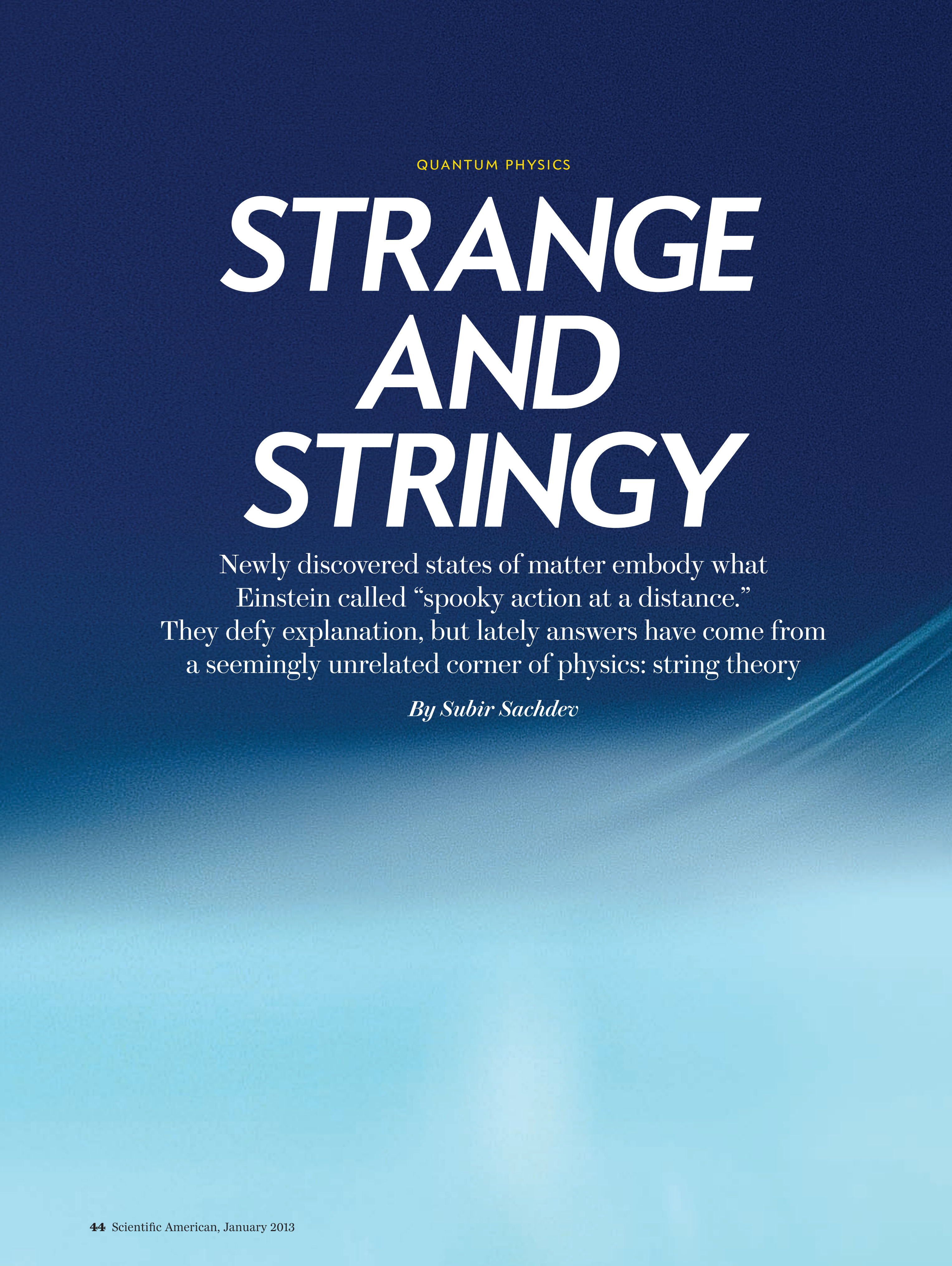
QUANTUM PHYSICS

STRANGE AND STRINGY

Newly discovered states of matter embody what Einstein called "spooky action at a distance." They defy explanation, but lately answers have come from a seemingly unrelated corner of physics: string theory

By Subir Sachdev



MAGNET is being levitated by an unseen superconductor in which countless trillions of electrons form a vast interconnected quantum state. Astoundingly, the quantum state of many modern materials is subtly related to the mathematics of black holes.

