Quantum criticality and the phase diagram of the cuprates

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Abstract

I discuss a proposed phase diagram of the cuprate superconductors as a function of temperature, carrier concentration, and a strong magnetic field perpendicular to the layers. I show how the phase diagram gives a unified interpretation of a number of recent experiments.
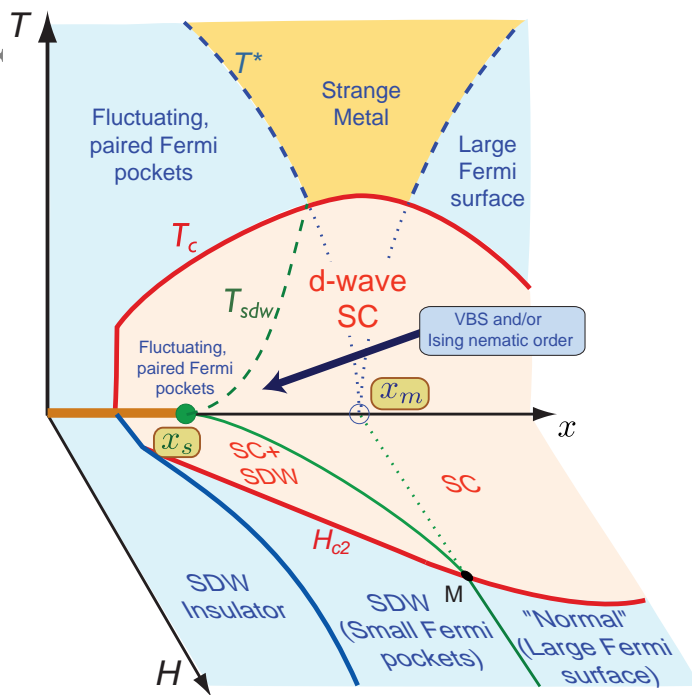
Figure 1: Proposed phase diagram of the cuprates showing the interplay between superconductivity (SC), spin density wave (SDW) order, and Fermi surface configuration as a function of carrier density (x), temperature (T), and magnetic field (H) perpendicular to the layers. Full lines are thermal or quantum phase transitions, dashed lines are crossovers, and dotted lines are guides to the eye. The phase transitions associated with valence bond solid (VBS) (or "charge") and nematic order are not shown. The superconducting regions are colored pink. We have assumed the absence of interlayer coupling, and so the SDW order is long-ranged only at $T = 0$: it is present in the regions labeled "SDW" and on the thick orange line for $x < x_s$. In the blue normal regions, the 'pseudogap' is between T_c and T^*, the 'Strange Metal' has an in-plane resistivity which is measured to be linear in T, and the "Large Fermi surface" has a conventional T^2 resistivity.

This brief note contains a summary of the key aspects of the proposed phase diagram of the cuprate superconductors shown in Fig. 1, and the central role played by ideas of quantum criticality. A more detailed discussion can be found in another recent review by the author [1], which also contains more complete citations to the literature. Here, I will focus on the central physical ideas and highlight support from recent experiments.

The phase transitions and crossovers in Fig. 1 appear quite intricate. However, they can be understood simply by focusing first on the quantum critical point (QCP) at doping density, $x = x_m$, temperature $T = 0$, magnetic field $H = 0$. As indicated in Fig. 1, this quantum critical point is pre-empted by the onset of superconductivity.

The QCP at $x = x_m$ is a transition between two metallic (hence the subscript m) Fermi liquid phases. At $x > x_m$ we have the full symmetry of the square lattice, and a "large" Fermi surface metal consisting of a hole-like Fermi surface enclosing the area $1 + x$ expected from the Luttinger theorem (this is for hole doping; with electron doping, x, the area enclosed is $1 - x$). At $x < x_m$ we have the onset of spin density wave (SDW) order, and this breaks apart the large Fermi surface into "small" Fermi pockets. Nevertheless the Luttinger theorem continues to be obeyed, after accounting for the large unit cell created by the SDW order. The ultimate theory of this quantum critical point is not fully understood, despite much theoretical attention [2].

