related notion of universality. The details of the microscopic theory do not affect the outcome of experiments involving low-energy probes of a system, enabling a principled focus on a small number of relevant terms in the Lagrangian of the theory. There is an exception: ‘dangerously irrelevant’ phenomena that become important in the presence of long times or large fields.

Reversing this flow is generally fraught with ambiguity, since one cannot determine the microscopic physics from a few relevant terms at low energy. But in a new development[3–5], certain irrelevant deformations are solvable and universal in a new sense. These deformations are specified via a differential equation, through a step-by-step procedure. At each step, the Lagrangian of the theory is deformed by a certain bilinear of the stress energy tensor known as $T\overline{T}$; the effect of this tensor on the energy spectrum is calculable. Joining this to a trajectory including the addition of a cosmological constant Λ at each step leads to distinct, but equally tractable results[4]. Since any quantum field theory has a stress energy tensor and a cosmological term, one can deform any seed theory in this way: it is universal.

This approach can be applied to quantum gravity: the energy spectrum and other features fit precisely with those of finite patches of spacetime, either anti de Sitter[5] or de Sitter (with Λ). This suggests a more concrete definition of the gauge/gravity duality for more realistic systems than the original anti de Sitter/CFT correspondence. Entanglement entropy calculations for large central charge — many degrees of freedom — agree on both sides of the duality and provide a specific interpretation of the Gibbons–Hawking–de Sitter entropy. However, many questions remain about the nature of this theory, which is currently an active research area.

The contributors

Sean Hartnoll is Associate Professor of Physics at Stanford University. He received his PhD from Cambridge University in 2005 and did postdocs at Cambridge, KITP Santa Barbara and Harvard. He won the New Horizons prize in 2015. He has co-authored a book on ‘Holographic Quantum Matter’ and works on a range of topics in high energy, gravitational and condensed matter physics.

Subir Sachdev is Herchel Smith Professor of Physics at Harvard University. He was educated at the Indian Institute of Technology, the Massachusetts Institute of Technology and Harvard. His honours include the Dirac Medal and the Onsager Prize. His research on many-particle quantum entanglement connects to the properties of quantum matter and black holes.

Tadashi Takayanagi received his PhD from the University of Tokyo in 2002. He worked in Harvard University and in KITP Santa Barbara as a post-doctoral fellow. He is currently a professor at Yukawa Institute for Theoretical Physics, Kyoto University and has been working on the deep connections between quantum gravity and quantum information, especially from the viewpoint of holography in string theory.

Xie Chen received her PhD from the Massachusetts Institute of Technology in 2012. She was a Miller research fellow at the University of California, Berkeley for two years before joining the faculty of the California Institute of Technology in 2014. Dr Chen is interested in studying non-trivial emergent phenomena in strongly interacting quantum many-body systems, such as topological order, symmetry anomaly, fracton order and others.

Eva Silverstein is Professor of Physics at Stanford University. Much of her research connects the mathematical structure of string theory to predictions for cosmological observables, with broader implications for primordial cosmology. She and her collaborators developed a framework for upgrading the gauge/gravity duality to the realistic case of a positive cosmological constant. At the interface with condensed matter theory, along with other researchers, she derived examples of non-Fermi liquids in theoretically controlled systems.

Julian Sonner received his PhD from the University of Cambridge (UK) where he also held a Research Fellowship at Trinity College. After postdoctoral work in London and Cambridge (USA), he became a professor of theoretical physics at the University of Geneva. He works on holographic duality, most recently on the connection between quantum chaos and black holes, as well as quantum simulation of simple holographic dualities.

Q *Quantum simulation*

Julian Sonner. The very properties that make quantum matter supporting holographically emergent gravity so rich and fascinating[6] also imply that classical computers are ultimately powerless in deciphering the underlying quantum states. A way to overcome the exponential complexity causing this impasse is to turn instead to another quantum system that emulates, or 'simulates', a many-body Hamiltonian of interest. This idea — known as quantum simulation — has evolved into the field of synthetic matter, where highly controllable quantum systems, such as superconducting qubits or ultra-cold atomic gases, are used to construct the desired Hamiltonian from simple ingredients, such as quantum gates and optical lattice potentials.

The convergence of high-energy and condensed-matter physics has resulted in new models of many-body systems, simple enough to lend themselves to a synthetic matter approach, yet intricate enough to give rise to holographic descriptions, as exemplified by the Sachdev–Ye–Kitaev model and its holographic dual, a two-dimensional black hole. The quantum simulation of these holographic states of matter opens up experimental avenues to investigate, for example, the relationship between quantum chaos (information 'scrambling') and non-Fermi-liquid transport, as well as building a new path to 'quantum gravity in the lab', where the holographic state of matter serves as an analogue gravity system that experimenters can manipulate and interrogate in their laboratory (see REFS[7,8] for two early examples).

In this way, the holographic language suggests new interpretations of many-body experiments where, for example, quantum teleportation is understood in terms of information travelling across wormholes in the gravitational picture[9,10]. Such insights will guide future fruitful interactions between theory and experiment.

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Competing interests

The authors declare no competing interests.

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