Photograph by Zachary Zavislak

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SEVERAL YEARS AGO I FOUND MYSELF WHERE I WOULD never have expected: at a conference of string theorists. My own field is condensed matter: the study of materials such as metals and superconductors, which we cool in the laboratory to temperatures near absolute zero. That is about as far as you can possibly get from string theory without leaving physics altogether. String theorists seek to describe the universe at energies far in excess of anything experienced in a lab or indeed anywhere else in the known universe. They explore the exotic physics governing black holes and putative extra spacetime dimensions. For them, gravity is the dominant force in nature. For me, it is an irrelevance.

This difference in subject matter is mirrored by a cultural gap. String theorists have a formidable reputation, and I went to the meeting in awe of their mathematical prowess. I had spent several months reading their papers and books, and I often got bogged down. I was certain I would be dismissed as an ignorant newcomer. For their part, string theorists had difficulty with some of the simplest concepts of my subject. I found myself drawing explanatory pictures that I had only ever used before with beginning graduate students.

So what was I doing there? In recent years many of us who specialize in condensed matter have discovered our materials doing things we never thought they could. They form distinctively quantum phases of matter, the structure of which involves some of the weirdest features of nature. In a famous paper in 1935 Albert Einstein, Boris Podolsky and Nathan Rosen drew attention to the fact that quantum theory implied a "spooky" connection between particles such as electrons—what we now call quantum entanglement. Somehow the activities of the particles are coordinated without mediation by a direct physical linkage. EPR (as Einstein and his co-authors are widely known) considered pairs of electrons, but metals and superconductors involve vast numbers of electrons—some 10^{23} of them, for a typical sample of material in the lab. In some materials, the complexity is mind-boggling, and I have spent much of my career trying to wrap my head around it. The problem is not merely academic: superconductors have become important technologically, and physicists have struggled to make sense of their properties and capabilities.

Then my colleagues and I came to realize that string theory could offer a completely unforeseen approach to such problems. In seeking to unify the theory of elementary particles with Einstein's theory of gravitation, string theorists had stumbled across "dualities"—hidden connections between far-flung areas of physics [see "The Illusion of Gravity," by Juan Maldacena; Scientific American, November 2005]. The dualities relate theories that work where quantum effects are weak but gravity is strong to theories that work where quantum effects are strong

IN BRIEF

Matter can assume many forms other than solid, liquid and gas. The electrons that perfuse materials can undergo their own transitions, which involve inherently quantum properties of matter. Superconductors are the best-known example.

These states of matter arise from an unimaginably complex web of quantum entanglement among the electrons—so complex that theorists studying these materials have been at a loss to describe them.

Some answers have come from an entirely separate line of study, string theory, typically the domain of cosmologists and high-energy particle theorists. On the face of it, string theory has nothing to say about the behavior of materials—no more than an atomic physicist can explain human society. And yet connections exist.

but gravity is weak. So they let us take insights from one realm and apply them to the other. We can translate our entanglement problem into a gravitational problem and avail ourselves of the efforts that string theorists have put into understanding black holes. It is lateral thinking at its finest.

HIDDEN PHASES

TO UNDERSTAND this circle of ideas, step back to high school physics, where teachers spoke of the phases of matter in terms of solids, liquids, gases. We have an intuitive grasp of the distinctions among these phases. Solids have a fixed size and shape; liquids take the shape of their container; and gases are like liquids, but their volume can be changed easily. Simple as these distinctions are, it was not until the early 20th century that a complete scientific understanding of the phases of matter around us emerged. Atoms have a regular, rigid arrangement in crystalline solids but are mobile in liquids and gases.

Yet these three phases do not begin to exhaust the possibilities. A solid is not just an array of atoms but also a swarm of electrons. Each atom offers up a few electrons, which roam across the entire crystal. When we connect a specimen to a battery, an electric current flows. Essentially all materials satisfy Ohm's law: the current is proportional to the voltage divided by the resistance. Electrical insulators such as Teflon have a high resistance; metals such as copper, a low resistance. Most remarkable are the superconductors, which have an immeasurably small resistance. In 1911 Heike Kamerlingh Onnes discovered them when he cooled solid mercury below –269 degrees Celsius. Today we know of superconductors that work at a comparatively balmy –138 degrees C.