Strong evidence for the QCP at $x = x_m$, and its associated $T > 0$ crossovers comes from recent experiments on Nd-LSCO [3, 4]. They detected the crossover between "Strange Metal" and "Fluctuating paired Fermi pockets" regions of Fig. 1. The latter region is our identification of the popular 'pseudogap' phase [5], and so the crossover temperature is T^*. The experiments identified T^* by deviations from linear resistivity in the in-plane resistance, or by an upturn in the c-axis resistivity, and showed these were correlated with signatures of changes in the

area enclosed by the Fermi surface. Further, they were able to track crossovers down to $T = 0$ by suppressing superconductivity by an applied magnetic field. This corresponds to locating the QCP along the green line beyond the point M in the $T = 0$ plane in Fig. 1.

Earlier evidence for the QCP at $x = x_m$ came indirectly from Panagopoulos and collaborators [6, 7], who used muon spin relaxation and ac-susceptibility measurements on a series of pure and Zn-substituted hole-doped cuprates to observe a glassy slowing down of spin fluctuations. This glassy behavior vanished above a critical doping which we identify as $x = x_m$.

Recent thermoelectric and Nernst effect experiments [8] and theory [9, 10] have also provided support for the Fermi surface transformations associated with the QCP at $x = x_m$. Associated measurements of the anisotropy in the Nernst co-efficient [11] have been proposed to be explained by the influence of nematic order in the Fermi surface [12]; this nematic order can be regarded as a remnant of a fluctuating SDW state, as is suggested by neutron scattering observations [13].

Finally, I also note the recent quantum oscillation observations [14] in the electron-doped cuprates, which show striking direct evidence for the sudden change in Fermi surface area at a large H. We identify this as a signature of the $T = 0$ green transition line beyond the point M at $x = x_m$ in Fig. 1.

Now, let us consider the onset of superconductivity at $H = 0$. This occurs in a dome-shaped region around $x = x_m$ [15]. Here a crucial effect is that the competition between the SC and SDW orders *shifts* the position of the SDW-ordering QCP to $x = x_s$ (the subscript s refers to the presence of superconductivity). Loosely speaking, the competition is for the Fermi surface: both the SDW and SC orders want to induce gaps in the same regions of the Fermi surface. We have presented a theory [15] which shows how such a competition leads to the shift in the position of the QCP.

Evidence for the QCP at $x = x_s$ appeared in early neutron scattering studies [16], and its signatures were addressed in initial theories for quantum criticality [17]. Also, in the electron-doped cuprates, the value of x_s has been quite precisely identified [18] — the high field quantum oscillation experiments we noted above [14] were done on the same material, and indeed found as $x_m > x_s$.

With the shift in the QCP from x_m to x_s, the crossovers as T is lowered for $x_s < x < x_m$ at $H = 0$ are quite complex, but simple to deduce from the topology of our phase diagram. As T is reduced below T^*, the electrons start to develop signatures of the onset of local SDW (and associated nematic) order. However, as T approaches T_c, the competition with SC halts the march towards to stronger SDW ordering. We sketch a crossover temperature T_{sdw} in Fig. 1, below which the electrons abandon SDW ordering, and the physics of the underlying large Femi surface can reappear *i.e.* the spectrum of the Bogoliubov quasiparticle excitations of the d-wave superconductor can lose signature of the Fermi pockets. Note that superconductivity competes mainly with SDW order, and will have a weaker suppression effect on the associated tendencies to VBS/nematic ordering [15]. Indeed, these orderings can survive at $T = 0$, as has been discussed in some toy models [19]. Low T evidence

for such ordering, and their connection to the pseudogap phase, has appeared in scanning tunnelling microscopy experiments [20, 21].

Now let us consider phase diagram at $T = 0$ in the x, H plane. The general structure of the phase transitions here appeared in early theoretic work [22], and indeed motivated the $T > 0$ portion of the phase diagram already discussed. A key prediction of this work was that the shift in the QCP from x_m to x_s implies the presence of a line of quantum phase transition within the SC phase which connects the point x_s to the point M in Fig. 1. This line marks the onset of long-range SDW order. A number of recent experiments [23, 24, 25] have presented strong evidence for this transition, in both LSCO and YBCO.

Moving to stronger fields, we loose superconductivity at $H = H_{c2}$ and cross into the normal state. The crucial, recent observation of high field quantum oscillations [26, 27, 28, 29, 30, 31, 32, 33] lead us to identify their small Fermi pockets with those of the normal phase region for $x < x_m$ in the x,H plane.

Also shown in the x,H plane of Fig. 1 is a metal-insulator transition to a low-doping SDW insulator. We believe this transition is associated with the localization of the small Fermi pockets, and is related to a number of experimental observations [34, 33].

Acknowledgements

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