Although it is not obvious just by looking at them, conductors, insulators and superconductors are different phases of matter. In each, the swarm of electrons takes on a different form. Over the past two decades physicists have discovered additional phases of electrons in solids. A particularly interesting example does not even have a proper name: physicists have defaulted into calling it the strange metal. It betrays itself by the unusual way its electrical resistance depends on temperature.

The differences among these phases arise from the collective behavior of electrons. Whereas the motion of atoms in solids, liquids and gases can be described by the classical principles of Newtonian mechanics, the behavior of electrons is ineluctably quantum. The key quantum principles governing the electrons are scaled-up versions of those governing the electrons in an atom. An electron orbits the nucleus, and its motion is described as a wave that propagates around the proton. The electron can reside in an infinite number of possible states with specific observable properties such as energy. Crucially, the electron not only orbits the nucleus but also spins around its own axis. This spin can be either clockwise or anticlockwise and cannot be slowed down or sped up; we conventionally label these two spin states as "up" and "down."

In atoms with more than one electron, the most important rule governing the electrons is the Pauli exclusion principle: no two electrons can occupy the same one-electron state. (This principle applies to all particles of matter, which physicists call fermions.) If you add electrons to an atom, each new electron settles into the lowest-energy state it can, like filling a glass with water from the bottom up.

The same reasoning also applies to the 10^{23} electrons in a piece of metal. The itinerant electrons, once detached from their original host atoms, occupy states that extend across the entire crystal. These states can be thought of as sinusoidal waves with a certain wavelength that is related to their energy. Electrons occupy states with the lowest-allowed energy that is consistent with the exclusion principle. Together they typically fill all the states up to an energy smaller than a threshold known as the Fermi energy.

Applying a voltage gives some electrons enough energy to transfer from an occupied state to a previously unoccupied one with an energy larger than the Fermi energy [*see box on next page*]. That electron can then flow freely. In an insulator, the density of electrons results in all accessible states being occupied already; even if we apply a voltage, there is no place for an electron to go, so no current can flow.

In superconductors, things get more complicated. The electrons in them cannot be understood one at a time. They bind into pairs, as described by the theory of superconductivity developed in 1957 by theoretical physicists John Bardeen, Leon Cooper and John Robert Schrieffer (also referred to as BCS). On the face of it, the particulate buddy system is odd because two electrons should repel each other. Vibrations of the crystal lattice, however, indirectly create an attractive force that overcomes the innate repulsion. Each pair behaves not as a fermion but as a different type of quantum particle known as a boson, which does not obey the Pauli exclusion principle. The electron pairs can all condense into the same state having the barest minimum amount of energy—a phenomenon known as Bose-Einstein condensation. It is like pouring water into a glass and, instead of filling up the glass, forming a thin layer of ice across the bottom that can absorb as much water as you care to add without getting thicker.

If you apply a voltage across such a material, that voltage pushes the electron pairs into a state with ever so slightly higher energy, yielding an electric current. This higher-energy state is otherwise empty, leaving nothing to impede the flow of the paired electrons. In this way, a superconductor transmits current with zero resistance.

GOING CRITICAL

THESE SUCCESSES of quantum theory in explaining metals, insulators, superconductors and other materials such as semiconductors (the basis of modern electronics) led many physicists in the early 1980s to conclude that they were approaching a full understanding of electrons in solids, with no remaining major discoveries to be made. This confidence was spoiled by the discovery of high-temperature superconductors.

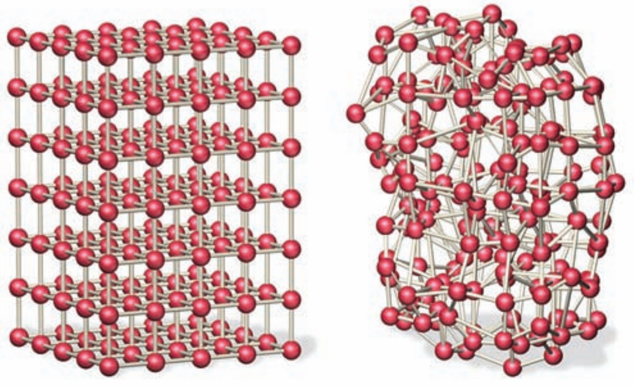
An example is barium iron arsenide, in which experimentalists have replaced some fraction of the arsenic with phosphorus. At low temperatures, this material is a superconductor, and physicists believe it obeys a theory similar to that of BCS except that the attractive force between electrons does not originate from vibrations of the crystal lattice but from physics associated with the electron spin. With a small amount of phosphorus, the material forms a state known as a spin–density wave [see "Charge and Spin-Density Waves," by Stuart Brown and George Grüner; SCIENTIFIC AMERICAN, April 1994]. On half of the iron sites, the electron spin is more likely to be up than

Just a Phase They're Going Through

Quantum physics gives the building blocks of matter radically new ways to organize themselves. The classical solid, liquid and gaseous phases involve different arrangements of atoms or molecules, governed by temperature. Quantum phases involve different arrangements of particles such as the electrons that perfuse materials. These phases are governed by features such as the electric field strength, which determines the forces that particles exert on one another and hence the way they assemble.

Classical Phases Solids have a fixed size and shape. In crystalline solids, molecules are arranged in a stable, regular, rigid lattice. In amorphous solids, molecules are jumbled, as in a liquid, but maintain their positions over long periods. Liquids have a fixed volume and variable shape. Their molecules are mobile but still bound together. Gases have a variable volume and shape. Their molecules are fully mobile and not bound to one another (or bound only very loosely). Beyond a certain temperature and pressure called the critical point, gas and liquid blend together and become indistinguishable from each other [*see phase diagram on opposite page*].

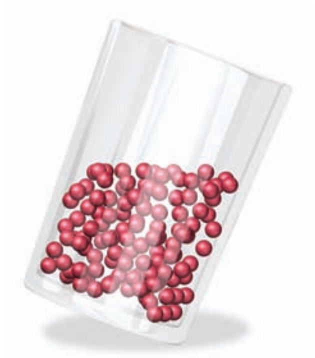
Solid



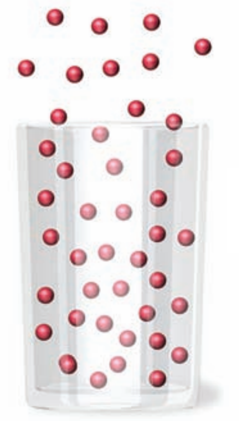
Crystalline

Amorphous

Liquid



Gas



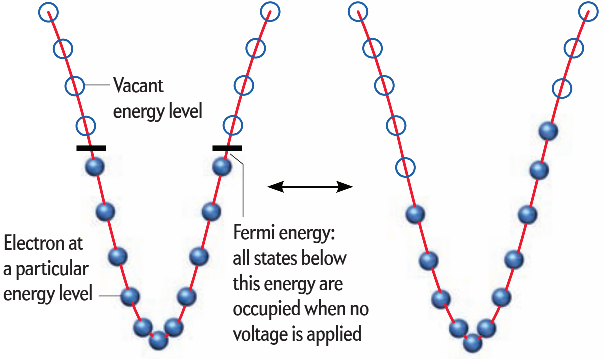
Quantum Phases Metals conduct electricity readily. Their electrons hopscotch from atom to atom, and if the atoms provide enough sites for the electrons to occupy, the electrons move freely, as if in a gas. Quantum effects limit the number of electrons that can have a given amount of energy.

Insulators scarcely conduct electricity at all. The atoms do not provide enough vacancies for the itinerant electrons, so electrons remain trapped in place, as if in a solid. They occupy all the available energy slots.

Superconductors are gases not of electrons but of pairs of electrons that behave as single particles. The electrons pair up under the effects of quantum spin or the influence of waves rippling through the atomic substrate. These pairs evade the quantum rules governing electrons. All of them can have the same amount of energy—lifting the restrictions that trap electrons in place and letting them flow without electrical resistance.

A spin-density wave (*not shown*) is a material (sometimes an insulator, sometimes a superconductor) with a peculiar pattern of electron spins. Half are spinning up, half down—say, in alternating rows. Sometimes a strange metal—a spin-density wave taken to extremes—forms [*see phase diagram on opposite page*]. The probability of each individual electron spinning up or down is 50–50, with no broader patterns. All the electrons in a strange metal are entangled and behave neither as individual particles nor even as pairs but as masses of trillions or more particles.

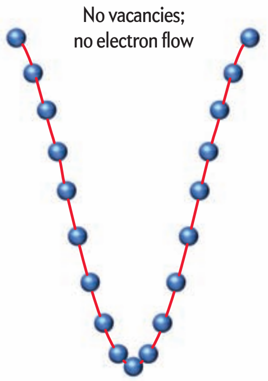
Conductor (metal)



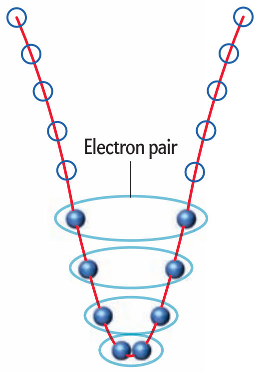
Nonconducting state

Conducting state

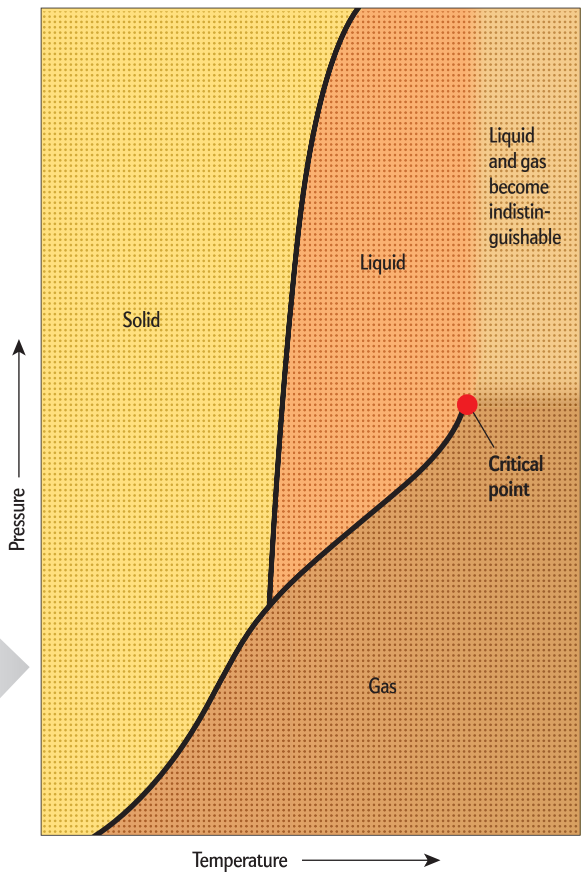
Insulator

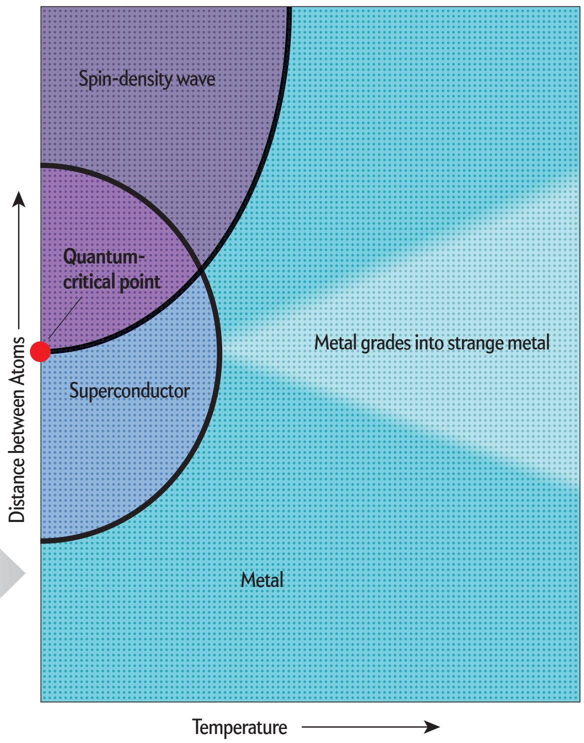


Superconductor



Illustrations by George Retseck





down, and vice versa on the other half. As you increase the amount of phosphorus, the strength of the spin-density wave diminishes. It disappears altogether when you have replaced a critical amount of arsenic, about 30 percent. At that point, the electron spin is equally likely to be up or down on each site, which has important consequences.

The first indication of the mysterious nature of this quantum critical state is the behavior of the system as experimentalists hold the amount of phosphorus fixed at 30 percent and raise the temperature. The result is neither a superconductor nor a spin-density wave but a strange metal.

The main new idea needed to describe the quantum-critical point, and the superconductors and strange metals close to it, is precisely the feature of quantum mechanics that so disturbed Einstein, Podolsky and Rosen: entanglement. Recall that entanglement is the superposition of two states—such as when one electron is spinning up and the other down, and vice versa. Imagine single electrons at two iron sites. Electrons are indistinguishable even in principle, so it is impossible to say which electron is up and which is down; both are equally likely to be up or down. All we can say is that if we measure one electron as up, the other is guaranteed to be down. They are perfectly anticorrelated: if we know one, we know the other.

At first glance, entanglement might not seem strange. Anticorrelation is common: if you have a pair of shoes and you put one in the front hallway and the other by the back door, then if you find a left shoe in one place, it is no mystery that the other shoe is right. Yet the quantum situation differs in an essential way. A shoe is either left or right even if you do not know which it is, but an electron has no fixed spin until the act of measurement. (If it did, we could tell by conducting a certain sequences of measurements on it.) In a sense, the electron is both up and down until forced to choose.

The mystery is how the electrons remain anticorrelated. When one electron chooses its spin, so does the other. How do they know to choose opposite directions? It seems that information on the quantum state of atom 1 is instantaneously known to atom 2, no matter how far away it is. Indeed, neither atom has a quantum state on its own; only the pair of them does. That is the nonlocality, the spooky action at a distance, that Einstein found so unpalatable.

Unpalatable or not, nonlocality has been verified numerous times in actual experiments. Einstein and his co-authors clearly put their finger on the most counterintuitive and unexpected aspect of quantum mechanics. And in the past decade physicists are beginning to appreciate that it accounts for the bizarre properties of strange metals. Close to the quantum-critical point, the electrons no longer behave independently or even in pairs but become entangled en masse. The same reasoning that EPR applied to two electrons now applies to all 10^{23} of them. Neighboring electrons are entangled with each other; this pair, in turn, is entangled with neighboring pairs, and so on, creating an enormous network of interconnections.

The same phenomenon occurs in other materials as well. Classifying and describing such entangled states is the forbidding challenge we face in developing the theory that describes the new materials. The network is so complex that it is beyond our ability to describe directly.

My colleagues and I used to worry that a theory of these quantum phases of matter would forever elude us. That was before we learned about string theory.

TANGLED UP IN STRINGS

ON THE FACE OF IT, string theory has nothing to do with entangled states of many electrons. It involves microscopic strings that vibrate like miniature guitar strings; the different modes of vibrations represent different elementary particles. The stringy nature of matter becomes evident at extremely high energies, found only moments after the big bang and near very dense black holes. In the mid-1990s string theorists such as Joseph Polchinski of the Kavli Institute for Theoretical Physics at the University of California, Santa Barbara, realized that their theory predicts more than strings. It also implies the existence of "branes": surfaces to which strings stick like bugs on flypaper. These membranes represent a vast kingdom of physics, beyond the high-energy particles the theory originally addressed.

What looks like a particle—a mere point—to us might actually be the end point of a string stretching from a brane through a higher spatial dimension. We can view the universe either as made up of point particles moving in a four-dimensional spacetime with a complex set of interparticle interactions or as made up of strings moving in a five-dimensional spacetime attached to branes. These two perspectives are equivalent, or dual, descriptions of the same situation. Remarkably, these two descriptions are complementary. When the point particles are hopelessly complex, the strings may behave simply. Conversely, when the particles are simple, the strings are clumsy and cumbersome.

For my purposes, the picture of strings dancing in some higher-dimensional spacetime is not important. It does not even matter to me whether string theory is a correct explanation of particle physics at very high energies. What is significant is that the duality lets me exchange a mathematically intractable problem for an easy one.

Until a few years ago, I mainly attended meetings of condensed-matter physicists in which we argued about the different entangled quantum states that electrons could form in newly discovered crystals. Now I find myself sipping coffee with string theorists, trying to make sense of their abstract and fanciful description of strings and branes and applying those ideas to down-to-earth issues raised by tabletop measurements on the new materials. Moreover, it is a two-way street. I think that our intuition and experimental experience with the quantum phases of electrons is helping string theorists to describe black holes and other exotica.

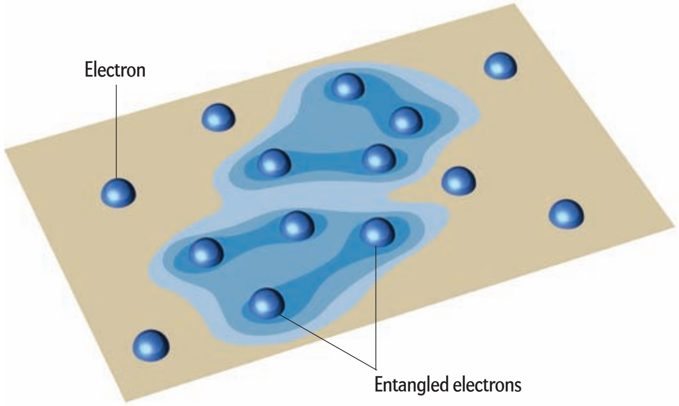
When electrons in crystals have only a limited degree of entanglement, they can still be thought of as particles (either the original electrons or pairs of them). When large numbers of electrons become strongly entangled with one another, however, they can no longer be viewed as particles, and conventional theory struggles to predict what happens. In our new approach, we describe these systems in terms of strings that propagate in an extra dimension of space.

My Harvard University colleague Brian Swingle has drawn an analogy between the extra spatial dimension and the network of quantum entanglement [*see box above*]. Moving up and down the network is mathematically just like moving through space. The strings can wriggle and fuse together within the extra dimension, and their motion mirrors the evolving entanglement of particles. In short, the spooky connections that troubled Einstein make sense when you think of the degree of entanglement as distance through an extra spatial dimension.

CONNECTION TO STRING THEORY

Web of Entanglement

For reasons that physicists have yet to understand, quantum phases of matter contain a latent spatial dimension that can appear at phase transitions like a figure in a pop-up book. This dimension becomes evident in the mathematical description of interparticle relations, or entanglement.



Electron Entanglement

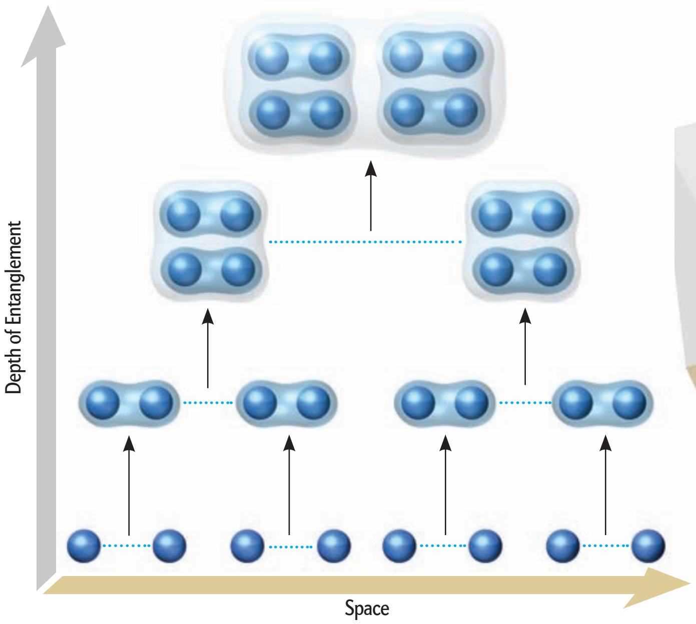
Entanglement means that multiple quantum particles act together as a single indivisible whole. Typically physicists talk about the entanglement of two particles or perhaps a handful of them, but in materials that transition from one quantum phase to another, huge numbers of electrons are entangled.

STRANGE COUSINS

THE PRACTICAL ADVANTAGE of these dualities is that string theorists have built up a large library of mathematical solutions to problems ranging from particle dynamics in the furnace of the big bang to the undulations of quantum fields on the lip of black holes. Those of us studying quantum phases of matter can go to the library, look up a possible solution to a specific problem and translate it (using the mathematics of duality) from the stringy situation to the entanglement situation.

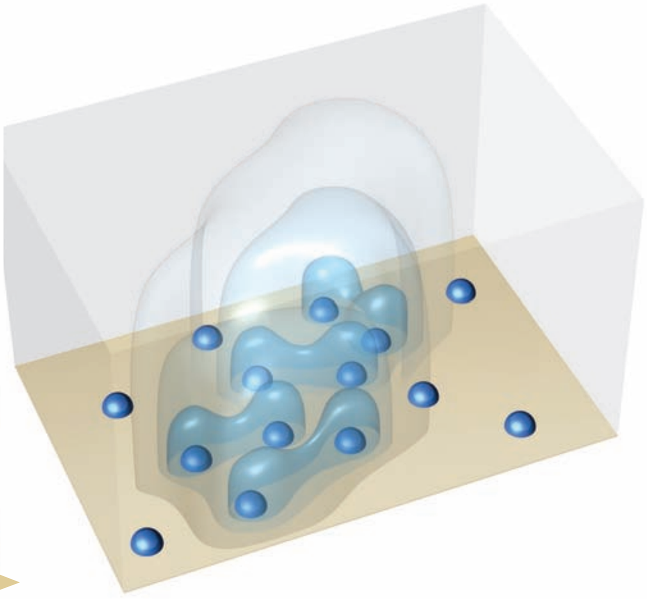
Typically we focus on the lowest-energy state at absolute zero, but we can readily describe matter at nonzero temperatures using a technique that might seem drastic: we imagine adding a black hole to the string situation. The fact that black holes are involved indicates how extraordinary these dualities are. No one is suggesting that the quantum phases of matter literally contain black holes; the connection is more subtle. Stephen Hawking of the University of Cambridge famously showed that every black hole has a certain temperature associated with it. From the outside, a black hole looks like a glowing hot coal. By the logic of duality, the corresponding condensed-matter system must also be hot, which has the effect of turning a spin-density wave or a superconductor into a strange metal.

These methods have made some progress in explaining



Hierarchy of Entanglement

Weirdly, the entanglement process behaves, mathematically, just like spatial distance. Much as moving through space entails passing through intervening points, getting from the entanglement of two particles up to trillions requires the two to combine with two others, and the resulting four with four more, and so on.



Extra Spatial Dimension

Thus, the depth of entanglement itself acts as an implicit spatial dimension, above and beyond the three dimensions where the electrons reside. By using this mathematical resemblance, theorists studying quantum phases of matter can draw on the findings of string theorists, who study extra spatial dimensions.

strange metals and other states of matter, but they have helped the most with the transition from a superfluid to an insulator. A superfluid is just like a superconductor, except it is made up of electrically neutral atoms; it reveals itself not by having zero electrical resistance but by flowing without any friction. In recent years experimentalists have developed remarkable new methods of creating artificial superfluids. They create a lattice of crisscrossing lasers and pour in trillions of extremely cold atoms. The atoms initially behave like a superfluid: they move freely from one lattice site to another. As experimentalists dial up the intensity of the laser, the atoms become less mobile and the superfluid abruptly turns into an insulator.

Experimentalists follow this transition by measuring how the atoms flow under external pressure. In the superfluid phase, they flow without resistance; in the insulator phase, they hardly flow at all; and at the transition, they flow but in a peculiar way. For instance, if experimentalists remove the external disturbance, the atoms come to a halt at a rate that depends on the temperature and on Planck's constant, the fundamental parameter of quantum theory, which does not enter into the behavior of the other phases. We have explained this behavior by imagining the quantum-critical fluid as the dual, or stringy doppelgänger, of a black hole.

The duality has a downside. By its very nature, it transforms the complex to the simple. We do not always want to transform the problem, however: we also want to understand the complexity for what it is. The duality is a mathematical black box and leaves us somewhat in the dark about the details of the complicated entangled states or about how these states occur in actual materials. Explaining what is really going on is still in its infancy. For those of us accustomed to thinking about the dynamics of electrons in crystals, string theory has given a fresh new perspective on the dynamics of complex quantum states involving entanglement. For string theorists, it has piqued interest in the phases of quantum materials, phenomena far removed from the physics of the early universe or that occurring in high-energy particle accelerators. The strange confluence of these currents of thought has shown us the wonderful unity of nature.

MORE TO EXPLORE

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